The Krugman model of international trade with process innovation

Model assumptions

- Countries and goods can be ranked according to their technological level

- An "advanced" country has technological advantages in the production of high-tech goods

- A "TECHNOLOGIC FRONTIER" defines the"state of the art" of technology

- Industries are characterized by different speed of introduction of technological progress

- Industries ranking is stable and is measured on the basis of their time distance from the frontier

- Labour is the only factor of production

Two countries model (1 e 2)

the labour requirement in production of good *z* in country paese *i* is:

$$a_i(z) \quad i = 1,2$$

$a^*(z)$ is the "best" (lower) labour requirement in production of good z (it defines the"best technology") labour input (hours) required by the “best technology” continuously decreases through time

$$a^{*}(z) = \exp\left[-g_z t\right] = e^{-g_z t} \quad \left[e = 2{,}71828\right]$$

g_z is the growth rate of technological progress in industry Z

Each country has a time lag τ with respect to the frontier:

$$a_1(z) = \exp\left[-g_z\left(t - \tau_1\right)\right]$$

If country 2 has a technological lag with respect to country 1, then: $\tau_2 > \tau_1$

Country 1 is more efficient in every industry

Productivity advantage is equal to:

$$\frac{a_2(z)}{a_1(z)} = \frac{\exp\left[-g_z\left(t-\tau_2\right)\right]}{\exp\left[-g_z\left(t-\tau_1\right)\right]} = \exp\left[-g_z\left(t-\tau_2\right)+g_z\left(t-\tau_1\right)\right]$$

$$\frac{a_2(z)}{a_1(z)} = \exp\left[g_z\left(\tau_2-\tau_1\right)\right]$$

Productivity advantage is increasing in g_z

Consumers' demand is identical in the two countries

In a continuum of goods ranked in decreasing order of technological efficiency, country 1 productivity advantage is given by

$$A(z) = \frac{a_2(z)}{a_1(z)} \quad A'_z > 0$$

$\bar{z}$ is the good for which production cost is the same in the two countries 1 and 2

$$w_1 a_1(z) = w_2 a_2(z)$$

$$\frac{w_1}{w_2} = \frac{a_2(z)}{a_1(z)} = A(\bar{z})$$

Goods $z < \bar{z}$ are produced (are cheaper) in 2

Goods $z > \bar{z}$ are produced (are cheaper) in 1

The income share S spent in purchases of goods of country 2 grows with the range of good that are produced in that country:

$$S = S(\overline{z}) \quad S' > 0$$

S is the income share of countries 1 and 2 spent buying goods of country 2

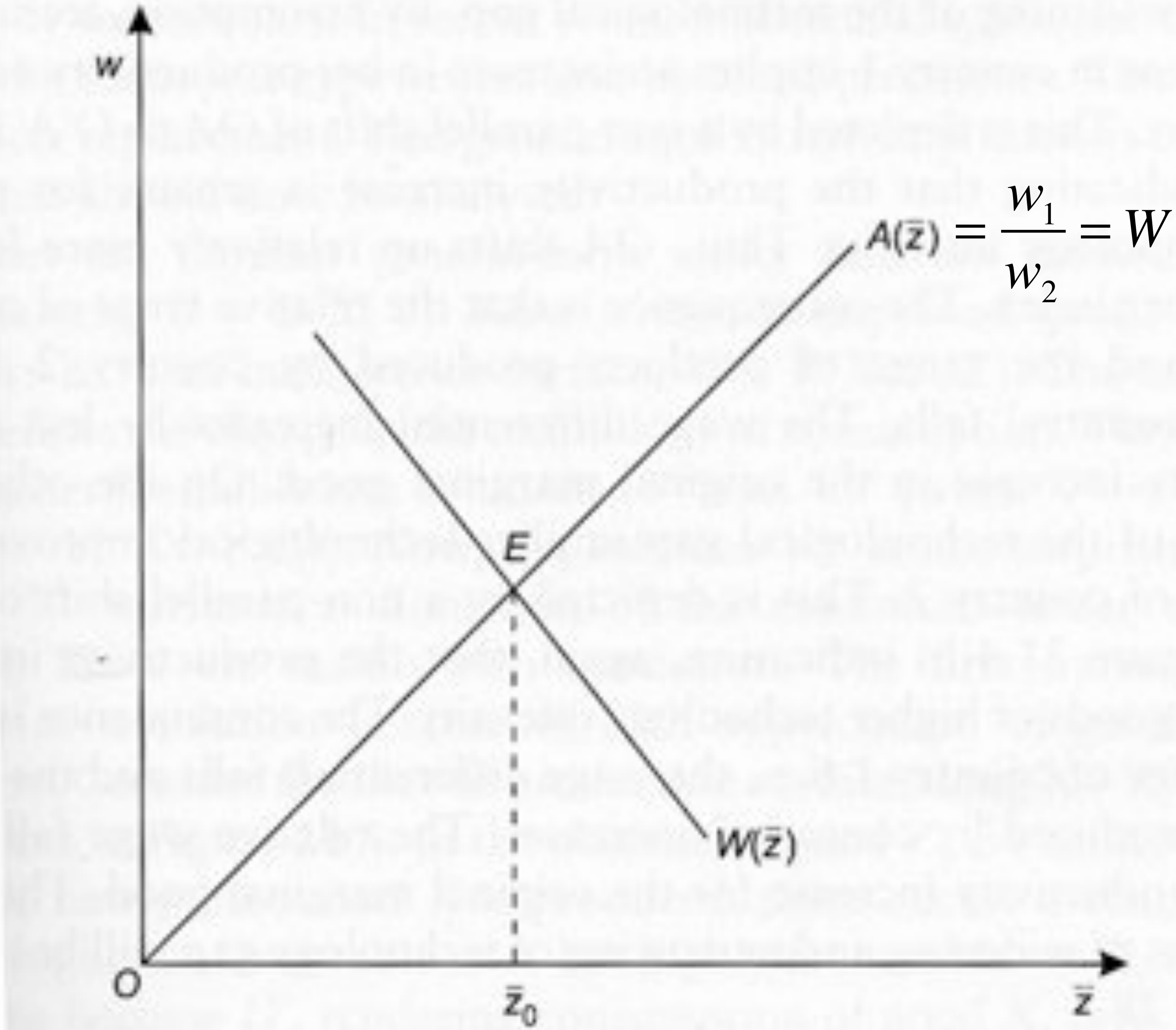
In equilibrium, the output value of country 2 is equal to world demand:

