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Unambiguous qualitative comparative static results for a *NARP*-maximising processing cooperative in short-run equilibrium

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Abstract:

The adjustment of a *NARP-maximising* processing cooperative facing an aggregated raw agricultural supply schedule with a positive slope due to decreasing returns to scale is revisited. When the peak coordinator being responsible for the daily management of the processing cooperative is able to control members' supplies of the raw agricultural product, unambiguous short-run qualitative comparative static results can be derived for the cooperative's relative choice functions on the basis of the unrestricted NARP function proven to be convex in final output price and processing production factor prices. In general, no unambiguous qualitative comparative static results are forthcoming for the processing cooperative's relative choice functions on the basis of the stable-equilibrium NARP function when the peak coordinator is unable to restrict members' supplies of the raw agricultural product.

Key words: Firm behaviour, processing cooperatives, duality theory.

JEL Codes: L29, P13, Q13.

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1. Introduction

The analysis of agribusiness processing cooperatives experienced a breakthrough in the middle of the 1950`s when agricultural economists started to adapt the «neoclassical» theory of the firm in order to investigate the adjustment of different types of agribusiness cooperatives operating in a perfectly competitive market environment without uncertainty. Richard Sexton, Brooks Wilson and Joyce Wann (1989) and Gerry Boyle (1998), independently of each other, extended the «neoclassical» theory of processing cooperatives further by developing a duality methodology based on Daniel McFadden's (1978) restricted profit function. Arvid Senhaji (2008a) proves that the unambiguous qualitative comparative static results derived by Boyle (1998) are not correct unless the inverse aggregated raw agricultural supply schedule is completely vertical. However, in the standard processing cooperative model analysed by Paavo Kaarlehto (1954, 1956), Oddvar Aresvik (1952, 1955), and Peter Helmberger and Sidney Hoos (1962), among others, the inverse aggregated raw agricultural supply schedule exhibits an ordinary positive slope. Senhaji (2008a) illustrates that within such a market setting the gradient of the restricted gross revenue product function does not provide us with the processing cooperative's ordinary behavioural functions as claimed first by Sexton et al. (1989: 60) and later by Boyle (1998, 2004).

In the current paper I revisit the short-run analysis of a processing cooperative facing an aggregated inverse raw agricultural supply schedule that exhibits a general positive slope. Unambiguous qualitative comparative static results are derived for the (relative) choice functions of a processing cooperative in two different scenarios depending on whether the daily manager is able to control members' supply of the raw agricultural product or not. The number of cooperative members is fixed, and members are presumed to be able to reach an agreement saying that business shall be managed so that the net average revenue product (*NARP*) is maximised. If the peak coordinator, being responsible for the daily management of the processing cooperative, is able to control members' supplies of the raw agricultural product and secure them the maximum *NARP* achievable, unambiguous qualitative comparative static results can be derived on the basis of the unrestricted NARP function proven to be convex in final output price and processing production factor prices. In general, no unambiguous qualitative comparative static results for the relative choice functions are forthcoming on the basis of the stable-equilibrium NARP function when the peak coordinator is unable to restrict members' supplies of the raw agricultural product.

2. The conceptual framework and the main assumptions

In this paper I focus on the adjustment of a regional processing cooperative established by farmers that want to restore and secure their own local processing facility, or that wish to avoid being exploited by profit-maximising investor-owned processing firms (IOF) with oligopsony power as outlined in Richard T. Rogers and Richard J. Sexton (1994). The processing cooperative only collects and processes the raw agricultural produce supplied by the fixed number of members equal to n^p. Cooperative farmers achieve a uniform raw agricultural price, maximise profits, and operate as price takers in all markets. Thus, the processing cooperative faces an aggregated raw agricultural supply schedule comprising the n^p members' profit-maximising supply functions. Whenever the cooperative's financial structure is patronage based, the aggregated raw agricultural supply schedule will be a function of the total payment that members of the cooperative society receive per unit of the raw agricultural product equal to *NARP*, together with D strictly positive farming production factor prices in the vector $\mathbf{w^d}$, and a policy support index denoted by T_a:

$$x_a^S = \sum_{i=1}^{n^p} \mathrm{S}_\mathrm{i}^\mathrm{a}(NARP, \mathbf{w^d}, T_a) = \mathrm{S}_\mathrm{a}^\mathrm{C}(NARP, \mathbf{w^d}, T_a) \tag{2.1}$$

A peak coordinator elected by the n^p members is responsible for the daily management of the processing and marketing business. The n^p members collectively constitute a «principal» and are presumed able to reach an agreement saying that the peak coordinator must run the processing business so as to maximise the *NARP* paid to members per unit of the raw agricultural product.

A processing cooperative that is able to restrict members' supplies of the raw agricultural product is distinguished from a processing cooperative that is not able to do so. The former type of processing cooperatives is labelled a restricting processing cooperative while the latter is labelled an unrestricting processing cooperative. All net return is rebated to members according to patronage defined as a farmer's share of the aggregated supply of the raw agricultural product. In the subsequent analysis I do not consider retaining of funds.

