Periodic seasonal Reg-ARFIMA-GARCH models for daily electricity spot prices

Abstract

Novel periodic extensions of dynamic long memory regression models with autoregressive conditional heteroskedastic errors are considered for the analysis of daily electricity spot prices. The parameters of the model with mean and variance specifications are estimated simultaneously by the method of approximate maximum likelihood. The methods are implemented for time series of $1,200$ to $4,400$ daily price observations. Apart from persistence, heteroskedasticity and extreme observations in prices, a novel empirical finding is the importance of day-of-the-week periodicity in the autocovariance function of electricity spot prices. In particular, daily log prices from the Nord Pool power exchange of Norway are modeled effectively by our framework, which is also extended with explanatory variables. For the daily log prices of three European emerging electricity markets (EEX in Germany, Powernext in France, APX in The Netherlands), which are less persistent, periodicity is also highly significant.

Keywords: Autoregressive fractionally integrated moving average model; Generalised autoregressive conditional heteroskedasticity model; Long memory process; Periodic autoregressive model; Volatility.

1 Introduction

Electricity supply has been the responsibility of public-private companies in many OECD countries until recently. It is anticipated that the private trading of electricity will intensify further in future and eventually move towards fully privatised electricity markets. In such markets large volumes of electricity power will be traded for the short and long term together with future contracts and options. Although similarities with financial markets exist with respect to its operations, the price formation at electricity markets is more complex since it strongly depends on the short-term characteristics of the energy supply function. The instantaneous nature of electricity and the availability of different plant technologies lead to atypical supply functions. On the other hand, electricity demand functions typically depend on weather variables, seasons in the year, day-of-week effects and holidays. These characteristics of electricity supply and demand functions determine the specific behaviour of electricity prices that is encountered in empirical work. The dynamic behaviour of prices is important for derivative pricing and real option analysis. Therefore, the empirical time series modeling of electricity prices is important for financial traders and investors.

Following the standard practice of modeling volatility in financial returns, we are interested in the conditional mean and variance of price innovations. For many efficient financial and commodity markets, log prices are assumed to behave as a random walk and price innovations are simply obtained by taking first differences of log prices. The mean process of electricity log prices can not simply be described by a random walk because of its specific characteristics, see Escribano, Peña, and Villaplana (2002) and Bunn and Karakatsani (2003) for reviews of the salient features of electricity prices. The following characteristics are often considered: (i) *Seasonality* in prices is due to the strong dependence of electricity demand on weather conditions but also on social and economic activities leading to different holiday and seasonal effects; (ii) *Mean-reversion* in electricity prices exists since weather is a dominant factor and influences equilibrium prices through changes in demand; (iii) *Jumps and spikes* can be due to the difficulty in storing large quantities of electricity so that supply and demand shocks cannot easily be smoothed out; (iv) *Volatility clustering* is regarded as a typical feature in financial markets where heavy trading takes place on underlying assets.

The literature on modeling and analyzing electricity prices is growing quickly, see the collection of articles in Bunn (2004) where different linear and nonlinear time series techniques are adopted in empirical work. Particular contributions of interest in the literature are by Lucia and Schwartz (2002) and Knittel and Roberts (2005) who argue for a mean-reversion model with deterministic seasonal mean functions and apply it to daily prices from the Nord Pool electricity power exchange and to Californian hourly electricity prices, respectively. Escribano et al. (2002) focus on volatility aspects using generalized autoregressive conditional heteroskedasticity (GARCH) models with possibly a jump-diffusion intensity parameter for daily spot prices from different electricity markets. Knittel and Roberts (2005) also include GARCH and jump processes in their model specification for hourly electricity prices.

In this paper the importance of regression effects, periodicity, long memory and volatility in electricity prices is highlighted and a simultaneous model for these features is proposed. The parameters in this model are jointly estimated by the method of approximate maximum likelihood using daily electricity spot prices from different exchange markets. The importance of periodicity is acknowledged by Wilkinson and Winsen (2002) and Hernáez et al. (2004) who point out that the pattern of prices varies across day-types. We go further and look at periodicities for the deterministic yearly seasonal effect and for the day-of-week effects in the mean, variance and autocovariance structures. Therefore, the parameters associated with the dynamics in the model are different for different days of the week. Haldrup and Nielsen (2004) estimate nonlinear nonperiodic long memory models for hourly Nord Pool prices. Periodic seasonal long memory models have not been considered in electricity prices earlier. It will be shown that seasonal long memory is an important feature in daily electricity prices. Although volatility has been considered in most empirical studies on electricity prices, we argue that volatility is not only a function of past squared price innovations. Seasonal factors and other fixed effects in the variance equation are also important.

Equal attention is given to the modeling of the mean, variance and autocovariance functions of the daily time series. The mean process includes deterministic effects, explanatory and intervention variables. Most coefficients in the mean are allowed to vary with day of the week (periodic). The variance process depends on day-of-week levels and yearly and half-yearly cosine waves with deviations from these deterministic functions modeled by a GARCH process with a Student-t density. The autocovariances are determined by seasonal long memory dynamics and lagged dependent variables with periodic coefficients. The different sets of parameters are treated simultaneously during the estimation process based on approximate maximum likelihood. The likelihood function is constructed as follows. The time series is corrected for the mean and the autocovariance features using the appropriate recursive filter for which the initial observations are treated as fixed and known. The resulting innovations are used as input for the conditional GARCH likelihood function for a t-distribution. The empirical study focuses on Nord Pool daily electricity spot prices between 4 January 1993 and 10 April 2005, that is more than 12 years of data (640 weeks) and equals 4,480 daily observations. To show the robustness of the new modeling framework for daily electricity spot prices, we extend the model with explanatory variables capturing significant and interpretable demand and supply effects in the Nord Pool market. We also present and discuss the results for three emerging electricity markets in Europe.

The paper is organised as follows. Section 2 describes the markets and data sets, and provides the motivation for our modeling approach. Section 3 discusses the specification of the model and develops a simultaneous approach for estimation and inference regarding the parameters of the model. Section 4 reviews the empirical results for the daily prices of the Nord Pool market with and without nondeterministic explanatory variables. Section 5 shows that the same modeling approach can be successfully applied to emerging electricity markets in mainland Europe. Section 6 concludes.

2 European electricity markets and daily spot prices

2.1 Some facts about electricity markets

First, we examine the time series of daily spot electricity prices from the Nord Pool exchange market in Norway. Subsequently, we analyse price data from three other European emerging electricity markets: European Energy Exchange (EEX) in Germany, Powernext in France and the Amsterdam Power Exchange (APX) in The Netherlands. These markets have started in different years and therefore the four daily time series are of different length. The oldest market is Nord Pool that started in 1991 for the trading of hydro electricity power generated in Norway. In 1996 Sweden, in 1998 Finland and in 1999 Denmark joined. In this paper we only consider the Norwegian electricity prices. Most of this electricity is generated in hydro electric power stations and therefore supply depends heavily on weather conditions. The average production capability of Norway's hydro power plants is about 113 Terawatt hours (TWh=10^9 KWh) per year. However, this production depends on precipitation levels. The EEX market is the largest electricity market in mainland Europe and the volume traded was 60 TWh in 2004. Powernext in France started in November 2001 and the volume traded in 2004 reached 14.1 TWh, 3% of France's electricity consumption. The spot market APX has been operational since May 1999 and in 2004 a total of 13.4 TWh was traded on this market. All four markets operate as "day-ahead" markets that concentrate on daily trade for electricity delivered on the next day. Daily series are constructed as the average of 24 price series for the different hours of the day. The resulting prices are referred to as spot prices.

2.2 Time series descriptives of Nord Pool electricity spot prices

We consider spot prices from the Nord Pool electricity market in the period January 4, 1993 until April 10, 2005. Figure 1 plots the daily spot prices, denoted by P_t and computed as the average of the 24 hourly prices, together with the daily first differences of $p_t = \log P_t$. The spot prices vary over the years and are subject to yearly cycles, weekly patterns, persistent level changes and spikes. The first differences of log prices (returns) show clear patterns of volatility clustering. It is tempting to conclude from these graphs that electricity spot prices exhibit the typical features of daily prices from other financial markets. However, upon closer inspection, there is clear evidence that the dynamic properties of electricity spot prices are more intricate.

The upper panel of Table 1 presents summary statistics for the first differences of log-prices for all data points (All) and for data points associated with a particular day of the week (Mon, Tue, ...). Notice that for Monday, the first difference is with respect to the Sunday spot price and Tuesday's first difference is with respect to the Monday price, etcetera. The reported periodic autocorrelations are computed as described in McLeod (1994). For example, the third column shows $r_{\text{Mon}}(7) = \text{corr}(\Delta p_t, \Delta p_{t-7}) = 0.26$. The large day-to-day differences in such autocorrelations motivate a periodic time series modeling approach. Further, the persistence of the periodic autocorrelations at the seasonal lags $7, 14, 21, 28$ is pronounced and needs to be modeled explicitly. The inclusion of long autoregressive polynomials in the model may capture these dynamics. A parsimonious alternative is to model the persistence by a seasonal fractional integration process. These findings of periodicity pertain both to the deterministic part and the dynamic part as we will illustrate in the following sections. Due to space considerations, other statistics are not presented here but the autocorrelations remain periodic when nonstationarities due to other day-of-the-week effects and yearly weather cycles have been removed from the data by regression or by seasonally differencing. This is also evident from model estimates presented below.

