$$X(L,T) = L^{\frac{1}{2}} T^{\frac{1}{2}}$$ $\boxed{\text{Es. 17.1}}$

a) Se $Y = 4$ l'isoquanto è

$$4 = L^{\frac{1}{2}} T^{\frac{1}{2}}$$

$$T^{\frac{1}{2}} = \frac{4}{L^{\frac{1}{2}}}$$

$$T = \frac{4^2}{\left(L^{\frac{1}{2}}\right)^2} = \frac{16}{L}$$

L	T	$Y=4$
4	4	$\sqrt{4} \cdot \sqrt{4} = 4$
1	16	$\sqrt{1} \cdot \sqrt{16} = 4$
16	4	$\sqrt{16} \cdot \sqrt{1} = 4$
⋮	⋮	

b) la somma degli esponenti è $\frac{1}{2} + \frac{1}{2} = 1$
→ RENDIMENTI COSTANTI

17.1

d) $\begin{cases} \text{Se } L=1 \text{ e } T=1 \\ y = 1^{\frac{1}{2}} \cdot 1^{\frac{1}{2}} = 1 \end{cases}$

$\begin{cases} \text{Se } \Delta L = 1 \to L=2 \\ y = 2^{\frac{1}{2}} \cdot 1^{\frac{1}{2}} = \sqrt{2} \cong 1{,}41 \\ \Delta y \cong 1{,}41 - 1 \cong 0{,}41 \end{cases}$

$\begin{cases} \text{Se } L = 4 \to y = \sqrt{4} \cdot \sqrt{1} = 2 \end{cases}$

$\begin{cases} \dfrac{\partial Y}{\partial L} = \dfrac{1}{2} L^{\frac{1}{2}-1} T^{\frac{1}{2}} = \dfrac{1}{2} L^{-\frac{1}{2}} T^{\frac{1}{2}} = \dfrac{\sqrt{T}}{2\sqrt{L}} = \text{MPL } (\ ) \end{cases}$

Es. 17.2

$$f(x_1, x_2) = \min\{x_1, x_2\}$$

a) Se $x_1 < x_2$

$MPx_1 = 1$ → COSTANTE per piccole Δx_1

$MPx_2 = 0$ → " "

$TRS = \frac{MPx_1}{MPx_2} = \frac{1}{0} \to \infty$

→ la Tecnologia presenta rendimenti di scala COSTANTI!

$$t f(x_1, x_2) = \min\{t x_1, t x_2\}$$

b) Se $x_1 = x_2 = 20$ $MPx_1 = ?$ $[Y = 20]$

Se $\Delta x_1 > 0 \to Y = 20 \to MPx_1 = 0$

Se $\Delta x_2 > 0 \to Y = 20 \to MPx_2 = 0$

Se $\Delta x_2 > 0$, MPx_1 aumenta! $[MPx_2 > 0]$

17,3

$f(x_1, x_2) = x_1^{\frac{1}{2}} \; x_2^{\frac{3}{2}}$ (1)

a) $MPx_1 = ?$

$MPx_1 = \dfrac{\partial f(x_1, x_2)}{\partial x_1} = \dfrac{1}{2} x_1^{\left(\frac{1}{2}-1\right)} \cdot x_2^{\frac{3}{2}}$

$MPx_1 = \dfrac{1}{2} x_1^{-\frac{1}{2}} \cdot x_2^{\frac{3}{2}} = \dfrac{1}{2} \dfrac{x_2^{\frac{3}{2}}}{\sqrt{x}}$

b) Se $\Delta x > 0$ $\Delta MPx_1 = ?$

$\dfrac{\partial MPx_1}{\partial x} = -\dfrac{1}{2} \cdot \dfrac{1}{2} x_1^{-\frac{1}{2}-1} \cdot x_2^{\frac{3}{2}}$

$= -\dfrac{1}{4} x^{-\frac{3}{2}} \cdot x_2^{\frac{3}{2}} < 0$!