Let the production process of the cooperative processing firm be described by the production function

$$y = \mathrm{f}(x_1^b, ..., x_B^b, x_a^C, K) = \mathrm{f}(\mathbf{x^b}, x_a^C, K) = \mathrm{f}(\mathbf{x}, K). \tag{2.2}$$

In expression (2.2) y denotes quantity produced of the final output, K is the fixed amount of productive capital, and x_j^b is the j[th] processing production factor in the processing input vector $\mathbf{x^b}$. Labour, fuels, energy, and non-agricultural materials are examples of processing inputs with a corresponding processing input price vector equal to $\mathbf{w^b} = (w_1^b, \ldots, w_B^b)$ containing B strictly positive processing input prices. The vector $\mathbf{x}$ includes all inputs given by the B

processing factors together with the raw agricultural quantity x_a^C. The production function is assumed to be continuous from above, and to exhibit weak monotonicity in the processing production factor vector $\mathbf{x^b}$. The raw agricultural quantity x_a^C is a strictly essential production factor. The strictly positive final output price equals *P*, and the cooperative processing firm is a price taker in the output market as well as in all processing production factor markets. Let *F* denote fixed cost. In order to alleviate notational clutter I will not include the fixed amount of productive capital as an argument in the short-run equilibrium choice functions analysed in the subsequent chapters.

The relationship between the *GRP* defined in expression (2.3) as total revenue minus variable processing cost, and the net revenue product (*NRP*), the net average revenue product (*NARP*), and the average revenue product *(ARP)* are as follows:

$$GRP = \left(Py - \left(\sum_{j=1}^{B} w_j^b x_j^b \right) \right), \tag{2.3}$$

$$NRP = \left(Py - \left(\sum_{j=1}^{B} w_j^b x_j^b \right) - F \right) = \left(GRP - F \right), \tag{2.4}$$

$$NARP = \left(\frac{\left(Py - \left(\sum_{j=1}^{B} w_j^b x_j^b \right) - F \right)}{x_a^{\mathrm{C}}} \right) = \left(\frac{NRP}{x_a^{\mathrm{C}}} \right) = \left(\frac{\left(GRP - F \right)}{x_a^{\mathrm{C}}} \right), \text{ and} \tag{2.5}$$

$$ARP = \left(\frac{\left(Py - \left(\sum_{j=1}^{B} w_j^b x_j^b \right) \right)}{x_a^{\mathrm{C}}} \right) = \left(\frac{(NRP + F)}{x_a^{\mathrm{C}}} \right) = \left(\frac{GRP}{x_a^{\mathrm{C}}} \right). \tag{2.6}$$

With these definitions made, I am ready to derive the set of unambiguous short-run qualitative comparative static results for a processing cooperative maximising *NARP* depicted in figure 1 below as a concave function of the raw agricultural quantity x_a^C. Numerous other objectives that a processing cooperative could pursue are described in Clare LeVay (1983), Sexton (1984), and Senhaji (2008b), among others.

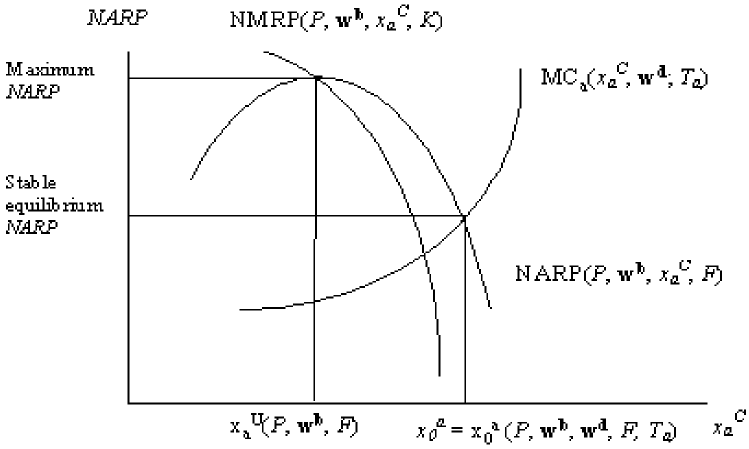

Figure 1

A restricting processing cooperative secures members the maximum *NARP* by collecting and processing the raw agricultural quantity $x_a^U(P, \mathbf{w^b}, F)$. When the processing cooperative is unrestricting, the adjustment will be characterised by the stable equilibrium *NARP* together with the equilibrium raw agricultural quantity x_0^a.

In the first scenario analysed in the subsequent chapter I assume that the peak coordinator is able to control members' supplies of the raw agricultural product x_a^C by means of for example production quotas. In the second scenario analysed in chapter 4, the peak coordinator is no longer able to control members' supplies of the raw agricultural product. In both scenarios the inverse aggregated raw agricultural supply schedule exhibits a general positive slope due to decreasing returns to scale in agriculture.