$$w_1 L_1 S(\overline{z}) + w_2 L_2 S(\overline{z}) = w_2 L_2$$

$$w_1 L_1 S(\overline{z}) = w_2 L_2 \left[1 - S(\overline{z})\right]$$

If $W = {}^{w_1}\!/_{w_2}$ is relative wage, then

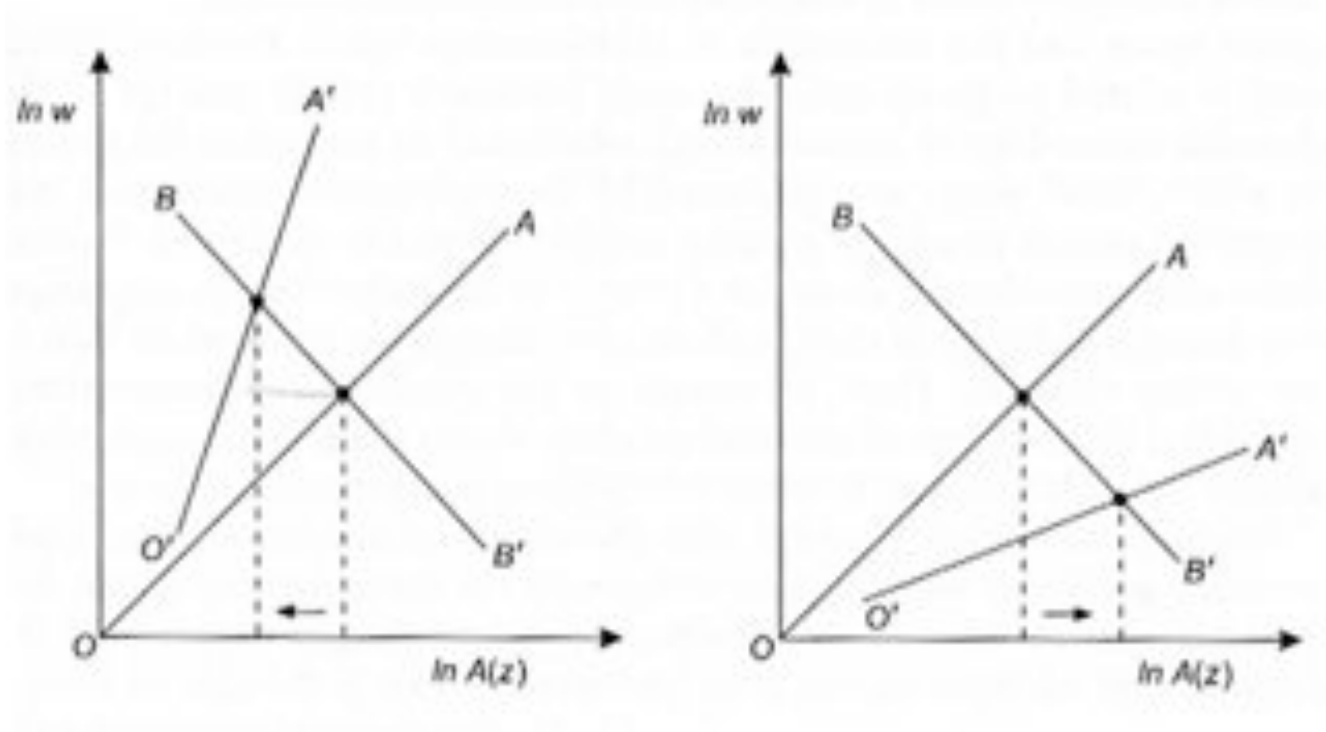
$$W = \frac{L_2}{L_1} \frac{1 - S(\overline{z})}{S(\overline{z})} \qquad \frac{dW}{d\overline{z}} = \frac{L_2}{L_2} \cdot \frac{-S'(\overline{z})}{[S(\overline{z})]^2} < 0$$


w
$A(\bar{z}) = \frac{w_1}{w_2} = W$
E
$W(\bar{z})$
O
$\bar{z}_0$
$\bar{z}$

Effects of "technological gap" changes

We use a logarithmic version of the model:

"Gap" increases "Gap" decreases



- When country 1 increases its technological advantage, the range of goods produced in country 1 enlarges
  - Relative wages increases too but *less* than the productivity gap
- If "gap" reduces (catching-up), then we have opposite conclusions.