2.3 Explanatory variables for Nord Pool prices

Although univariate time series modeling of electricity prices is important in its own right, it is interesting to extend the analysis using publicly available data on the determinants of power demand and supply. The two most relevant and closely watched variables for the hydropower market of Nord Pool are daily data on Norwegian power consumption and weekly measurements of the overall water reservoir levels in Norway. The first graph of Figure 2 shows a time series plot of the water reservoir levels as a percentage of total Norwegian capacity for 1993-2005. The second graph of Figure 2 presents daily aggregate power consumption data, which are only available for 2001-2005. Both series are dominated by yearly cycles. In addition, the reservoir levels seem to exhibit long memory and the power consumption clearly shows a varying weekly pattern. These features might explain some of the dynamic characteristics of electricity prices in a meaningful way. We expect a negative effect on prices of (unexpected) positive shocks in water levels and a positive effect of (unexpected) positive shocks in consumption. In this paper we do not attempt to model power consumption and water reservoir levels.

2.4 Time series descriptives of other electricity spot prices

Time series of the logarithms of spot prices (in Euros/MWh) from three European emerging electricity markets are presented in Figure 3. The time series from EEX, Powernext and APX have different lengths and are shorter than the time series of Nord Pool. Further the dynamic properties of the three time series are different from the behaviour of Nord Pool. This is most probably due to the type of electricity traded on the different markets. Most of the electricity traded on the Nord Pool market is produced by hydro power generation and therefore it depends on long run weather conditions. In the APX, most of the electricity traded is thermal (via the burning of coal and gas) while the EEX and Powernext markets trade electricity produced mainly by nuclear power plants. These new markets are also strongly linked to each other by high voltage power lines. This leads to clear comovements in prices, unrelated to the price swings in the Nord Pool market. However, we do not model the three series simultaneously.

Table 2 contains periodic descriptive statistics for the EEX, Powernext and APX markets. As these prices are less persistent than the Nord Pool prices, we present descriptive statistics for log prices, rather than for returns. Table 2 shows that the mean and variance of these daily electricity prices depend on the day of the week and so does the autocorrelation structure. There are several similarities across the markets. For example, the mean is larger on Tuesdays and smaller on Sundays while the correlation between Wednesdays and Tuesdays (0.78, 0.88 and 0.84 for EEX, Powernext and APX, respectively) is higher than the correlation between Mondays and preceding Sundays (0.64, 0.60 and 0.46) as one would expect. Turning to periodic long memory characteristics, the 4-week autocorrelations for Mondays (0.22, 0.22 and 0.06) are considerably lower than for Saturdays (0.41, 0.44, 0.40).

3 The periodic seasonal Reg-ARFIMA-GARCH model

3.1 Model specification

Consider a time series of electricity prices y_t for $t = 1, \dots, T$ and with s periods or seasons. Period j is a modulus function of the time index t, that is $j = j(t) = 1 + t \bmod s$. In the empirical section, we analyse daily electricity spot prices where the seasonal variation is mainly due to weekly patterns and therefore the seasonal length is $s = 7$. Seasonal variations due to monthly and quarterly patterns can be captured by specific covariates in the model. We first describe our model before relating it to the existing literature.

An effective model for capturing the salient features of electricity price series as discussed in the previous section is the regression model with seasonal periodic autoregressive fractionally integrated moving average (ARFIMA) disturbances, that is

$$\Phi_j(L^s)(1 - L^s)^{D_j}(y_t - \mu_t) = \Theta_j(L^s)\varepsilon_t, \qquad \varepsilon_t|\mathcal{F}_{t-1} \sim NID(0, \sigma_t^2), \quad t = 1, \dots, T, \qquad (1)$$

with $j = j(t)$ and where $\mu_t = E(y_t|\mathcal{F}_{t-1})$ and $\sigma_t^2 = Var(y_t|\mathcal{F}_{t-1})$ are, respectively, the conditional mean and variance functions for an appropriate filtration $\mathcal{F}_t$. The periodic polynomials $\Phi_j(L^s)$ and $\Theta_j(L^s)$ are in the lag operator L that is defined by $L^k y_t = y_{t-k}$. These polynomials do not play a role in the empirical analysis of this paper and therefore we will not consider these lag polynomials further and have $\Phi_j(L^s) = \Theta_j(L^s) = 1$. The scalar periodic coefficient D_j determines the order of seasonal fractional integration for which the stationarity and invertibility conditions apply, that is $|D_j| < 0.5$ for period $j = 1, \ldots, s$. Using a binomial expansion we formally define

$$(1 - L^s)^{D_j} = \sum_{i=1}^{\infty} \frac{\Gamma(i - D_j)}{\Gamma(-D_j)\Gamma(i+1)} L^{is}, \qquad (2)$$

where $\Gamma(\cdot)$ is the gamma function. It follows from (2) that fractional integration implies an infinite order lag polynomial.

The conditional mean function is given by

$$\mu_t = \phi_{1j} y_{t-1} + \ldots + \phi_{pj} y_{t-p} + \sum_{k=1}^{K} \left(\delta_{k0j} x_{kt} + \delta_{k1j} x_{k,t-1} + \ldots + \delta_{krj} x_{k,t-r} \right), \qquad (3)$$

for $t = \max(p, r)+1, \ldots, T$, where ϕ_{ij} for $i = 1, \ldots, p$ and δ_{kij} for $k = 1, \ldots, K$ and $i = 0, 1, \ldots, r$ are periodic regression coefficients for periods $j = 1, \ldots, s$ when y_t or x_{kt} are observed in period $j = j(t)$. The set of coefficients ϕ_{ij} implies a periodic autoregressive polynomial for y_t that is assumed causal. In practice, we take $p < s$ and $r < s$ so that seasonal lags do not play a role in the conditional mean equation. Seasonal lags only play a role in the ARFIMA specification. The covariates x_{kt} can either be assumed deterministic or weakly exogenous, e.g. $E(x_{k,t-i}\varepsilon_t) = 0$ for $k = 1, \ldots, K$ and $i = 0, 1, \ldots, r$. The mean function μ_t is referred to as conditional since it is only properly defined when past values of y_t and concurrent and past x_{kt}'s are treated as known. Therefore, μ_t is only properly defined for $t = \max(p, r) + 1, \ldots, T$.

The conditional time-varying variance process for $\sigma_t^2 = Var(y_t|\mathcal{F}_{t-1})$ is specified by the generalized autoregressive conditional heteroskedasticity (GARCH) model with regression effects and scaled by seasonal factors, that is

$$\sigma_t^2 = \exp(\lambda_j) h_t, \qquad h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 h_{t-1} + \sum_{k=1}^{K^*} \gamma_k z_{kt}, \qquad t = \max(p, r) + 2, \ldots, T, \quad (4)$$

with $j = j(t)$ and with unknown coefficients $\alpha_0, \alpha_1, \beta_1$ and $\gamma_1, \ldots, \gamma_{K^*}$. The seasonal factors $\lambda_1, \ldots, \lambda_s$ are also unknown coefficients but not all can be identified and therefore we restrict $\lambda_1 = 0$. Higher order lags for h_t and ε_t^2 in (4) can be considered as well, but do not play a role in our empirical analysis. The covariates z_{kt} are assumed deterministic for $k = 1, \ldots, K^*$ and typically consist of time-functions and dummy variables. The autoregressive process for h_t is initialized by its unconditional mean, which is replaced by the corresponding sample mean in the estimation process, discussed below. We assume and impose $0 \leq \alpha_1 + \beta_1 \leq 1$.

Initially, the distribution of ε_t in (1) is assumed Gaussian. Given the fact that time series of electricity spot prices are usually fat-tailed, we consider the Student-t distribution for the disturbances ε_t, that is

$$\varepsilon_t|\mathcal{F}_{t-1} \sim \mathrm{t}_\nu(0, \sigma_t^2), \qquad t = 1, \ldots, T, \tag{5}$$

with degrees of freedom ν, zero mean and time-varying variance σ_t^2. The disturbances $\varepsilon_1, \ldots, \varepsilon_T$ are assumed to be serially uncorrelated. In the case of t-disturbances for model (1) with μ_t and σ_t^2 given by (3) and (4), respectively, the time index t takes the values $t = \max(p, r) + 2, \ldots, T$.

The periodic seasonal regression ARFIMA model with seasonal heteroskedasticity and GARCH disturbances (the periodic seasonal Reg-ARFIMA-GARCH model) has a conditional mean equation with periodic coefficients. This suggests that each day of the week can be described by a different model. If all coefficients are periodic, including the ones of the conditional variance function, we can isolate the periods from each other and estimate separate time-invariant models for the different periods, see Tiao and Grupe (1980). However, we do not pursue this approach since our focus will be on more subtle periodic formulations in which only certain parameters are periodic. Before discussing estimation details, we shortly describe the main origins of our model in the existing literature.