3. The unrestricted NARP-maximising behavioural functions

In order to derive the unrestricted NARP-maximising choice functions denoted by superscript «U», I first present the maximisation problem facing the peak coordinator that is now assumed to be able to restrict members' supplies in order to secure them the maximum *NARP* achievable:

$$\max_{y,\mathbf{x^b},x_a^C} \left\{NARP\right\} = \max_{y,\mathbf{x^b},x_a^C} \left\{ \left(\frac{Py - \left(\sum_{j=1}^{B} w_j^b x_j^b \right) - F}{x_a^C} \right) \mid \mathrm{f}(\mathbf{x^b}, x_a^C) \geq y \right\}. \tag{3.1}$$

Without loss of generality, I divide the maximisation problem facing the peak coordinator of the restricting processing cooperative into two separate stages. On stage one the peak coordinator treats the raw agricultural quantity x_a^C as an exogenous variable equal to x_0^a. As suggested by Sexton et al. (1989: 56), and Boyle (1998, 2004), maximising *NARP* with regards to output supply y and the processing production factor vector $\mathbf{x^b}$ is then equivalent to maximising *GRP* with regards to y and $\mathbf{x^b}$:

$$\max_{y,\mathbf{x^b},x_a^C} \left\{NARP \mid x_a^C = x_0^a\right\} \Leftrightarrow \max_{y,\mathbf{x^b}} \left\{GRP \mid x_a^C = x_0^a\right\}. \tag{3.2}$$

The restricted variable processing cost function $V^R(\mathbf{w^b}, y, x_0^{\ a})$ and the restricted GRP function $GRP^R(P, \mathbf{w^b}, x_0^a)$ are defined directly by the following two maximisation problems:

$$V^R\left(\mathbf{w}^b, y, x_0^a\right) = \min_{\mathbf{x^b}} \left\{ \left(\sum_{j=1}^{B} w_j^b x_j^b \right) \mid \mathrm{f}\left(\mathbf{x^b}, \mathrm{x}_0^a\right) \geq y \right\}, \text{ and} \tag{3.3}$$

$$\begin{aligned} GRP^R\left(P, \mathbf{w^b}, x_0^a\right) &= \max_{y, \mathbf{x^b}} \left\{ Py - \sum_{j=1}^{B} w_j^b x_j^b \mid \mathrm{f}\left(\mathbf{x^b}, x_0^a\right) \geq y \right\} \\ &= \max_{y} \left\{ Py - V^R\left(\mathbf{w^b}, y, x_0^a\right) \right\}. \end{aligned} \tag{3.4}$$

The restricted variable processing cost function $V^R(\mathbf{w^b}, y, x_0^a)$ in expression (3.3) is the support function of the implicit processing input requirement set $L^*(y, x_0^a)$[1] identical to:

$$L^*\left(y, x_0^a\right) = \left\{ \mathbf{x^b} : \left(\sum_{j=1}^{B} w_j^b x_j^b \right) \geq V^R\left(\mathbf{w^b}, y, x_0^a\right) \text{ for all } \mathbf{w^b} > \mathbf{0_B}. \right\} \tag{3.5}$$

Likewise, the restricted GRP function in expression (3.4) is the support function of the implicit production possibilities set $T^*(x_0^a)$[2] equal to:

$$T^*\left(x_0^a\right) = \left\{ \begin{array}{l} \left(y \ \mathbf{x^b}\right) : \left(Py - \left(\sum_{j=1}^{B} w_j^b x_j^b \right) \right) \leq GRP^R\left(P, \mathbf{w^b}, x_0^a\right) \text{ for all} \\ \mathbf{w^b} > \mathbf{0_B} \text{ and } x_0^a > 0. \end{array} \right\} \tag{3.6}$$

The restricted NARP function in expression (3.7) specifies the maximum price the processing cooperative can pay per raw agricultural unit after processing cost and fixed cost are paid:

[1] $L^*(y, x_0^a)$ is either identical or a monotonised convexification of the original restricted input requirement set given by $L\left(y, x_0^a\right) = \left\{ \mathbf{x^b} : \mathrm{f}\left(\mathbf{x^b}, x_0^a\right) \geq y, \mathbf{x^b} > \mathbf{0_B}, x_0^a > 0 \right\}$ whenever $L(y, x_0^a)$ contains nonconvex configurations that are never utilised by a rationale cost-minimising processing firm. See Chambers (1994:86) for a further discussion of this point.

[2] $T^*(x_0^a)$ is either identical or a monotonised convexification of the original restricted production possibilities set given by $T\left(x_0^a\right) = \left\{ \mathbf{x^b} : \mathrm{f}\left(\mathbf{x^b}, x_0^a\right) \geq y, x_0^a > 0 \right\}$ whenever $T(x_0^a)$ contains nonconvex configurations that are never utilised by a rationale *NARP*-maximising processing firm.

$$\mathrm{NARP}^{\mathrm{R}}\left(P,\mathbf{w}^{\mathbf{b}},x_0^a,F\right)=\left(\frac{\mathrm{GRP}^{\mathrm{R}}\left(P,\mathbf{w}^{\mathbf{b}},x_0^a\right)-F}{x_0^a}\right). \tag{3.7}$$

Arvid Senhaji (2008a) derives the following six properties for the restricted NARP function:

1. $\mathrm{NARP}^{\mathrm{R}}(P, \mathbf{w}^{\mathbf{b}}, x_0^a, F) \geq (-F / x_0^a)$;

2. If $P_2 \geq P_1$, then $\mathrm{NARP}^{\mathrm{R}}(P_2, \mathbf{w}^{\mathbf{b}}, x_0^a, F) \geq \mathrm{NARP}^{\mathrm{R}}(P_1, \mathbf{w}^{\mathbf{b}}, x_0^a, F)$;

3. If $\mathbf{w_2}^{\mathbf{b}} \geq \mathbf{w_1}^{\mathbf{b}}$,[3] then $\mathrm{NARP}^{\mathrm{R}}(P, \mathbf{w_2}^{\mathbf{b}}, x_0^a, F) \leq \mathrm{NARP}^{\mathrm{R}}(P, \mathbf{w_1}^{\mathbf{b}}, x_0^a, F)$;

