

A moment symbolic representation of Lévy processes with applications

I. Oliva

Dept. of Economic Sciences, University of Verona

Abstract

The term *classical umbral calculus* represents a symbolic method consisting of a syntax with an alphabet of symbols, called umbrae, and a suitable linear functional, called evaluation.

The main devices of this method are essentially two. The first key point is to associate a unital number sequence $1, a_1, a_2, \dots$ to a sequence $1, \alpha, \alpha^2, \dots$ of powers of an umbra α by means of the evaluation functional. The second key point is that distinct umbrae could represent the same sequence $\{a_n\}$, so they are called similar.

By using such a moment symbolic calculus, we present a symbolic version of Lévy processes with some applications to the family of the so-called *time-space harmonic* polynomials, that is, a family of polynomials $\{P(x, t)\}_{t \geq 0}$ such that $E[P(X_t, t) | \mathfrak{F}_s] = P(X_s, s)$, for all $s \leq t$, where $\mathfrak{F}_s = \sigma(X_\tau : \tau \leq s)$ is the natural filtration associated with the stochastic process.

What is more, the symbolic representation of multivariate Lévy processes has some connections with the so-called moment representation of various families of multivariate polynomials sequences.

The usefulness of time-space harmonic polynomials with respect to Lévy processes is that the stochastic process $\{P(X_t, t)\}$ is a martingale, whereas $\{X_t\}$ does not necessarily have this property. Therefore to find polynomials such that it is a martingale the stochastic process obtained by replacing the indeterminate x with the Lévy process $\{X_t\}_{t \geq 0}$, could be particularly meaningful in several area and in particular in mathematical finance.