Our model combines ideas from different strands of the statistical, geophysical and econometric literature. Periodic autoregressive models were first applied by Hannan (1955) and Jones and Brelsford (1967). Gladysev (1961) first analysed the periodic correlation function and multivariate representation, while Tiao and Grupe (1980) discussed the consequences for traditional autoregressive moving average (ARMA) modeling if the underlying process really follows a periodic ARMA model. Estimation methods and algorithms for periodic ARMA models are developed by Pagano (1978), Vecchia (1985) and Li and Hui (1988), amongst others. McLeod (1994) discussed the empirical identification of periodic AR models. Further developments on likelihood evaluation and asymptotic theory for different estimators of periodic ARMA models are discussed by Lund and Basawa (2000) and Basawa and Lund (2001).

The fractional differencing model introduced by Adenstedt (1974) has become a standard model for long memory behaviour. The generalisation towards the ARFIMA model (1) with $s = 1$, $\sigma_t^2 = \sigma^2$ and no periodic coefficients was introduced by Granger and Joyeux (1980) and Hosking (1981). Statistical properties and inference for ARFIMA and other long memory processes are extensively discussed in the monograph by Beran (1994b), in the overview article of Baillie (1996) and, more recently, in the edited volume of Robinson (2003). Carlin, Dempster, and Jonas (1985) provided an early analysis of ARFIMA models with seasonal fractional integration parameter D. As a final step Ooms and Franses (2001), let the seasonal fractional D be periodic.

A novelty in this paper is the introduction of a GARCH process for the variance of a periodic seasonal Reg-ARFIMA model. The GARCH model is developed by Engle (1982) and Bollerslev

(1986). The statistical properties of GARCH processes are well established, see, for example, Bollerslev, Engle, and Nelson (1994). Further, Bollerslev and Ghysels (1996) introduced a periodic version of the GARCH model, which is slightly different from ours. The inclusion of regression effects in the variance specification of a non-seasonal and non-periodic AR-GARCH model in the context of modeling electricity prices is considered by Byström (2005).

Our paper extends ARFIMA-GARCH-t models with seasonal and periodic features. Baillie, Chung, and Tieslau (1996) first applied nonperiodic ARFIMA-GARCH models to price indexes. Ling and Li (1997) derived conditions for asymptotic normality of the approximate (Gaussian) ML estimator in the ARFIMA-GARCH model.

3.2 Maximum likelihood estimation

The exact Gaussian loglikelihood function of the standard ARFIMA model (1) with $\mu_t = \mu$, $\sigma_t^2 = \sigma^2$ and $s = 1$ is given by

$$\log L(y;\psi) = -\frac{T}{2}\log 2\pi\sigma^2 - \frac{1}{2}\log|V_y| - \frac{1}{2\sigma^2}(y-\mu)'V_y^{-1}(y-\mu), \tag{6}$$

where the parameter vector ψ collects all unknown coefficients of the model (1) and

$$y = (y_1, \ldots, y_T)', \qquad \sigma^2 V_y = Var(y),$$

with the variances and autocovariances in V_y for an ARFIMA process computed by efficient methods such as the ones developed by Sowell (1992) and Doornik and Ooms (2003), who also discuss efficient methods for the computation of $\log L(y;\psi)$ for the ARFIMA model using Durbin-Levinson methods for the necessary Choleski decomposition of V_y. The generalisation towards an ARFIMA model with $s > 1$, periodic coefficients and seasonal lags, can in principle be implemented for the evaluation of the loglikelihood function, However, the computation of V_y is intricate and not practical for large T as no analytical expressions for V_y exist in the case of a periodic seasonal ARFIMA model.

The time-varying conditional mean function (3) for the model (1) does not lead to further complexities if x_{kt} for $k = 1, \ldots, K$ is completely deterministic. In the case any x_{kt} is correlated with ε_t, the likelihood approach requires a multivariate approach to appropriately deal with the autocovariance structure in V_y. This is however beyond the scope of this paper. Furthermore, when the time-varying conditional variance σ_t^2 is modeled as the GARCH process (4), other complexities for a full-information maximum likelihood approach arise. Therefore, we adopt the approximate likelihood approach of Beran (1994a) that effectively amounts to removing the log determinant term $\log|V_y|$ from the loglikelihood function (6) and truncating the infinite autoregressive representation (2). The resulting estimator belongs to a wider class of M-estimators for which it can be shown that the estimator converges almost surely to its true value at the rate of $\sqrt{T}$, see Beran (1994a) for more details. In the empirical study of the next section, the sample size is sufficiently large so that we can rely on these asymptotic results.

3.3 Approximate maximum likelihood estimation and inference

In the case of model (1) with μ_t given by (3) and $\sigma_t^2 = \sigma^2$, the estimation of ψ is carried out as follows. For realisations $y_1, \ldots, y_T$, with a given ψ and by truncating the infinite autoregressive polynomial (2), we compute $\varepsilon_t = \varepsilon_t(\psi)$ in a standard way as implied by the model (1) for $t = \max(p, r) + 1, \ldots, T$. Since the truncated autoregressive polynomial is long, the earlier disturbances ε_t for $t = \max(p, r) + 1, \max(p, r) + 2, \ldots$ are based on polynomials of varying and lower dimensions. Finite sample modifications like those of Haslett and Raftery (1989) are not implemented since T is large. However, the periodic nature of the coefficients is taken into account. The estimation of ψ is then based on

$$\widehat{\psi} = \arg\min_{\psi} S(\psi), \qquad S(\psi) = \sum_{t=\max(p,r)+1}^{T} \varepsilon_t^2, \tag{7}$$

with $\varepsilon_t = \varepsilon_t(\psi)$. This is an M-estimator discussed by Beran (1994a).

In the case that σ_t^2 is modeled by the GARCH specification (4), the disturbances $\varepsilon_t(\psi)$ are obtained in the same way and taken as input for the GARCH likelihood function that is given by

$$\ell(\psi) = -\frac{1}{2} \sum_{t=\max(p,r)+1}^{T} \left(\log 2\pi + \log \sigma_t^2 + \sigma_t^{-2} \, \varepsilon_t^2 \right), \tag{8}$$

where $\sigma_t^2 = \sigma_t^2(\psi)$ is given by (4) for $t = \max(p, r) + 1, \ldots, T$. Since the process of h_t in (4) is defined by a recursion, their values for $t = \max(p, r) + 2, \ldots, T$ can be computed conditionally on the initial value $h_{\max(p,r)+1}$ that is set equal to the estimated sample variance of $\varepsilon_t = \varepsilon_t(\psi)$ for $t = \max(p, r) + 1, \ldots, T$, as is common in the literature. Asymptotically the choice of initialisation is negligible, see Francq and Zakoïan (2004). It should be noted that σ_t^2 is computed recursively and also requires the input of $\varepsilon_t = \varepsilon_t(\psi)$. The estimation of ψ in this case is based on $\widehat{\psi} = \arg\max_{\psi} \ell(\psi)$.

Finally, we consider model (1) with disturbances ε_t modeled by the t-distribution with variance σ_t^2 and number of degrees of freedom ν for $t = \max(p, r) + 1, \ldots, T$. The GARCH likelihood function for t-disturbances is given by

$$\ell^*(\psi) = \{T - \max(p, r)\} \log c(\nu) - \frac{1}{2} \sum_{t=\max p,r+1}^{T} \left[\log d_t(\nu) + (\nu + 1) \log\{1 + d_t(\nu)^{-1} \, \varepsilon_t^2\} \right], \tag{9}$$

where

$$c(\nu) = \frac{\Gamma\left(\frac{\nu}{2} + \frac{1}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right)}, \qquad d_t(\nu) = (\nu - 2) \, \sigma_t^2, \qquad \nu > 2,$$

with $j = j(t)$ for $t = \max(p, r) + 1, \ldots, T$. The shape coefficient ν is also part of the parameter vector ψ and can be estimated.

The actual ML estimation of ψ amounts to the maximisation of $\ell(\psi)$ or $\ell^*(\psi)$ with respect to ψ using numerical optimisation methods such as the quasi-Newton method, see Fletcher (1987). We employed the `MaxBFGS()` routine in Ox, see Doornik (1999). Starting values for the optimisation can be obtained from the M-estimates for ϕ_{ij}, δ_{ij} and D_j based on (7) and on QML estimates for the GARCH-parameter based on (8). Standard errors of the estimates and Wald test statistics are obtained from numerical second order derivatives of (9).

We found interesting differences between the inefficient (Gaussian) M estimates and efficient ML estimates, although our samples are large. The strong persistence in the volatility, measured as $\widehat{\alpha}_1 + \widehat{\beta}_1$ being close to unity, has a profound influence on the estimation of the autoregressive parameters. As a consequence the efficiency of estimates and tests is increased. Boswijk and Klaassen (2004) discussed the empirical relevance of this efficiency gain for AR-GARCH-t models. Under the assumption that $E|\varepsilon_t|^4$ exists, Jensen and Rahbek (2004) show that the asymptotic behaviour of the QML estimator of the GARCH(1,1) parameters is continuous around $\alpha_1 + \beta_1 = 1$. The estimator is also asymptotically normal if $\alpha_1 + \beta_1 > 1$. Francq and Zakoïan (2004) derive asymptotic normality of QML estimators of stable ARMA-GARCH(p,q) models under weak conditions. In our case, the innovations of electricity prices are fat tailed and therefore we cannot use the QML estimator and the inference is directly based on the Student-t likelihood. This approach is also applicable in stable GARCH models when $E|\varepsilon_t|^4$ does not exist, and which is more efficient, see Berkes and Horvath (2004, example 2.4).