4. $\mathrm{NARP}^{\mathrm{R}}(P, \mathbf{w}^{\mathbf{b}}, x_0^a, F)$ is positively linearly homogeneous in the vector $(P\ \mathbf{w}^{\mathbf{b}}\ F)$;

5. $\mathrm{NARP}^{\mathrm{R}}(P, \mathbf{w}^{\mathbf{b}}, x_0^a, F)$ is convex and continuous[4] in the price vector $(P\ \mathbf{w}^{\mathbf{b}})$; and finally

6. If $\mathrm{GRP}^{\mathrm{R}}(P, \mathbf{w}^{\mathbf{b}}, x_0^a)$ is differentiable in the price vector $(P\ \mathbf{w}^{\mathbf{b}})$, $\mathrm{NARP}^{\mathrm{R}}(P, \mathbf{w}^{\mathbf{b}}, x_0^a, F)$ is also differentiable in these (B+1) strictly positive prices since the latter function is a positive linear transformation of the former. The gradient of the restricted NARP function in the price vector $(P\ \mathbf{w}^{\mathbf{b}})$ is equal to (*The Hotelling-McFadden lemma*):

$$\nabla_{\left(P,\mathbf{w}^{\mathbf{b}}\right)}\mathrm{NARP}^{\mathrm{R}}\left(P,\mathbf{w}^{\mathbf{b}},x_0^a,F\right)=\left(\left(\frac{\mathrm{S}^{\mathrm{R}}\left(P,\mathbf{w}^{\mathbf{b}},x_0^a\right)}{x_0^a}\right)\quad\cdots\quad-\left(\frac{\mathrm{D}_{\mathrm{B}}^{\mathrm{R}}\left(P,\mathbf{w}^{\mathbf{b}},x_0^a\right)}{x_0^a}\right)\right),\text{ and} \tag{3.8}$$

[3] Vector inequalities follow the subsequent convention throughout the paper: $\mathbf{w_2}^{\mathbf{b}} > \mathbf{w_1}^{\mathbf{b}}$ means that every element of $\mathbf{w_2}^{\mathbf{b}}$ is strictly greater than the corresponding element of $\mathbf{w_1}^{\mathbf{b}}$; $\mathbf{w_2}^{\mathbf{b}} \geq \mathbf{w_1}^{\mathbf{b}}$ means that every element of $\mathbf{w_2}^{\mathbf{b}}$ is at least as large as the corresponding element of $\mathbf{w_1}^{\mathbf{b}}$ and that at least one element of $\mathbf{w_2}^{\mathbf{b}}$ is strictly greater than the corresponding element in $\mathbf{w_1}^{\mathbf{b}}$.

[4] Furthermore, the restricted NRP- and NARP function are both convex and continuous in the extended vector $(P\ \mathbf{w}^{\mathbf{b}}\ F)$.

$$\left(\frac{\nabla_{(P,\mathbf{w^b})} \mathrm{NARP^R}(\cdot)}{-\frac{\partial \mathrm{NARP^R}(\cdot)}{\partial F}} \right) = \left(\mathrm{S^R}\left(P,\mathbf{w^b},x_0^a\right) \quad -\mathrm{D_1^R}\left(P,\mathbf{w^b},x_0^a\right) \ldots \quad -\mathrm{D_B^R}\left(P,\mathbf{w^b},x_0^a\right) \right) \Leftrightarrow \tag{3.9}$$

$$\left(\frac{\nabla_{(P,\mathbf{w^b})} \mathrm{NARP^R}\left(P,\mathbf{w^b},x_0^a,F\right)}{-\frac{\partial \mathrm{NARP^R}\left(P,\mathbf{w^b},x_0^a,F\right)}{\partial F}} \right) = \nabla_{(P,\mathbf{w^b})} \mathrm{NRP^R}\left(P,\mathbf{w^b},x_0^a\right) = \nabla_{(P,\mathbf{w^b})} \mathrm{GRP^R}\left(P,\mathbf{w^b},x_0^a\right). \tag{3.10}$$

$$\mathrm{NARP}\left(P,\mathbf{w^b},x_a^C,F\right) = \mathrm{NARP^R}\left(P,\mathbf{w^b},x_a^C,F\right) = \left(\frac{\mathrm{GRP^R}\left(P,\mathbf{w^b},x_a^C\right) - F}{x_a^C} \right). \tag{3.11}$$

On stage two the peak coordinator maximises the NARP function defined in expression (3.11) with regards to the raw agricultural quantity x_a^C:

$$\mathrm{NARP^U}\left(P,\mathbf{w^b},F\right) = \max_{x_a^C} \left\{ \mathrm{NARP}\left(P,\mathbf{w^b},x_a^C,F\right) \right\}. \tag{3.12}$$

