Probability and Statistics for Economics and Finance

A.A. 2010/2011
PHD PROGRAMME IN ECONOMICS AND FINANCE

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Teaching
The course will take place over 6 weeks (from the 16 of November 2010 to the 21 of December 2010) for a total amount of 40 hours. The lessons will take place according to the following calendar:
Tuesday, 14.30-18.30 (4 hours);
Friday, 14.30-18.30 (4 hours).

Availability
The course is intended for 1st year students on PhD in Economics and Finance.

Pre-requisites
Introduction to Mathematics and Elementary Statistical Theory. Attendance at more advanced courses such as Real Analysis, Probability, Distribution Theory and Inference would be highly desirable.

Objectives of the course
The purposes of this course are: (i) to explain the formal basis of abstract probability theory, and the justification for basic results in the theory; (ii) to explore those aspects of the theory most used in advanced analytical models in economics and finance; (iii) to give an introduction to some concepts of statistical inference. The approach taken will be formal, but the topics will be illustrated and explained through many examples.

COURSE CONTENT

Probability Spaces and Kolmogorov axioms
Sample spaces, events, algebras of events, sigma-algebras of events, probability functions and Kolmogorov axioms, probability spaces, event trees.
Conditional probability with respect to a non null event: definitions, properties, product law, theorem of total probabilities, Bayes theorem, independence between events, random walk, Simpson's paradox.

Random variables
Measurability, distribution functions, discrete random variables, (absolutely) continuous random variables, probability density functions, singular random variables.
Transformations of random variables: one-to-one transformations, log-normal distribution, other transformations, probability integral transform.

Expected value
Theory of integration, Riemann integral, Riemann-Stieltjes integral, (generalized) Lebesgue integral, expected value, examples and paradoxes.
Expected value of a function of a random variable, variance, properties of the expected value and of the variance.
Some important inequalities: Markov inequality, Tchebycheff inequality, Jensen inequality.
Moments: absolute moments, central moments, moment gene rating function.

Multidimensional random variables
Joint cumulative distribution functions, discrete and continuous multidimensional random variables, joint density functions.
Conditional distributions: conditional cumulative distribution functions, conditional distributions for discrete random variables, conditional densities for continuous random variables.
Independence between random variables: definitions, properties, examples.

Functions of multidimensional random variables
Functions of multidimensional random variables, expected value of functions of random variables, expected value of a linear combination of random variables, covariance, correlation coefficient of Bravais, Cauchy-Schwarz inequality, variance of a linear combination of random variables, expected value and variance of the sample mean.
Conditional expectation with respect to another random variable, conditional probability with respect to a random variable, conditional variance with respect to another random variable.
Mixed absolute moments, mixed central moments, joint moment generating function.
Bivariate normal distribution: joint density, joint moment gene rating function, expected values, variances, covariance and correlation coefficient of Bravais, marginal densities, conditional densities.

Distributions of functions of multidimensional random variables
Method of the cumulative distribution function: some examples.
Distribution of the minimum and of the maximum: some examples.
Distribution of the sum and of the difference of two random variables: convolution formula, examples.
Distribution of the product and of the ratio of two random variables: examples.
Distribution of the sum of a random number of random variables: examples.
Method of the moment generating function: sum of independent random variables, sum of Gaussian random variables, examples.

Convergence of sequences of random variables and principal limit theorems
Principal modes of convergence: almost sure convergence (with probability one), convergence in probability, convergence in distribution, convergence in quadratic mean, examples and counterexamples.
Weak law of large numbers: theorem of Tchebycheff, weak law of large numbers of Bernoulli for relative frequences, some examples.
Central limit theorem: proof with the moment generating function, limiting distributions and asymptotic distributions, approximation of the binomial distribution to the normal distribution, some examples.
Strong law of large numbers: some examples.
Order statistics: definitions, marginal distributions, asymptotic distributions, examples.
Empirical distribution function: some properties and examples.

Statistical inference
Statistical models; parametric models.
Principles of statistical inference; the repeated sampling principle; counterexamples; Bayesian inference (basics).
Point estimation, confidence intervals and test of hypothesis in the decision theoretic approach.

Likelihood inference
Likelihood function; likelihood principle.
Sufficient statistics; exponential families (basics).
Maximum likelihood estimators; likelihood equations; observed Fisher information; expected Fisher information; properties of maximum likelihood estimators.

INDICATIVE READING

The following books may prove useful. A set of exercises with solutions will be distributed by the teacher.

A. M. Mood, F. A. Graybill, D. C. Boes (1974). ***Introduction to the Theory of Statistics*****. McGraw-Hill.**

CHAPTER 1. Probability
[no Example 1.20, Example 1.28]

CHAPTER 2. Random Variables, Distribution Functions, and Expectations

CHAPTER 3. Special Parametric Families of Univariate Distributions
[no 3.2.3, Theorem 3.7, Theorem 3.8, just the definition of the negative binomial, 3.2.6, 3.4.1]

CHAPTER 4. Joint and Conditional Distributions, Stochastic Independence, More expectation

CHAPTER 5. Distributions of Functions of Random Variables
[no Formula 5.13, Theorem 5.4, Theorem 5.8, Example 5.13]

CHAPTER 6. Sampling and Sampling Distributions
[no 6.2, 6.3.5, 6.3.6, 6.3.7, 6.4, 6.5.2, Theorem 6.16]

CHAPTER 11. Nonparametric Methods
[just 11.1, 11.2, 11.2.1 (no Formula 11.7)]

APPENDIX A. Mathematical Addendum
[no A.2.3, no multinomial theorem]

A. Azzalini (1996). ***Statistical Inference Based on the Likelihood*****. Chapman & Hall.**

CHAPTER 1. Introduction and Overview
[no 2.4]

CHAPTER 2. Likelihood

CHAPTER 3. Maximum Likelihood Estimation

Other useful books on the theory of probability and statistical inference

G. Casella, R. L. Berger (2002). *Statistical Inference, 2nd Editon*. Duxbury Advanced Series.
W. Feller (1968). *An Introduction to Probability Theory and Its Applications, 3rd Edition, Volume 1*. Wiley.
G. R. Grimmett, D. R. Stirzaker (2001). *Probability and Random Processes*. Oxford University Press.
R. V. Hogg, A. T. Craig (1994). *Introduction to Mathematical Statistics, 5th Edition*. Macmillan.
J. Jacod, P. Protter (2000). *Probability Essentials*. Springer, New York.
L. Pace, A. Salvan (1997). *Principles of Statistical Inference from a Neo-Fisherian Perspective*, Advanced Series on Statistical Science and Applied Probability, Vol.4, World Scientific, Singapore.
J. S. Rosenthal (2006). *A First Look at Rigorous Probability Theory, 2nd Edition*. World Scientific Publishing, Singapore.
T. A. Severini (2000). *Likelihood Methods in Statistics*. Oxford University Press, Oxford.
D. Williams (1991). *Probability with Martingales*. Cambridge University Press.

ASSESSMENT

A two-hour written paper and an oral exam at the end of the course.