4 Empirical results for Nord Pool

In this section we present empirical results for the daily Nord Pool data, for which data characteristics are summarised in section 2.2. It is hinted that a seasonal periodic heteroskedastic long memory model may be adequate to capture the dynamics in the conditional mean of the series. The Nord Pool series is sufficiently long which makes a parametric long memory analysis using the approximate maximum likelihood method feasible.

4.1 Time Series Model for Nord Pool prices

The time series model (1)-(5) using deterministic functions of time for the x_{kt} and z_{kt} variables is estimated using the method described in sections 3.2-3.3 with $p = 3$. A yearly cycle is part of the x_{kt}s as in Lucia and Schwartz (2002). This cycle captures the smooth seasonal swings in the supply and demand functions of electricity. The prices are subject to significant holiday effects in demand that lead to low returns on holidays and high returns thereafter. The AR parameters ϕ_{ij} for holidays differ from normal weekend days. The degrees of freedom are not sufficient to estimate ϕ_{ij} separately for each type of holiday. Instead, dummy variables for each type of holiday are included and their effects on prices are measured, contemporaneously and for a maximum of p lags. For a holiday that takes place on the same day-of-the-week each year (e.g. Ascension day), we have $p+1$ parameters in the model to measure its effect. For other type of holidays we have $(p+1)\times s$ parameters in the model as these holiday effects may depend on the day of the week. Finally, the conditional variance is explained by both a yearly and a half-yearly cycle in z_{kt} following Byström (2005).

The estimation results are presented in Table 3. To economize on the estimation output we omit standard errors. Instead, we present Wald test statistics for the nullity and for the nonperiodicity of sets of parameters. The seasonal integration parameters D_j are largest for Monday and Saturday, as expected from the autocorrelations presented in Table 1. These estimated parameters are significant, clearly periodic and smaller than 0.5. The AR parameters ϕ_{ij} are also clearly periodic. The third order lag is particularly important for the Monday. For Thursday and Friday the AR polynomial of the model reduces to the difference operator as is usual in models for returns in stock markets. The periodic AR polynomial of the model however is stable. The largest inverse root of the characteristic polynomial equals 0.95, see Boswijk and Franses (1996) for unit root tests in periodic AR models.

The yearly cycle and the holiday effects measured by δ_{kij} are significant. For example, the electricity price on Ascension Day is approximately 18% lower than on a normal Thursday. This effect is based on 12 Ascension Day observations. The periodic effect of a holiday with a fixed calendar date (e.g. May 1) is more difficult to measure since it varies with the day of the week and its estimate is sometimes based on only one or two observations. As far as the volatility equation is concerned, significant periodicity is found for the log-variance scalar λ_j. Monday and Saturday are more volatile than other days. Furthermore, the χ^2 tests show that significant yearly and half-yearly cycles in the volatility are detected in our analysis. The estimates of the GARCH paramaters α_1 and β_1 are on the boundary of the admissable parameter space. As a result we have high persistence in the conditional variance, a typical finding in many financial applications. The t distribution of the errors is fat-tailed with estimated degrees of freedom $\widehat{\nu} = 3.98$.

The last rows of Table 3 and Figure 4 present diagnostics for the standardised residuals $\widehat{\eta}_t = \widehat{\varepsilon}_t/\widehat{\sigma}_t$ that are normalised by the transformation $\widehat{\eta}_t^* = F_G^{-1}\left[F_{\mathrm{t}}(\widehat{\eta}_t)\right]$, for $t = 1,\ldots,n$, where $F_G()$ and $F_{\mathrm{t}}()$ are the cumulative density functions of the standard normal and the t distributions, respectively. Since the standard diagnostic statistics and graphs are designed for residuals that are assumed normal and since the model disturbances are assumed to come from a t density, this transformation for the residuals is justified. The Ljung-Box Q() test statistics for serial correlation in the normalised scaled residuals do not exhibit significant serial correlation while evidence of erratic behaviour in $\widehat{\eta}_t^*$ is limited and evidence of non-normal behaviour of $\widehat{\eta}_t^*$ is not apparent. The high number of zeros in the empirical histogram stems from holiday variables that occur once and put associating residuals to zero. Strong evidence of serial correlation in $\widehat{\eta}_t^{*2}$ is not present, especially in the short run.

4.2 Explanatory variables for Nord Pool prices

Prediction errors of the pure time series model can be taken as stemming from changes in the electricity supply and demand functions. To verify this proposition, we extend the analysis for Nord Pool prices by adding explanatory variables to the time series model. First, we consider weekly data on Monday's water reservoir levels, both in demeaned levels and in demeaned weekly differences, as a proxy for supply. Levels and first differences are less correlated than levels and lagged levels and associating test statistics are easier to interpret. Further, the coefficients of first differences measure short run effects while those of levels capture long run effects, see Johnston and Dinardo (1997, Ch. 8). Table 4 presents selected estimation results from which we learn that, ceteris paribus, a change in the water levels has a significant negative effect on electricity prices, except on Mondays when the measurements for the new week are not yet publicly available. The parameter estimates of the pure time series model are not much affected by the introduction of the water levels. The noticeable exception is the effect of the deterministic yearly cycle in the conditional mean of prices, that is largely replaced by the effect of the yearly cycle in changing water levels.

Second, we consider levels, daily differences and lagged daily differences of demeaned log power consumption, that we take as a proxy for electricity demand. Periodic sample means of this variable are reported in Table 1. Since power consumption is only available for a short period, a separate analysis is carried out for the model with both water supply and power consumption as explanatory variables. The estimation results are presented in Table 5. The effect of water level changes on prices remains significantly negative for all days of the week except Monday. Positive changes in log consumption have very significant positive effects on electricity prices. Long run effects of water levels and consumption are not significant in this basic model that does not allow for feedbacks from prices to consumption. The fractional integration and AR parameters remain jointly significant and periodic.

These empirical results for the Nord Pool case show that the Reg-ARFIMA-GARCH model describes the conditional mean and variance of the price process successfully. The estimation results are easy to interpret and make economic sense. However, a more extensive analysis should take account of the facts that aggregate demand and supply functions are nonlinear and vary with the hour of each day. We should further note that historical data on aggregate supply curves for the Nord Pool are not publicly available.

5 Empirical results for other European markets

To investigate the robustness of the periodic seasonal Reg-ARFIMA-GARCH model, we repeat the analysis for daily electricity spot prices from three younger mainland European markets: the EEX in Germany, Powernext in France and the APX in the Netherlands. The countries have six holidays in common. We take account of 12 French, 10 German and 8 Dutch holidays. Table 6 reports selected parameter estimates for the models of these three new markets. The estimates of the holiday effects are not reported but are significant despite the fact that they are based on a relative small number of observations. For the APX and Powernext, extra intervention dummies are introduced for the week of July 11-17, 2003 (in this week, a lack of cooling water in rivers threatened the nuclear power production and prices were more than $e^{2.1} = 8.2$ times higher than expected on July 11).

The estimated dynamic parameters and disturbance variances vary from day to day in all three markets. For example, Mondays produce particularly low AR(1) coefficients. The periodic patterns of the dynamic parameters vary significantly from market to market. Monday's AR(3) coefficients for Powernext deviate from the EEX and APX market. Sunday's AR(3) coefficients for the APX differ from the EEX and Powernext. Powernext and APX show long memory behaviour for Saturdays, in concordance with the autocorrelations reported in Table 2, whereas the EEX estimates indicate fractional integration for Sundays. Apparently, slowly evolving changes in the weekly seasonal patterns occurred in these markets in the first years of their existence. The AR parts of models for these new market models show stronger mean reversion than in the Nord Pool. The largest inverse roots of the characteristic polynomial of the AR component are 0.47, 0.36 and 0.20 for EEX, Powernext and APX, respectively. Finally, the volatility persistence as measured by $\alpha_1 + \beta_1$ is lower than unity for the EEX but the volatilities for Powernext and APX are persistent.

Figure 5 presents graphs of the normalised scaled residuals $\widehat{\eta}_t^*$ together with the correlograms of $\widehat{\eta}_t^*$ and $\widehat{\eta}_t^{*2}$ and the histogram of $\widehat{\eta}_t^*$ for the three markets. The last row of Table 6 presents the Box-Ljung $Q()$ statistics for $\widehat{\eta}_t^*$ and $\widehat{\eta}_t^{*2}$. The residual diagnostics are satisfactory but not perfect. Long run autocorrelations are still present for the EEX and Powernext markets. The normalised APX residuals are skewed to the left and long run autocorrelations in the squared residuals are apparent. The basic model specification can be improved by taking account of market specific features.