The first- and second-order conditions for this maximisation problem read:

$$\left(\frac{\partial \mathrm{NARP}\left(P,\mathbf{w^b},x_a^C,F\right)}{\partial x_a^C} \right) = \left(\frac{\mathrm{NMRP}\left(P,\mathbf{w^b},x_a^C,K\right) - \mathrm{NARP}\left(P,\mathbf{w^b},x_a^C,F\right)}{x_a^C} \right) = 0, \text{ and} \tag{3.13}$$

$$\left(\frac{\partial^2 \mathrm{NARP}\left(P,\mathbf{w^b},x_a^C,F\right)}{\partial x_a^{C2}} \right) = \left(\frac{\partial \mathrm{NMRP}\left(P,\mathbf{w^b},x_a^C,K\right)}{\partial x_a^C} \right) \left(\frac{1}{x_a^C} \right) < 0. \tag{3.14}$$

The net marginal revenue product NMRP(P, $\mathbf{w^b}$, x_a^C, K) in expression (3.13) measures the increase in *GRP* and *NRP* from processing an additional unit of the raw agricultural product, and is identical to the raw agricultural shadow price:

$$\left(\frac{\partial \mathrm{GRP}\left(P,\mathbf{w^b},x_a^C\right)}{\partial x_a^C}\right)=\left(\frac{\partial \mathrm{NRP}\left(P,\mathbf{w^b},x_a^C,F\right)}{\partial x_a^C}\right)=\mathrm{NMRP}\left(P,\mathbf{w^b},x_a^C,K\right). \tag{3.15}$$

When the raw agricultural shadow price is declining in the aggregated raw agricultural quantity $x_a{}^C$, the second-order condition in expression (3.14) is fulfilled. The optimal demand of the raw agricultural product for the restricting processing cooperative is implicitly defined by the first-order condition in expression (3.13) as a function of final output price *P*, the processing production factor prices $\mathbf{w^b}$, and fixed cost *F*:

$$x_a^U = \mathrm{x}_\mathrm{a}^\mathrm{U}\left(P,\mathbf{w^b},F\right). \tag{3.16}$$

Notice the important fact that the processing cooperative's raw agricultural input function defined in expression (3.16) is not a function of the input prices in the vector $\mathbf{w^d}$. But this result is contingent on the fact that the aggregated raw agricultural supply schedule is sufficiently large to allow for the realisation of the maximum *NARP* in the first place. Thus, it is assumed here that the aggregated marginal cost function $\mathrm{MC_a}(\mathbf{w^d}, x_a{}^C; T_a)$ intersects with the NARP function to the right of, or at the latter function's apex, as illustrated in figure 1 above.

The properties of the unrestricted NARP function defined in expression (3.12) that enables me to recapture both the relative and the ordinary unrestricted NARP-maximising choice functions are now stated (*III*):

1. $\mathrm{NARP^U}(P, \mathbf{w^b}, F) \geq (-F / (\mathrm{x_a}^\mathrm{U}(P, \mathbf{w^b}, F)\,)$;

2. if $P_2 \geq P_1$, then $\mathrm{NARP^U}(P_2, \mathbf{w^b}, F) \geq \mathrm{NARP^U}(P_1, \mathbf{w^b}, F)$;

3. if $\mathbf{w_2^b} \geq \mathbf{w_1^b}$, then $\mathrm{NARP^U}(P, \mathbf{w_2^b}, F) \leq \mathrm{NARP^U}(P, \mathbf{w_1^b}, F)$;

4. $\mathrm{NARP^U}(P, \mathbf{w^b}, F)$ is positively linearly homogeneous in the vector $(P\ \mathbf{w^b}\ F)$;

5. $\mathrm{NARP^U}(P, \mathbf{w^b}, F)$ is convex and continuous in the extended vector $(P\ \mathbf{w^b}\ F)$; and finally

6. if $\mathrm{NARP^U}(P, \mathbf{w^b}, F)$ is differentiable in the extended vector $(P\ \mathbf{w^b}\ F)$, the gradient of $\mathrm{NARP^U}(P, \mathbf{w^b}, F)$ in $(P\ \mathbf{w^b})$ is identical to (*The Viner-Wong envelope theorem*):

$$\nabla_{(P,\mathbf{w^b})}\mathrm{NARP^U}\left(P,\mathbf{w^b},F\right)=\left(\left(\frac{\mathrm{S^U}\left(P,\mathbf{w^b},F\right)}{\mathrm{x_a^U}\left(P,\mathbf{w^b},F\right)}\right)\quad \ldots \quad -\left(\frac{\mathrm{D}_B^{\mathrm{U}}\left(P,\mathbf{w^b},F\right)}{\mathrm{x_a^U}\left(P,\mathbf{w^b},F\right)}\right)\right), \text{ and} \tag{3.17}$$

$$\left(\frac{\nabla_{(P,\mathbf{w^b})}\mathrm{NARP^U}\left(P,\mathbf{w^b},F\right)}{-\left(\dfrac{\partial \mathrm{NARP^U}\left(P,\mathbf{w^b},F\right)}{\partial F}\right)}\right)=\left(\mathrm{S^U}\left(P,\mathbf{w^b},F\right)\ \ldots -\mathrm{D_B^U}\left(P,\mathbf{w^b},F\right)\right). \tag{3.18}$$

Proofs of statements (*III*) are found in the appendix. The crux of the short-run neoclassical theory for a restricting processing cooperative firm is presented in the following formal theorem:

THEOREM 1: *The unambiguous qualitative comparative static results of a restricting processing cooperative confronting an upward sloping inverse aggregated raw agricultural supply function and whose goal is to maximise the net average revenue product, can be summarised in the statement that the matrix of cross-partial derivatives of the type*

$\left(\dfrac{\partial^2 \text{ NARP}^{\text{U}}\left(P, \mathbf{w^b}, F\right)}{\partial Z_i \partial Z_j}\right)$, *where* Z_i, Z_j *are any prices in the price vector* (P $\mathbf{w^b}$), *is symmetric and positive semidefinite.*

The symmetry of the matrix in theorem 1 follows from Young's theorem. This symmetry immediately yields the following reciprocity relations related to the optimal relative final output (y^U/x_a^U) identical to the inverse of the optimal conversion factor, and the B optimal relative processing input demands (x_j^{bU}/x_a^U), j = 1,…, B.