MPx₁ è decrescente!

c) $MPx_2 = \dfrac{\partial f(x_1, x_2)}{\partial x_2} = \dfrac{3}{2} x_1^{\frac{1}{2}} \cdot x_2^{\frac{3}{2}-1} = \dfrac{3}{2} x_1^{\frac{1}{2}} x_2^{\frac{1}{2}}$

$\dfrac{\partial MPx_2}{\partial x_2} = \dfrac{1}{2} \cdot \dfrac{3}{2} x_1^{\frac{1}{2}} x_2^{-\frac{1}{2}} > 0$

MPx_2 è crescente!

d) Se $\Delta x_2 > 0 \quad \Delta MPx_1 \gtrless 0$?

$$\frac{\partial MPx_1}{\partial x_2} = \frac{3}{2} \cdot \frac{1}{2} x_1^{-\frac{1}{2}} x_2^{\frac{1}{2}} > 0 \,!$$

e)

$$TRS = -\frac{MPx_1}{MPx_2} = -\frac{\frac{1}{2} x_1^{-\frac{1}{2}} x_2^{\frac{3}{2}}}{\frac{3}{2} x_1^{\frac{1}{2}} x_2^{\frac{1}{2}}}$$

$$TRS = -\frac{1}{2} \cdot \frac{2}{3} x_1^{-\frac{1}{2}} \cdot x_1^{-\frac{1}{2}} \cdot x_2^{\frac{3}{2}} \cdot x_2^{-\frac{1}{2}}$$

$$TRS = -\frac{1}{3} x_1^{-\frac{1}{4}} \cdot x_2$$

f) Se $\Delta x_1 > 0 \quad \left| \frac{\partial TRS}{\partial x_1} \right| \gtrless 0$?

$$\left| \frac{\partial TRS}{\partial x_1} \right| = -\frac{1}{4} \cdot \frac{1}{3} x_1^{-\frac{1}{4}-1} \cdot x_2 < 0$$

$\boxed{\to \text{il TRS è decrescente!}}$

g) poiché $\frac{1}{2} + \frac{3}{2} = \frac{4}{2} = 2 > 1$ i rendimenti di scala sono CRESCENTI!

$\to$

$\rightarrow$ g) $\begin{cases} y = (\tau x_1)^{\frac{1}{2}} \cdot (\tau x_2)^{\frac{3}{2}} = \tau^{\frac{1}{2}} \cdot \tau^{\frac{3}{2}} \cdot x_1^{\frac{1}{2}} \cdot x_2^{\frac{3}{2}} \\ y = \tau^2 x_1^{\frac{1}{2}} x_2^{\frac{3}{2}} > \tau f(x_1, x_2) \,! \end{cases}$

18.1

1

$$y(L) = 6L^{\frac{2}{3}}$$

$$w = 6 \quad , \quad p = 3$$

---

a) isoprofitto

$$\overline{\pi} = p \cdot y - w \cdot L$$

$$y = \frac{\overline{\pi}}{p} + \frac{w}{p} L$$

$$y = \frac{\overline{\pi}}{3} + \frac{6}{3} L = \frac{\overline{\pi}}{3} + 2L$$

rette per $(0,12)$, $(0,8)$, $(0,4)$

$$\begin{cases} y = 12 + 2L \longrightarrow 12 = \frac{\pi}{3} \rightarrow \pi = 36 \\ y = 8 + 2L \longrightarrow 8 = \frac{\pi}{3} \rightarrow \pi = 24 \\ y = 4 + 2L \longrightarrow 4 = \frac{\pi}{3} \rightarrow \pi = 12 \end{cases}$$

- inclinazione é $\frac{w}{p} = 2$

(2)

L	$y(L) = 6L^{\frac{2}{3}}$
0	0
1	6
2	9,5
3	12,4
4	$\simeq 15$
6	$\simeq 20$
8	$\simeq 24$
10	$\simeq 28$