6 Conclusions

This paper has presented an empirical analysis of daily spot prices for four European electricity markets using periodic seasonal Reg-ARFIMA-GARCH models to explain the dynamics in the conditional mean and variance of log prices. The day-of-the-week periodic autocovariances for short run dynamics are modeled by lagged dependent variables and for long run dynamics by seasonal ARFIMA models. Regressors capture yearly cycles, holiday effects and possible interventions in mean and variance. The GARCH-t component takes account of volatility clustering and extreme observations. The model parameters are estimated simultaneously by approximate maximum likelihood methods. Given the persistent changes in volatility, simultaneous estimation of mean and variance parameters is preferred above two-step methods. Residual diagnostics show a good fit of the model. The resulting time series models allow for dynamic point forecasting and stochastic simulation. The Nord Pool market trades hydro power and it is shown that a significant part of the short term price movement can be explained by weekly water reservoir levels and daily electricity consumption. The inclusion of these explanatory variables in the model does not significantly change the estimated periodic heteroskedastic seasonal autocovariance structure in Nord Pool prices. The basic modeling framework is successful for Nord Pool prices while it can be somewhat improved for prices from other European markets.

Suggestions for future extensions are more flexible distributions for the error term, smoothly time-varying (periodic) parameters and a more extensive specification of the conditional variance equation. More parsimonious periodic AR components can be estimated and tested. The model can also be used for prices at a particular hour of the day. Finally, the strong interrelationships between prices and consumption may lead to multivariate modeling approaches. The empirical findings in this paper may have important consequences for the modeling and forecasting of mean and variance functions of spot prices for electricity and associated contingent assets.

Table 1: Descriptive statistics daily log-prices Nord Pool and explanatory variables

	First difference of daily log-prices							
	All	Mon	Tue	Wed	Thu	Fri	Sat	Sun
T	4466	638	638	638	638	638	638	638
Mean	0.0002	0.1124	0.0105	-0.0018	-0.0082	-0.0176	-0.0667	-0.0273
S.D.	0.102	0.127	0.095	0.079	0.067	0.073	0.100	0.057
$r(1)$	-0.03	-0.26	-0.22	0.17	-0.06	-0.15	-0.03	0.15
$r(2)$	-0.19	-0.54	-0.08	-0.14	0.04	0.03	0.13	0.14
$r(7)$	0.39	0.26	0.05	-0.08	-0.03	0.09	0.46	0.18
$r(14)$	0.34	0.21	-0.03	0.01	-0.06	0.05	0.31	0.07
$r(21)$	0.35	0.21	-0.03	0.01	-0.06	0.05	0.31	0.07
$r(28)$	0.33	0.15	0.07	-0.04	-0.07	0.04	0.25	0.05
Reservoir levels		**Daily Log power consumption**						
	Weekly	Mon	Tue	Wed	Thu	Fri	Sat	Sun
T	638	222	222	222	222	222	222	222
Mean	62.225	12.6714	12.6756	12.6718	12.6771	12.6636	12.6046	12.5891
S.D.	20.657	0.19890	0.19391	0.19372	0.19669	0.19895	0.20269	0.20683

T: Sample size. Mean: Sample means. S.D.: Sample standard deviations. $r(\tau)$: Periodic autocorrelation of y_t for a lag of τ days. Prices in NOK per MWh. Reservoir levels as a percentage of total Norwegian capacity. Daily power consumption in MWh (around 300,000 MWh per day). Sample log prices and Reservoir levels: January 18, 1993- April 10, 2005. Sample log power consumption: February 26, 2001- April 10, 2005. Source: www.statnett.no.

Table 2: Descriptive statistics for daily log prices new European Power Markets

	Mon	Tue	Wed	Thu	Fri	Sat	Sun
EEX							
Mean	3.362	3.428	3.412	3.390	3.328	3.084	2.821
S.D.	0.355	0.374	0.330	0.381	0.296	0.271	0.294
$r(1)$	0.64	0.62	0.78	0.74	0.77	0.73	0.87
$r(2)$	0.55	0.42	0.60	0.61	0.73	0.62	0.67
$r(7)$	0.37	0.23	0.44	0.29	0.45	0.46	0.71
$r(14)$	0.23	-0.13	0.19	0.16	0.25	0.42	0.63
$r(21)$	0.26	0.14	0.25	0.21	0.31	0.48	0.62
$r(28)$	0.22	0.25	0.28	0.20	0.29	0.41	0.59
Powernext							
Mean	3.308	3.402	3.390	3.358	3.309	3.066	2.806
S.D.	0.411	0.333	0.359	0.343	0.314	0.305	0.382
$r(1)$	0.60	0.67	0.88	0.76	0.85	0.84	0.80
$r(2)$	0.65	0.58	0.64	0.72	0.70	0.71	0.69
$r(7)$	0.32	0.41	0.43	0.44	0.48	0.58	0.55
$r(14)$	0.22	0.18	0.17	0.23	0.27	0.42	0.44
$r(21)$	0.27	0.34	0.31	0.34	0.38	0.44	0.46
$r(28)$	0.22	0.40	0.33	0.34	0.34	0.44	0.50
APX							
Mean	3.572	3.607	3.583	3.573	3.483	3.219	2.932
S.D.	0.506	0.504	0.481	0.438	0.368	0.293	0.325
$r(1)$	0.46	0.79	0.84	0.72	0.74	0.41	0.58
$r(2)$	0.39	0.33	0.74	0.64	0.62	0.41	0.34
$r(7)$	0.28	0.26	0.21	0.33	0.39	0.48	0.37
$r(14)$	0.13	0.06	0.06	0.22	0.20	0.43	0.32
$r(21)$	0.09	0.17	0.17	0.28	0.22	0.38	0.33
$r(28)$	0.06	0.14	0.19	0.32	0.22	0.40	0.28

$r(\tau)$: Periodic autocorrelation of y_t for a lag of τ days. Samples: EEX: 182 weeks, October 15/2002-April 10/2005, Powernext: 174 weeks, December 10/2002-April 10/2005, APX: 222 weeks, January 08/2001-April 10/2005.

Table 3: ML estimates daily log-prices NordPool Jan 18, '93-April 10, '05, Deterministic x_t.

Periodic parameters									m	χ^2_m	m	χ^2_m
	j	Mon,1	Tue, 2	Wed, 3	Thu, 4	Fri, 5	Sat, 6	Sun, 7		$\theta = 0$		$\theta = \iota\theta_0$
	D_j	0.146	0.005	-0.012	0.033	0.071	0.119	0.078	7	90.3	6	39.6
	$\phi_{1,j}$	0.682	1.011	1.150	1.097	1.035	1.181	1.097	7	4765	6	19.7
	$\phi_{2,j}$	-0.097	0.043	-0.178	-0.080	0.028	0.048	0.016	7	19.7	6	16.0
	$\phi_{3,j}$	0.381	-0.067	0.022	-0.016	-0.064	-0.214	-0.116	7	92.4	6	92.3
Constant	$\delta_{1,0,j}$	0.202	0.065	0.030	-0.010	-0.006	-0.102	0.008	7	42.6	6	39.5
CosYear	$\delta_{2,0,j}$	-0.175	-0.010	-0.042	0.002	0.026	0.115	0.083	7	50.0	6	49.6
SinYear	$\delta_{3,0,j}$	-0.014	-0.074	-0.066	-0.027	-0.035	-0.059	-0.038	7	44.4	6	6.8
Maundy	$\delta_{4,0,4}$				-0.025				1	14.7		
Good Fri	$\delta_{5,i,5}$					-0.033	0.062	-0.000	3	57.4		
Easter	$\delta_{6,i,1}$	-0.067	0.130	-0.013	0.002				4	173.5		
Ascension	$\delta_{7,i,4}$				-0.178	0.156	0.011	-0.008	4	188.5		
Pentecost	$\delta_{8,\ldots,1}$	-0.112	0.145	-0.010	0.012				4	107.8		
Dec 24	$\delta_{9,0,j}$	-0.169	-0.050	-0.096	-0.079	-0.016	-0.031	-0.004	7	82.3		
Dec 25	$\delta_{10,0,j}$	-0.135	-0.040	0.015	0.006	-0.010	0.054	-0.032	7	26.7		
Dec 26	$\delta_{11,0,j}$	-0.005	0.033	0.024	-0.014	0.039	0.064	0.011	28	138.4		
	$\delta_{11,1,j-1}$	-0.003	0.026	0.111	0.095	0.033	0.094	0.010				
	$\delta_{11,2,j-2}$	0.038	0.006	-0.009	-0.035	-0.012	0.007	-0.014				
	$\delta_{11,3,j-3}$	0.051	-0.014	0.001	0.019	0.051	0.021	-0.055				
Jan 1	$\delta_{12,0,j}$	-0.116	-0.016	-0.053	0.012	0.033	0.042	0.001	28	1502		
	$\delta_{12,1,j-1}$	0.110	0.165	0.608	0.125	0.055	0.058	-0.002				
	$\delta_{12,2,j-2}$	0.062	0.010	-0.056	-0.404	0.047	0.014	-0.011				
	$\delta_{12,3,j-3}$	-0.005	-0.026	0.043	0.004	-0.019	-0.015	-0.026				
May 1	$\delta_{13,0,j}$	-0.166	-0.144	-0.102	-0.049	-0.171	-0.124	-0.166	28	424.8		
	$\delta_{13,1,j-1}$	0.312	0.134	0.193	0.078	0.130	0.222	0.099				
	$\delta_{13,2,j-2}$	0.086	0.030	-0.014	-0.038	0.038	0.043	-0.105				
	$\delta_{13,3,j-3}$	0.192	0.016	0.096	-0.159	0.004	-0.030	-0.030				
May 17	$\delta_{14,0,j}$	-0.043	0.019	-0.197	-0.013	-0.071	-0.124	-0.217	28	425.7		
	$\delta_{14,1,j-1}$	0.101	0.055	0.362	0.224	-0.023	0.061	-0.064				
	$\delta_{14,2,j-2}$	0.016	0.072	-0.071	0.005	-0.050	-0.118	-0.008				
	$\delta_{14,3,j-3}$	0.025	-0.016	0.071	-0.048	-0.011	-0.173	-0.049				
logVar	λ_j	0	-0.371	-0.611	-0.742	-0.671	-0.018	-0.730	6	131.7		
Nonperiodic parameters:												
	CosY	SinY	CosHY	SinHY								
	γ_1	γ_2	γ_3	γ_4	$10^3\alpha_0$	α_1	β_1	ν				
	-0.240	0.089	0.145	.0079	.504	0.406	0.594	3.98				
χ^2_1	8.00	4.47	6.35	0.00	15.5		154					
LL:	7429.04	#pars:	205	T:	4466		AIC/T:	-3.2351	ρ_1:	0.951		
AC: $Q(28)$:	37.4	$Q(91)$:	86.3	in var:	$Q^*(28)$:	55.3	$Q^*(91)$:	151.5				