COROLLARY 1.1:

$$\left(\frac{\partial\left(\dfrac{y^U}{x_a^U}\right)}{\partial w_j^b}\right) = -\left(\frac{\partial\left(\dfrac{x_j^{bU}}{x_a^U}\right)}{\partial P}\right), \mathrm{j} = 1, ..., B, \tag{3.19}$$

$$\left(\frac{\partial\left(\dfrac{y^U}{x_a^U}\right)}{\partial F}\right) = -\left(\frac{\partial\left(\dfrac{1}{x_a^U}\right)}{\partial P}\right) = \left(\frac{\partial x_a^U}{\partial P}\right)\left(\frac{1}{x_a^{U\,2}}\right), \tag{3.20}$$

$$\left(\frac{\partial\left(\dfrac{x_j^{bU}}{x_a^U}\right)}{\partial F}\right) = \left(\frac{\partial\left(\dfrac{1}{x_a^U}\right)}{\partial w_j^b}\right) = -\left(\frac{\partial x_a^U}{\partial w_j^b}\right)\left(\frac{1}{x_a^{U\,2}}\right), j = 1, ..., B, \text{ and} \tag{3.21}$$

$$\left(\frac{\partial\left(\dfrac{x_i^{bU}}{x_a^U}\right)}{\partial w_j^b}\right) = \left(\frac{\partial\left(\dfrac{x_j^{bU}}{x_a^U}\right)}{\partial w_i^b}\right), \ \mathrm{i} \neq \mathrm{j},\ \mathrm{i,j} = 1, ..., B. \tag{3.22}$$

COROLLARY 1.2: *The unrestricted relative processing demand functions are downward sloping in their own price, i.e.*

$$\frac{\partial\left(\dfrac{\mathrm{D}_{\mathrm{j}}^{\mathrm{U}}\left(P,\mathbf{w}^{\mathbf{b}},F\right)}{\mathrm{x}_{\mathrm{a}}^{\mathrm{U}}\left(P,\mathbf{w}^{\mathbf{b}},F\right)}\right)}{\partial w_j^b} \leq 0,\ \mathrm{j}=1,\ldots,B. \tag{3.23}$$

PROOF:

The statement made in expression (3.23) is an immediate consequence of the unrestricted NARP function being convex in the price vector (P $\mathbf{w}^{\mathbf{b}}$). The diagonal entries in the symmetric matrix $[\mathbf{NARP}^{\mathbf{U}}{}_{\mathbf{ts}}]$**, t**,**s** = 1,…,$B$+1, are necessarily positive.

COROLLARY 1.3. *The own-price elasticity of a particular processing input demand function is less than (or equal to) the elasticity of the raw agricultural input with regards to the particular processing input price, i.e.*

$$\mathrm{EL}_{w_j^b}\,\mathrm{D}_{\mathrm{j}}^{\mathrm{U}}\left(P,\mathbf{w}^{\mathbf{b}},F\right) \leq \mathrm{EL}_{w_j^b}\,\mathrm{x}_{\mathrm{a}}^{\mathrm{U}}\left(P,\mathbf{w}^{\mathbf{b}},F\right),\ \mathrm{j}=1,\ldots,B. \tag{3.24}$$

PROOF:

This is an immediate consequence of corollary 1.2, and the quotient rule.

COROLLARY 1.4: *The unrestricted relative supply function is upward sloping in final output price P, i.e.*

$$\frac{\partial\left(\dfrac{\mathrm{S}^{\mathrm{U}}\left(P,\mathbf{w}^{\mathbf{b}},F\right)}{\mathrm{x}_{\mathrm{a}}^{\mathrm{U}}\left(P,\mathbf{w}^{\mathbf{b}},F\right)}\right)}{\partial P} \geq 0. \tag{3.25}$$

PROOF:

The statement made in expression (3.25) is another immediate consequence of the unrestricted NARP function being convex in the price vector (P $\mathbf{w^b}$).

COROLLARY 1.5: *The supply function is more (or equally) elastic than the raw agricultural input function with regards to the final output price P, i.e*

$$\mathrm{EL}_P\,\mathrm{S^U}\left(P,\mathbf{w^b},F\right) \geq \mathrm{EL}_P\,\mathrm{x_a^U}\left(P,\mathbf{w^b},F\right). \tag{3.26}$$

PROOF:

This result can be derived directly from corollary 1.4 using the quotient rule.