→ per $L \geq 1$ 2 punti sulla retta per $(0,4)$

b) $L = ?$ $y = ?$ $\pi^* = ?$

$$\pi = 3 \cdot 6 L^{\frac{2}{3}} - 6L$$

$$\pi = 18 L^{\frac{2}{3}} - 6L$$

$$\max_{L} \; 18 L^{\frac{2}{3}} - 6L$$

$$\frac{d\pi}{dL} = \frac{2}{3} \cdot 18 L^{\frac{2}{3}-1} - 6$$

$$\frac{d\pi}{dL} = \frac{12}{L^{\frac{1}{3}}} - 6 = 0$$

$$12 = 6 L^{\frac{1}{3}}$$

$$L^{\frac{1}{3}} = 2 \rightarrow \boxed{L^* = 2^3 = 8}$$

$$\boxed{y = 6 \cdot 8^{\frac{2}{3}} \simeq 24}$$

$$\boxed{\max \pi = 18 \cdot 8^{\frac{2}{3}} - 6 \cdot 8 \simeq 24}$$

c) Se $W = 4$ e $p = 3$

- le curve di isoprofitto diventano

$$y = \frac{\overline{\pi}}{3} + \frac{4}{3} L$$

→ l'inclinazione scende da 2 a $\frac{4}{3}$

→ aumentano l'uso del lavoro e l'output!

18.4 $\begin{cases} \text{acquisto di mele} \\ \text{vendita di cassette di mele e sidro} \end{cases}$ (1)

3 limiti: spazio nel magazzino (S)
numero di macchine per imballaggio (I)
" " spremimele (SM)

$$\begin{cases} (6S \text{ e } 2I) \to 1 \text{ cassetta } (C) \\ (3S \text{ e } 1SM) \to 1 \text{ vaso di sidro } (V) \end{cases}$$

funzioni di produzione:

$$\begin{cases} C(S, I) = \min\left\{\frac{1}{6}S, \frac{1}{2}I\right\} \\ V(S, SM) = \min\left\{\frac{1}{3}S, SM\right\} \end{cases}$$

input disponibili:

$S = 1200$, $I = 600$, $SM = 250$

2

a) Se unico limite fosse $S = 1200$

$$\begin{cases} C_{MAX} = \min\left\{\frac{1}{6}\cdot 1200, \frac{1}{2} I\right\} = \min\left\{200, \frac{1}{2} I\right\} \\ \boxed{C_{MAX} = 200} \\ V_{MAX} = \min\left\{\frac{1}{3}\cdot 1200, SM\right\} = \min\left\{400, \frac{1}{121} SM\right\} \\ \boxed{V_{MAX} = 400} \end{cases}$$

b) Se unico limite $I = 600$

$$\begin{cases} C_{MAX} = \min\left\{\frac{1}{6} S, \frac{1}{2}\cdot 600\right\} = \min\left\{\frac{1}{6} S, 300\right\} = 300 \\ V_{MAX} = \min\left\{\frac{1}{6} S, \frac{1}{2} 600, SM\right\} = \min\left\{\frac{1}{6} S, 300, SM\right\} = 300 \end{cases}$$

c) Se limite unico SM = 250

$$\begin{cases} C_{MAX} = \min\left\{\frac{1}{6}S, \frac{1}{2}I\right\} = \emptyset \ ! \\ V_{MAX} = \min\left\{\frac{1}{6}S, \frac{1}{2}I, 250\right\} = 250 \end{cases}$$

e) $P_C = 5 \quad P_V = 2$

$$R = 5 \cdot C + 2 \cdot V$$

$$1000 = 5 \cdot C + 2 \cdot V$$

$$2V = 1000 - 5C$$

$$\boxed{V = 500 - 2{,}5C}$$

→ Si ha il max R $C = 200$, $V = 0$ dato che la curva di isoricavo (isoprofitto) è più inclinata degli altri vincoli $2{,}5 > 2 > 1 > 0$!

MAX R = $5 \cdot 200 = 1000$!