NOTES: Model (1)-(5), §3.1. $\delta_j(L)'x_t$: Periodic effects in conditional mean. Deterministic Regressors x_t: Constant term, Yearly cycle $\cdot 10^{-1}$ and 11 holidays with nonoverlapping daily lags. $\delta_{k,i,j}$: coefficients for lags $0, 1, 2$ or $0, 1, 2, 3$ of day-of-the-week j. $\delta_{k,i,j-i}$: coefficients for lag i of day-of-the-week $j-i$, where j follows modulo 7 arithmetic. λ_j: periodicity parameters conditional variances. $\gamma' z_t$: Nonperiodic effects conditional variance. Deterministic regressors z_t: Yearly cycle $\cdot 10^{-3}$ and HalfYearly cycle $\cdot 10^{-3}$. m: degrees of freedom of asymptotic χ^2_m Wald-test statistics, $\theta = 0$: test for nullity of parameter set. $\theta = \iota\theta_0$: test for equality of parameters in corresponding row. Asymptotic critical values at 95% and 99% for $m = 7$: 14.1 and 18.5, for $m = 6$: 12.6 and 16.8. α_1, β_1: GARCH parameters. ν: parameter t-distribution. LL: approximate log likelihood. T: number of observations. AIC: $(-2\cdot\text{LL} + 2\#\text{par})$. ρ_1: largest inverse root of characteristic polynomial of periodic AR part. AC: $Q()$: Ljung-Box statistics on normalised residuals. $Q^*()$: idem for squared normalised residuals.

Table 4: Effect Water Reservoir Levels Nord Pool Jan 18, '93-April 10, '05

Periodic parameters									m	χ^2_m	m	χ^2_m
	j	Mon,1	Tue, 2	Wed, 3	Thu, 4	Fri, 5	Sat, 6	Sun, 7		$\theta = 0$		$\theta = \iota\theta_0$
	D_j	0.154	0.011	-0.022	0.037	0.050	0.110	0.073	7	86.7	6	41.8
	$\phi_{1,j}$	0.687	1.021	1.144	1.079	1.011	1.151	1.095	7	4635	6	47
	$\phi_{2,j}$	-0.101	0.032	-0.157	-0.073	0.028	0.048	0.011	7	16.0	6	12.6
	$\phi_{3,j}$	0.364	-0.073	0.009	-0.012	-0.049	-0.193	-0.107	7	79.2	6	79.2
Constant	$\delta_{1,0,j}$	0.280	0.096	0.019	0.026	0.041	-0.059	-0.003	7	33.7	6	25.8
CosYear	$\delta_{2,0,j}$	-0.013	-0.003	-0.011	-0.006	-0.003	0.001	0.004	7	24.8	6	15.0
SinYear	$\delta_{3,0,j}$	-0.019	-0.015	-0.007	-0.012	-0.017	-0.018	-0.002	7	33.2	6	6.9
...	...											
Δ_7Water	$\delta_{15,\ldots,1}$	0.046	-0.171	-0.321	-0.328	-0.351	-0.564	-0.185	7	56.1	6	11.1
Water	$\delta_{16,\ldots,1}$	-0.077	-0.032	0.008	-0.034	-0.050	-0.038	0.012	7	15.3	6	9.3
logVar	λ_j	0	-0.374	-0.630	-0.759	-0.702	-0.037	-0.721	6	132.2		
Nonperiodic parameters:												
	CosY	SinY	CosHY	SinHY								
	γ_1	γ_2	γ_3	γ_4	$10^3\alpha_0$	α_1	β_1	ν				
	-0.274	0.086	0.137	0.016	0.484	0.436	0.564	3.946				
χ^2_1	7.77	4.35	7.01	0.14	15.10		187.8					
LL:	7466.57	#pars:	219	T:	4466		AIC/T:	-3.2457				
AC: $Q(28)$:	29.2	$Q(91)$:	79.9	in var:	$Q^*(28)$:	50.3	$Q^*(91)$:	144.3	ρ_1:	0.906		

NOTES: See also notes Table 3. Coefficients $\delta_{4,i,j}, \ldots, \delta_{14,i,j}$ of holiday effects not reported. Water: Water reservoir levels for Norway as a fraction of total capacity, Water: Water levels in Nord Pool area as a fraction of total capacity minus 0.6245: the sample mean over 1993-2004 as reported on Mondays for the Nord Pool area. reported for Mondays. Source: www.statnett.no. Δ_7Water: change with respect to previous week.

Table 5: Effect Reservoir Levels and Power Consumption Nord Pool, Feb 26, '01-Apr 10, '05.

Periodic parameters									m	χ^2_m	m	χ^2_m
	j	Mon,1	Tue, 2	Wed, 3	Thu, 4	Fri, 5	Sat, 6	Sun, 7		$\theta = 0$		$\theta = \iota\theta_0$
	D_j	0.125	0.053	-0.124	0.006	-0.118	0.062	0.052	7	25.9	6	25.4
	$\phi_{1,j}$	0.460	0.892	0.995	0.983	0.790	0.966	1.056	7	1153	6	52.2
	$\phi_{2,j}$	0.094	0.076	0.018	0.138	-0.003	0.149	-0.142	7	6.79	6	5.41
	$\phi_{3,j}$	0.398	0.003	-0.014	-0.136	0.198	-0.097	0.112	7	37.1	6	36.3
Constant	$\delta_{1,0,j}$	0.295	0.158	0.000	0.072	0.056	-0.131	-0.162	7	19.3	6	17.5
CosYear	$\delta_{2,0,j}$	-0.016	-0.014	-0.016	0.005	-0.034	-0.010	-0.027	7	44.4	6	15.0
SinYear	$\delta_{3,j}$	-0.012	-0.013	-0.003	-0.016	-0.014	-0.013	-0.004	7	13.0	6	3.0
...	...											
Δ_7Water	$\delta_{15,.,j}$	0.365	-0.292	-0.391	-0.302	-0.549	-0.628	-0.478	7	60.6	6	15.7
Water	$\delta_{16,.,j}$	-0.057	-0.050	0.012	-0.072	-0.009	-0.003	0.038	7	9.3	6	7.8
ΔConsu	$\delta_{17,0,j}$	0.423	0.468	0.395	0.324	0.425	0.372	0.240	28	221.1		
	$\delta_{17,1,j-1}$	0.231	0.172	0.045	0.221	-0.049	0.058	-0.007				
	$\delta_{17,2,j-2}$	0.333	0.175	-0.051	0.025	0.082	0.086	0.202				
	$\delta_{17,3,j-3}$	0.100	0.043	-0.094	-0.055	-0.021	-0.229	-0.068				
Consu	$\delta_{18,0,j}$	0.071	0.035	0.028	-0.046	0.083	0.025	0.069	7	19.7		
logVar	λ_j	0	-0.410	-0.737	-0.654	-0.715	-0.054	-0.406	6	36.3		
Nonperiodic parameters:												
	CosY	SinY	CosHY	SinHY								
	γ_1	γ_2	γ_3	γ_4	$10^3\alpha_0$	α_1	β_1	ν				
	-0.274	-0.015	0.136	0.035	0.403	0.501	0.499	3.216				
χ^2_1	2.38	0.07	2.0	0.2	3.57		20.1					
LL:	3060.38	#pars:	200	T:	1505		AIC/T:	-3.8012				
AC: $Q(28)$:	39.1	$Q(91)$:	119.2	in var:	$Q^*(28)$:	40.3	$Q^*(91)$:	93.9	ρ_1:	0.923		

NOTES: See also notes Table 3 and 4. Coefficients $\delta_{4,i,j}, \ldots, \delta_{14,i,j}$ of holiday effects not reported. Number of coefficients for the 'fixed date' holidays is lower than in 3 due to the smaller sample. Consu: Log daily Norwegian Power consumption in MWh, corrected for day-of-the-week means over period 2002-2005, reported in Table 1.