COROLLARY 1.6: *The unrestricted conversion factor is downward sloping in final output price P, i.e.*

$$\frac{\partial\left(\dfrac{\mathrm{x_a^U}(\cdot)}{\mathrm{S^U}(\cdot)}\right)}{\partial P} = \left(\frac{\mathrm{x_a^U}(\cdot)}{\mathrm{S^U}(\cdot)P}\right)\left(\left(\frac{\partial\,\mathrm{x_a^U}(\cdot)}{\partial P}\right)\left(\frac{P}{\mathrm{x_a^U}(\cdot)}\right)-\left(\frac{\partial\,\mathrm{S^U}(\cdot)}{\partial P}\right)\left(\frac{P}{\mathrm{S^U}(\cdot)}\right)\right) \leq 0. \tag{3.27}$$

PROOF:

This result is an immediate consequence of corollary 1.5.

COROLLARY 1.7: *The unrestricted raw agricultural demand function is upward sloping in fixed cost F, i.e.*

$$\left(\frac{\partial\,\mathrm{x_a^U}\left(P,\mathbf{w^b},F\right)}{\partial F}\right) \geq 0. \tag{3.28}$$

PROOF:

The statement made in expression (3.28) is another immediate consequence of the unrestricted NARP function being convex in the vector (P $\mathbf{w^b}$ F).

THEOREM 2: *The elasticity of the raw agricultural input function* $\mathrm{x_a}^{\mathrm{U}}(P, \mathbf{w^b}, F)$ *with regards to final output price P is positive if the elasticity of the NMRP with regards to the final output price P is greater than the share* α_y^U.

PROOF:

In optimum, we have that $\mathrm{NMRP}(P, \mathbf{w^b}, \mathrm{x_a}^{\mathrm{U}}(P, \mathbf{w^b}, F), K) = \mathrm{NARP}^{\mathrm{R}}(P, \mathbf{w^b}, \mathrm{x_a}^{\mathrm{U}}(P, \mathbf{w^b}, F), F)$. Differentiating this equality with regards to the final output price P yields:

$$\left(\frac{\partial\, \mathrm{x_a^U}\left(P, \mathbf{w^b}, F\right)}{\partial P}\right) = \left(\frac{\left(\frac{\partial\, \mathrm{NARP^R}(\cdot)}{\partial P}\right) - \left(\frac{\partial \mathrm{NMRP}(\cdot)}{\partial P}\right)}{\left(\frac{\partial \mathrm{NMRP}(\cdot)}{\partial x_0^a}\right)}\right) \Leftrightarrow \tag{3.29}$$

$$EL_P\, \mathrm{x_a^U}\left(P, \mathbf{w^b}, F\right) = \left(\frac{EL_P\ \mathrm{NMRP}\Big(P, \mathbf{w^b}, \mathrm{x_a^U}\left(P, \mathbf{w^b}, F\right), K\Big) - \alpha_y^U}{-EL_{x_a^C}\ \mathrm{NMRP}\Big(P, \mathbf{w^b}, \mathrm{x_a^U}\left(P, \mathbf{w^b}, F\right), K\Big)}\right), \tag{3.30}$$

where $\alpha_y^U = \left(\dfrac{P\ y}{NARP\ x_U^a}\right)$.

THEOREM 3: *When the elasticity of the raw agricultural input function* $x_a^U(P, \boldsymbol{w^b}, F)$ *is positive with regards to the final output price P, the supply function is upward sloping in P, i.e.*

$$\left(\frac{\partial\, \mathrm{S^U}\left(P, \mathbf{w^b}, F\right)}{\partial P}\right) \geq 0. \tag{3.31}$$

PROOF:

This result is an immediate consequence of corollary 1.5.

THEOREM 4: *The elasticity of the raw agricultural input function* x_a^U*(P,* $\mathbf{w^b}$*, F) with regards to the processing input price* w_j^b*, j = 1,…, B, is negative if the sum equal to the elasticity of the NMRP with regards to* w_j^b *plus the value share* α_j^U*, is negative.*

PROOF:

In optimum we have that NMRP(P, $\mathbf{w^b}$, $x_a^U(P, \mathbf{w^b}, F)$, K) = NARP^R(P, $\mathbf{w^b}$, $x_a^U(P, \mathbf{w^b}, F)$, F). The result in theorem 4 can be derived by differentiating this equality with regards to a particular processing input price w^b_j, j = 1,…, B, which yields

$$\left(\frac{\partial\, \mathrm{x}_{\mathrm{a}}^{\mathrm{U}}\left(P,\mathbf{w^b},F\right)}{\partial w_j^b}\right)=\left(\frac{\left(\frac{\partial\,\mathrm{NARP}^{\mathrm{R}}(\cdot)}{\partial w_j^b}\right)-\left(\frac{\partial \mathrm{NM\,RP}(\cdot)}{\partial w_j^b}\right)}{\left(\frac{\partial \mathrm{NM\,RP}(\cdot)}{\partial x_0^a}\right)}\right)\Leftrightarrow \tag{3.32}$$

$$EL_{w_j^b}\,\mathrm{x}_{\mathrm{a}}^{\mathrm{U}}\left(P,\mathbf{w^b},F\right)=\left(\frac{EL_{w_j^b}\,\mathrm{NMRP}\left(P,\mathbf{w^b},\mathrm{x}_{\mathrm{a}}^{\mathrm{U}}\left(P,\mathbf{w^b},F\right),K\right)+\alpha_j^U}{-EL_{x_a^C}\,\mathrm{NMRP}\left(P,\mathbf{w^b},\mathrm{x}_{\mathrm{a}}^{\mathrm{U}}\left(P,\mathbf{w^b},F\right),K\right)}\right), \tag{3.33}$$

where $\alpha_j^U=\left(\frac{w_j^b\ x_j^b}{NARP\ x_a^U}\right)$, $\mathrm{j}=1,...,B.$