18.11 $\cancel{f}(x_1, x_2) = x_1^{\frac{1}{2}} \cdot x_2^{\frac{1}{2}}$

$p = 1$ ①, w_1, w_2

---

a) $MP_1 = \frac{\partial f}{\partial x_1} = \frac{1}{2} \cdot \frac{1}{\sqrt{x_1}} \cdot x_2^{\frac{1}{2}}$

$MP_1 = w_1$

$w_1 = \frac{1}{2} \frac{1}{\sqrt{x_1}} \cdot x_2^{\frac{1}{2}}$

$MP_2 = \frac{1}{2} x_1^{\frac{1}{2}} \frac{1}{\sqrt{x_2}} = w_2$

$w_1 = w_2$

$\Rightarrow \cancel{\frac{1}{2}} \frac{1}{\sqrt{x_1}} \cdot x_2^{\frac{1}{2}} = \cancel{\frac{1}{2}} x_1^{\frac{1}{2}} \cdot \frac{1}{\sqrt{x_2}}$

$x_2^{\frac{1}{2}} \cdot x_2^{\frac{1}{2}} = x_1^{\frac{1}{2}} \cdot x_1^{\frac{1}{2}}$

$x_2 = x_1$

$\boxed{\frac{x_1}{x_2} = 1}$

b) No

/ Es. 1 $f(x_1, x_2) = x_1 + 2x_2$ (1)

x_1 = lavoro non qualificato , x_2 = lavoro qualificato

a) ISOQUANTO PER $Y = 20$ e $Y = 40$

$$\begin{cases} 20 = x_1 + 2x_2 \\ 2x_2 = 20 - x_1 \\ x_2 = 10 - \frac{1}{2} x_1 \end{cases}$$

$$\begin{cases} 40 = x_1 + 2x_2 \\ 2x_2 = 40 - x_1 \\ x_2 = 20 - \frac{1}{2} x_1 \end{cases}$$

b) rendimenti di scala costanti!

x_1	x_2	$f(x_1, x_2)$
1	1	$1 + 2 \cdot 1 = 3$
2	2	$2 + 2 \cdot 2 = 6$
3	3	$3 + 2 \cdot 3 = 9$

$$t\, f(x_1, x_2) = f(t x_1, t x_2)$$

(19.1) (2)

c) $\begin{cases} \text{Se } x_2 = 0 \\ y = \cancel{2}x_{\cancel{2}1} \\ x_1 = y \end{cases}$

d) $\begin{cases} \text{Se } x_1 = 0 \\ y = 2x_2 \\ x_2 = \frac{1}{2}y \end{cases}$

e) $(w_1, w_2) = (1, 1)$ x_1 e x_2 minimi? per $y=20$?

Per produrre $y=20$

$$C = w_1 x_1 + w_2 x_2 = x_1 + x_2$$

→ ISOCOSTO

$$x_2 = \bar{C} - x_1$$

→ POICHÉ INCLINAZIONE ISOCOSTO MAGGIORE DELLA INCLINAZIONE DELL'ISOQUANTO $1 > \frac{1}{2}$ IN VALORE ASSOLUTO SI USA SOLO LAVORO QUALIFICATO!

→

e)

$\rightarrow \quad x_1 = 0 \quad x_2 = 10$!

f) Se $(w_1, w_2) = (1, 3)$

$$C = \cancel{x_1} + 3\cancel{x_2}$$

$$x_2 = \frac{\bar{C}}{3} - \frac{1}{3} x_1$$

$\rightarrow$ ORA $\quad \frac{1}{3} < \frac{1}{2}$

CONVIENE USARE SOLO LAVORO NON QUALIFICATO!

PER PRODURRE $y = 20$

$$\begin{cases} x_2 = \frac{1}{2} 20 = 10 & \rightarrow \quad 3 \cdot 10 = 30 \\ x_1 = 20 & \rightarrow \quad 1 \cdot 20 = 20 \; ! \end{cases}$$