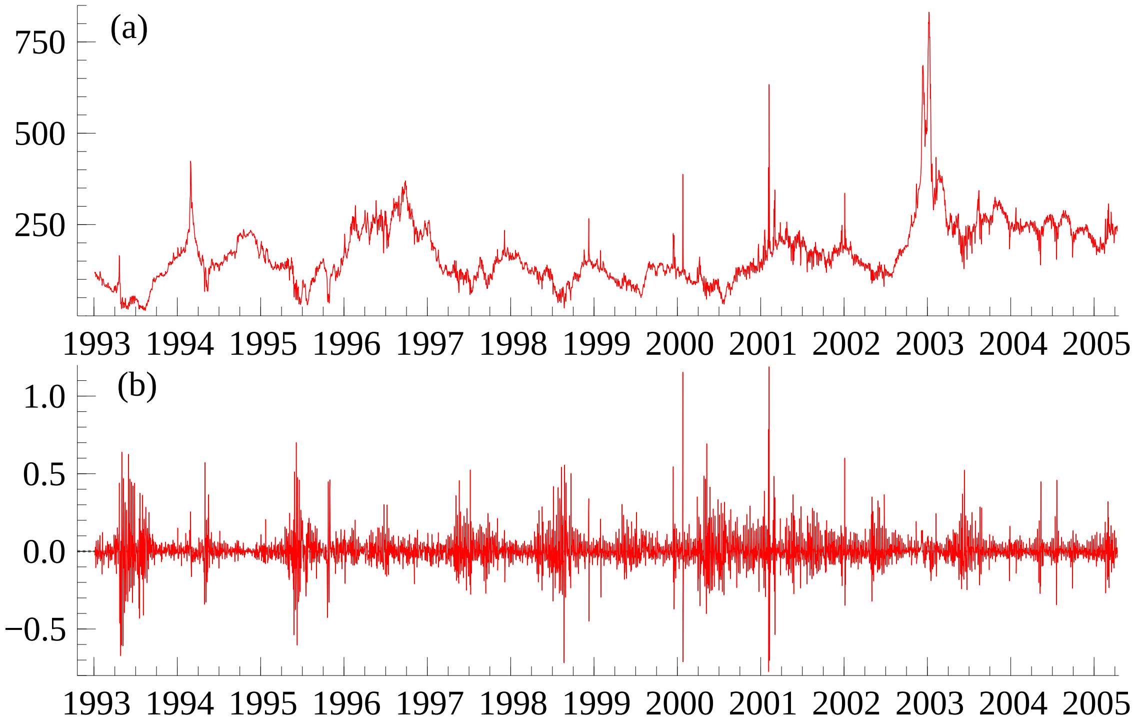
Table 6: ML estimates for daily log-prices EEX, Powernext and APX.

Periodic parameters	j	Mon,1	Tue, 2	Wed, 3	Thu, 4	Fri, 5	Sat, 6	Sun, 7	m	χ^2_m $\theta = 0$	m	χ^2_m $\theta = \iota\theta_0$
EEX												
	D_j	0.078	-0.047	-0.055	-0.015	0.178	0.101	0.372	7	57.1	6	43.0
	$\phi_{1,j}$	0.389	0.860	0.633	0.752	0.652	0.625	0.729	7	431	6	19.7
	$\phi_{2,j}$	-0.092	-0.086	0.259	0.024	0.263	0.049	-0.032	7	18.9	6	16.7
	$\phi_{3,j}$	0.511	0.053	0.016	0.149	-0.061	0.078	0.127	7	101	6	61.0
Constant	$\delta_{1,0,j}$	0.830	0.596	0.334	0.271	0.457	0.598	0.265	7	68.8	6	10.3
...	...											
logVar	λ_j	0	0.242	0.093	-0.056	-0.076	0.462	-0.274	6	21.2		
Powernext												
	D_j	-0.003	0.205	-0.089	-0.105	0.087	0.358	-0.255	7	54.8	6	52.7
	$\phi_{1,j}$	0.109	0.519	0.720	0.769	0.784	0.693	1.059	7	453	6	114
	$\phi_{2,j}$	0.578	-0.049	0.241	0.168	0.036	0.008	0.492	7	56.8	6	41.6
	$\phi_{3,j}$	0.151	0.187	-0.045	0.019	0.097	-0.032	-0.436	7	22.2	6	22.1
Constant	$\delta_{1,0,j}$	0.733	1.200	0.263	0.137	0.245	0.931	-0.608	7	96.4	6	71.6
...	...											
2003/07/11	$\delta_{16,0,j}$	2.154	-0.964	-0.482	-0.306	0.020	0.385	0.725	7	338	6	256
logVar	λ_j	0	-0.340	-0.847	-0.613	-0.786	-0.650	0.518	6	70.5		
APX												
	D_j	0.078	0.121	0.044	0.090	0.045	0.354	0.052	7	90.0	6	41.0
	$\phi_{1,j}$	0.350	0.520	0.689	0.692	0.541	0.286	0.785	7	438	6	36.4
	$\phi_{2,j}$	0.181	0.036	0.134	0.038	0.127	0.026	0.075	7	13.0	6	3.6
	$\phi_{3,j}$	0.318	0.262	0.011	0.078	0.113	-0.057	-0.014	7	34.3	6	26.3
Constant	$\delta_{1,0,j}$	0.693	0.678	0.573	0.657	0.721	2.312	0.259	7	237	6	76.0
...	...											
2003/7	$\delta_{12,0,j}$	2.735	0.738	0.924	-1.988	-1.005	0.171	-0.021	7	408	6	403
logVar	λ_j	0	-0.713	-1.127	-1.009	-0.832	-1.047	-0.486	6	36.6		

Nonperiodic parameters:						**Nonperiodic diagnostics:**						
	$10^3\alpha_0$	α_1	β_1	ν	LL	#pars	$Q(28)$	$Q(91)$	T	$Q^*(28)$	$Q^*(91)$	ρ_1
EEX	8.26	0.332	0.409	3.516	753.29	142	65.0	132.8	1274	38.5	132.4	0.468
Powernext	1.083	0.149	0.851	3.257	806.23	190	52.6	120.0	1218	23.2	87.7	0.364
APX	16.68	0.422	0.578	3.065	335.80	137	49.0	111.8	1554	39.5	140.5	0.196

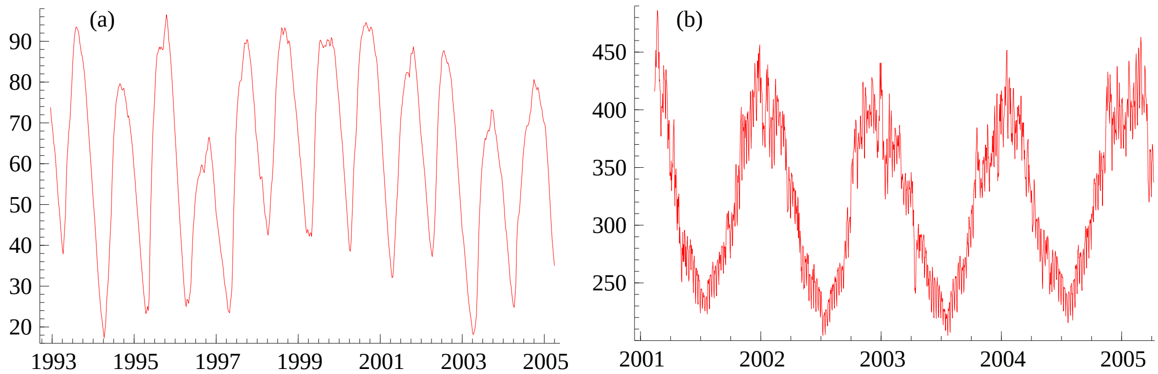
NOTES: See also notes Table 3, Coefficients $\delta_{2,0,j}, \ldots, \delta_{3,0,j}$ for periodic yearly cycle in mean and $\gamma_1, \ldots, \gamma_4$ for nonperiodic cycle in variance not reported. Other unreported coefficients: $\delta_{4,i,j}, \ldots, \delta_{13,i,j}$ for EEX (10 holidays), $\delta_{4,i,j}, \ldots, \delta_{15,i,j}$ for Powernext (12 holidays), $\delta_{4,i,j}, \ldots, \delta_{11,i,j}$ for APX (8 holidays). Common holidays for Germany (EEX) , France (Powernext) and the Netherlands (APX): Good Friday, Easter, Ascencion Day, Pentecost, Dec 25, Jan 1. EEX: November 1, May 1, October 3, December 24, December 26. Powernext: May 1, May 8, July 14, August 15, November 1, November 11. December 2003/7: dummy for week 11-17 July 2003.

Figure 1: Daily spot prices for the Nord Pool



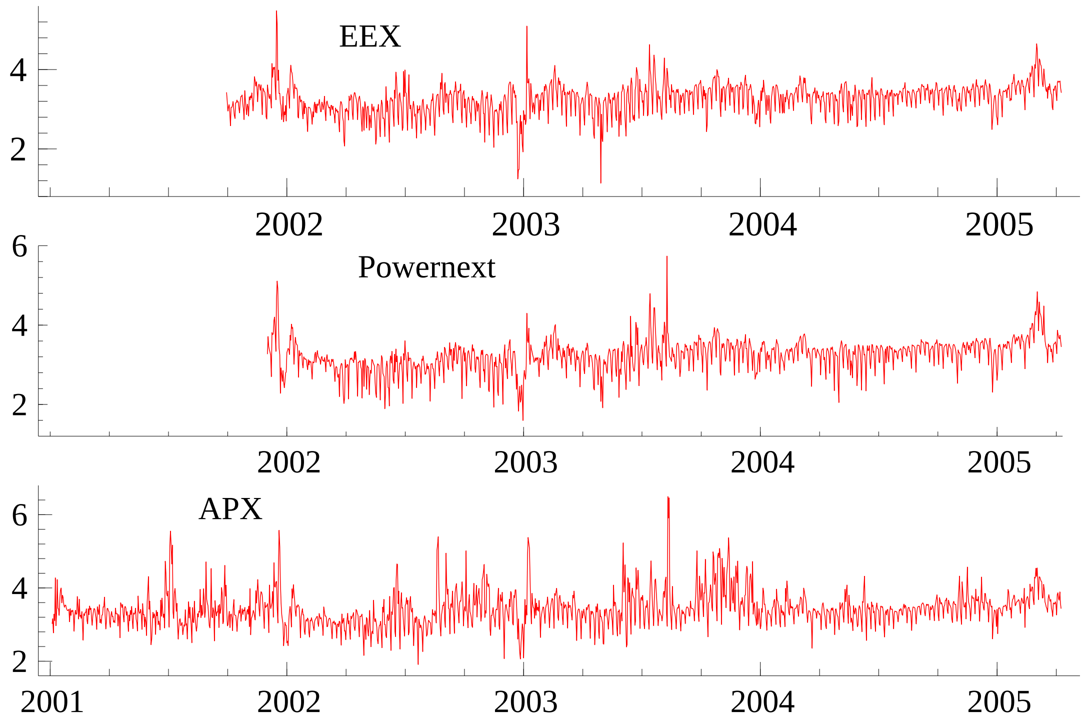
NOTES: (a): Prices. (b): Daily returns: changes in log prices.
Sample: January 4, 1993- April 10, 2005. Prices in Norwegian Kroner (NOK) Mwh. 1 Euro is approximately 8 NOK.

Figure 2: Weekly water reservoir levels and daily consumption for the Nord Pool



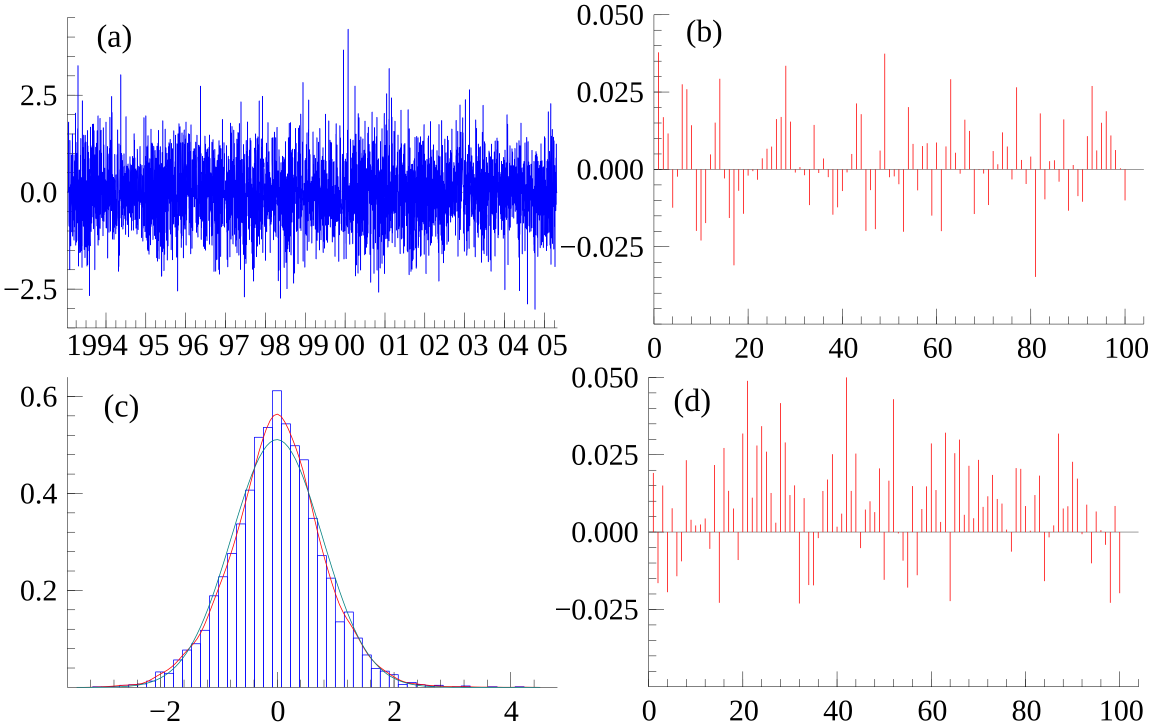
NOTES: (a): Reservoir levels as a percentage of total Norwegian capacity. Sample: January 4, 1993-April 10, 2005. (b): Daily Norwegian Power Consumption in GWh/day Sample: February 26, 2001-April 10, 2005. Source: www.statnett.no.

Figure 3: Log daily spot prices for three new European electricity markets



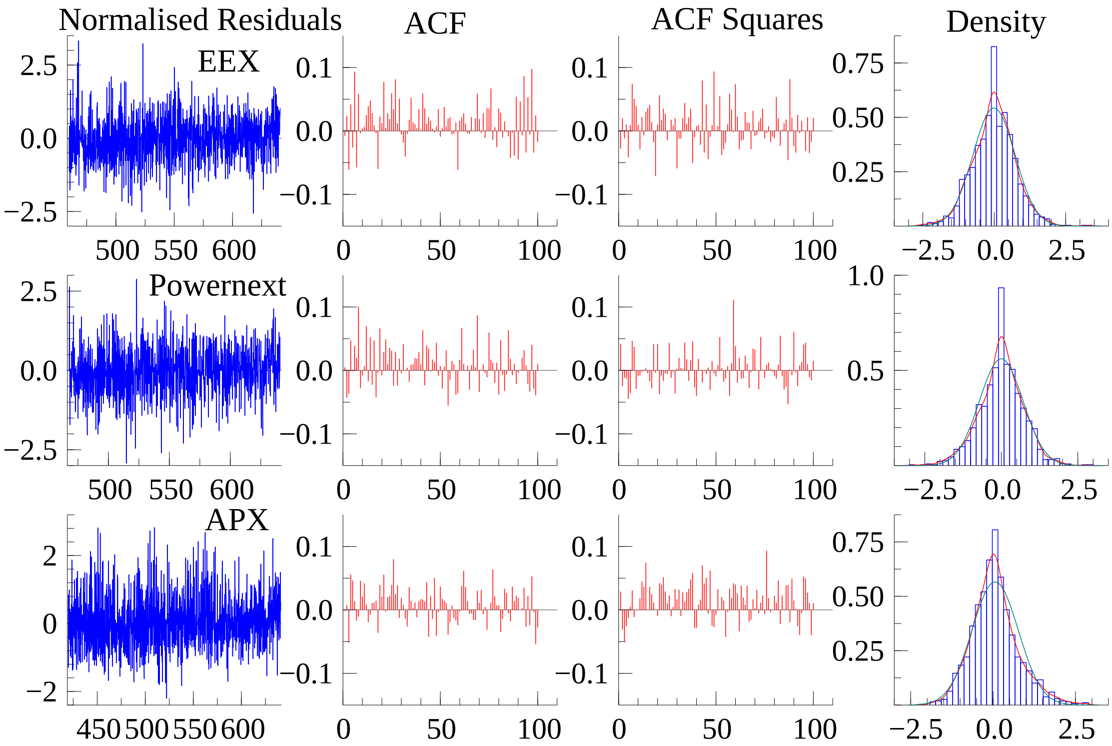
NOTES: Prices in Euros/MWh. Samples: EEX: October, 1, 2001 - April 10, 2005, Powernext: December, 3, 2001 - April 10, 2005, APX: January, 1, 2001 - April 10, 2005. Sources: www.eex.de, www.powernext.fr, www.apx.nl.

Figure 4: Diagnostics for Nord Pool Model with Deterministic x_t



NOTES: Model estimates presented in Table 3. (a): Daily Normalised scaled residuals against time. (b): Autocorrelations function normalised scaled residuals against lag in days. (c): Histogram and nonparametric density estimate of normalised scaled residuals. Thick line gives reference for normal distribution. (d): Autocorrelations function squared normalised scaled residuals against lag in days. Sample: Jan 18, 1993- April 10, 2005.

Figure 5: Residual Diagnostics for New Markets



NOTES: Model estimates presented in Table 6. First column: Daily normalised residuals against time in weeks. (1993.1 = 1). Second column: Autocorrelation function normalised residuals against lag in days. Third column: Autocorrelation function normalised squared residuals against lag in days. Fourth column: Histogram and nonparametric density estimate normalised residuals. Thick line gives reference for normal distribution. First row: results for EEX. Second row: results for Powernext. Third row: results for APX.

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