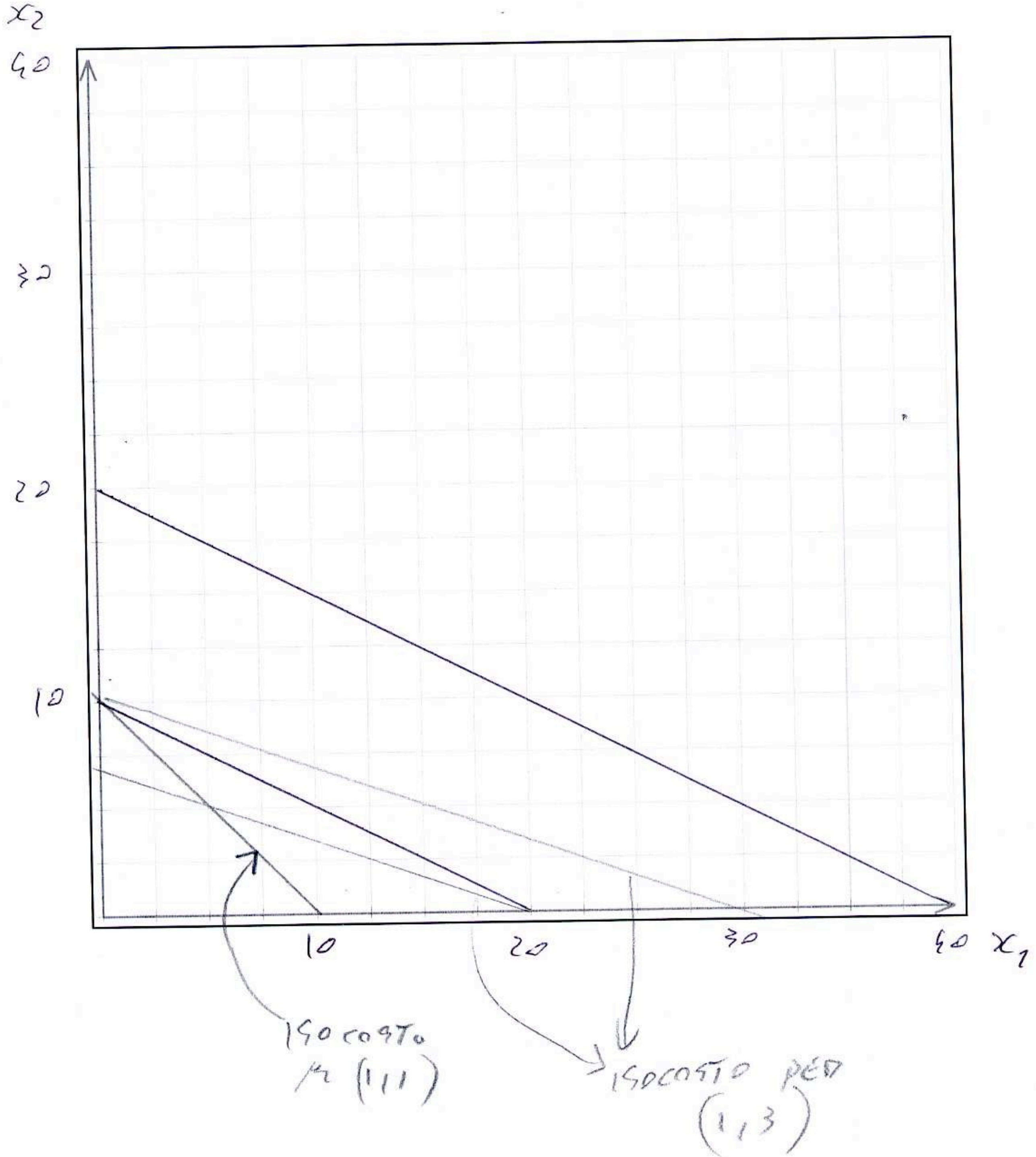
g) $\underset{min}{C} = \min \left\{ w_1 \cdot 20, \; w_2 \cdot \frac{1}{2} \cdot 20 \right\}$

h) $\underset{min}{C} = \min \left\{ w_1 y, \; w_2 \frac{1}{2} y \right\}$

Parameters:	Plotting Area	Axes Division	Printing Format
x-Axis:	(0.0 | 16.0)	1E = automatic	1E = 1 cm
y-Axis:	(0.0 | 16.0)	1E = automatic	1E = 1 cm

Functions:

Es. 13.1



— 19.2 $f(x_1, x_2) = \text{MIN}\{x_1, 2x_2\}$ 1

$\left.\begin{array}{l} x_1 = \text{RAME} \\ x_2 = \text{STAGNO} \end{array}\right\} Y = \text{BRONZO}$

a) ISOQUANTO

x_1	x_2	$\min\{x_1, 2x_2\}$
1	0,5	$\{1, 2 \cdot 0,5\} = 1$
2	1	$\{2, 2 \cdot 1\} = 2$
20	10	$\{20, 2 \cdot 10\} = 20$
⋮	⋮	

b) rendimenti di scala COSTANTI!

c) Se $Y = 10$ RAME $x_1 = ?$ STAGNO $x_2 = ?$

$$\min\{10, 2 \cdot 5\} = 10$$

$x_1 = 10$, $x_2 = 5$

②

d) $\begin{cases} \text{p/} \ (w_1, w_2) = (1, 1) \text{ e } y = 10 \\ x_1^* \text{ e } x_2^* \text{ p/ } C\text{min?} \\ x_1^* = 10 \ , \ x_2^* = 5 \end{cases}$

$C = 10 + 5 = 15$

e) Se (w_1, w_2)

$C(w_1, w_2, 10) = w_1 \cdot 10 + w_2 \cdot 5$

f) $C(w_1, w_2, y) = w_1 \cdot y + w_2 \cdot \frac{1}{2} y$

19.3) $f(L,M) = 4L^{\frac{1}{2}} M^{\frac{1}{2}}$ (1)

$W_L = 40 \quad W_M = 10$

---

a) $C = 40L + 10M$

$M = \frac{C}{10} - 4L$ [ISOCOSTO]

$$\begin{cases} C = 400 \rightarrow M = 40 - 4L \\ C = 200 \rightarrow M = 20 - 4L \end{cases}$$

b) per C_{min}, quale $\frac{M}{L}$?

$|TRS| = \frac{W_L}{W_M}$

$TRS = -\frac{MP_L}{MP_M}$

$$\begin{cases} MP_L = \frac{1}{2} 4 L^{-\frac{1}{2}} M^{\frac{1}{2}} = 2 L^{-\frac{1}{2}} M^{\frac{1}{2}} \\ MP_M = \frac{1}{2} 4 L^{\frac{1}{2}} M^{-\frac{1}{2}} = 2 L^{\frac{1}{2}} M^{-\frac{1}{2}} \end{cases}$$

(19.3) 2

b)
$$\frac{\cancel{k} L^{-\frac{1}{2}} M^{\frac{1}{2}}}{\cancel{k} L^{\frac{1}{2}} M^{-\frac{1}{2}}} = \frac{40}{10} = 4$$

$$\frac{M^{\frac{1}{2}} M^{\frac{1}{2}}}{L^{\frac{1}{2}} L^{\frac{1}{2}}} = 4 \rightarrow \boxed{\frac{M}{L} = 4}$$

c) ISOQUANTO PER $Y = 40$

$$\begin{cases} 40 = 4 L^{\frac{1}{2}} M^{\frac{1}{2}} \\ 10 = L^{\frac{1}{2}} M^{\frac{1}{2}} \end{cases}$$

min C per $Y = 40$, $w_L = 40$, $w_M = 10$

$\rightarrow$ nel punto di costo minimo $M = 4L$

$$10 = L^{\frac{1}{2}} \cdot (4L)^{\frac{1}{2}} = 2 L^{\frac{1}{2}} \cdot L^{\frac{1}{2}} = 2L$$

$$\begin{cases} L^* = \frac{10}{2} = 5 \\ M^* = 4 \cdot 5 = 20 \end{cases}$$

$C_{min}(40, 10, 40) = 40 \cdot 5 + 10 \cdot 20 = 400$

3

d) $\begin{cases} \bar{Y} = 4 L^{\frac{1}{2}} M^{\frac{1}{2}} \\ M = 4L \end{cases}$

$$\bar{Y} = 4 \cdot L^{\frac{1}{2}} \cdot 2 L^{\frac{1}{2}} = 8L$$

$$\begin{cases} L^* = \frac{\bar{Y}}{8} \\ M^* = 4 \cdot \frac{\bar{Y}}{8} = \frac{1}{2} \bar{Y} \end{cases}$$