THEOREM 5: *When the elasticity of the raw agricultural input function* x_a^U(P, $\mathbf{w^b}$, F) *is negative with regards to the processing factor price* w_j^b*, j = 1,…, B, the demand for processing production factor* x_j^b *is downward sloping in its own price, i.e.*

$$\left(\frac{\partial\,\mathrm{D}_{\mathrm{j}}^{\mathrm{U}}\left(P,\mathbf{w^b},F\right)}{\partial w_j^b}\right)\le 0,\ \mathrm{j = 1,..., B.} \tag{3.34}$$

PROOF:

This result is an immediate consequence of corollary 1.3.

THEOREM 6: *The elasticity of the unrestricted NARP function with regards to the final output price P, equals the value share* α_y^U .

PROOF:

Notice that:

$$\left(\frac{\partial\,\mathrm{NARP^U}(\cdot)}{\partial P}\right)\left(\frac{P}{\mathrm{NARP^U}(\cdot)}\right)=\left(\frac{\mathrm{S^U}\left(P,\mathbf{w^b},F\right)P}{\mathrm{x_a^U}\left(P,\mathbf{w^b},F\right)\mathrm{NARP^U}(\cdot)}\right)=\alpha_y^U. \tag{3.35}$$

THEOREM 7: *The elasticity of the unrestricted NARP function with regards to processing input price* w_j^b*, equals the negative of the value share* α_j^U , j = 1,..., *B.*

PROOF:

Notice that:

$$\left(\frac{\partial\,\mathrm{NARP^U}(\cdot)}{\partial w_j^b}\right)\left(\frac{w_j^b}{\mathrm{NARP^U}(\cdot)}\right)=-\left(\frac{\mathrm{D_j^U}\left(P,\mathbf{w^b},F\right)w_j^b}{\mathrm{x_a^U}\left(P,\mathbf{w^b},F\right)\mathrm{NARP^U}(\cdot)}\right)=-\alpha_j^U,\ \mathrm{j}=1,..,B. \tag{3.36}$$

THEOREM 8: *The elasticity of the unrestricted NARP function with regards to fixed cost F, equals the negative of the value share* α_F^U .

Proof:

Notice that:

$$\left(\frac{\partial \mathrm{NARP}^{\mathrm{U}}(\cdot)}{\partial F}\right)\left(\frac{F}{\mathrm{NARP}^{\mathrm{U}}(\cdot)}\right)=-\left(\frac{F}{\mathrm{x}_{\mathrm{a}}^{\mathrm{U}}\left(P,\mathbf{w}^{\mathbf{b}},F\right)\mathrm{NARP}^{\mathrm{U}}(\cdot)}\right)=-\alpha_F^U. \tag{3.37}$$

After having derived eight theorems pertaining to a processing cooperative with a peak coordinator able to restrict members' supplies of the raw agricultural quantity, I now turn my focus towards the adjustment of an unrestricting processing cooperative.

4. The adjustment of an unrestricting processing cooperative

The decisive characteristic of an unrestricting processing cooperative is that the peak coordinator is unable to control members' supplies of the raw agricultural product x_a^C. Without loss of generality, I once again divide the optimisation problem into two separate stages. Let the first stage be equal to the primary stage analysed in chapter 3 where the peak coordinator maximises *GRP* for a given raw agricultural quantity x_0^a.

On stage two the peak coordinator of the unrestricting processing cooperative collects and processes the equilibrium raw agricultural quantity identical to the quantity where the NARP function intersects with the inverse of the aggregated raw agricultural supply schedule as depicted in figure 1 above:

$$\mathrm{NARP}^{\mathrm{R}}\left(P,\mathbf{w}^{\mathbf{b}},x_0^a,F\right)=\mathrm{MC}_{\mathrm{a}}\left(\mathbf{w}^{\mathbf{d}},x_0^a;T_a\right). \tag{4.1}$$

Stability requires that the inverse of the aggregated raw agricultural supply function must cut the NARP function from below, implying that:

$$\left(\frac{\partial \mathrm{MC}_{\mathrm{a}}\left(\mathbf{w}^{\mathbf{d}},x_0^a;T_a\right)}{\partial x_a^C}-\frac{\partial \mathrm{NARP}^{\mathrm{R}}\left(P,\mathbf{w}^{\mathbf{b}},x_0^a,F\right)}{\partial x_a^C}\right)=\beta>0. \tag{4.2}$$

As stated in Senhaji (2008), the equilibrium raw agricultural quantity x_0^a is defined implicitly by the equality in expression (4.1) as a function of the output price P, the processing production factor prices in the vector $\mathbf{w^b}$, the farming input prices in the vector $\mathbf{w^d}$, fixed cost F, and the agricultural policy variable T_a:

$$x_0^a = \mathrm{x}_0^{\mathrm{a}}\left(P, \mathbf{w^b}, \mathbf{w^d}, F, T_a\right). \tag{4.3}$$

Based on the equality in expression (4.1) and the stability condition in expression (4.2), four theorems related to the equilibrium raw agricultural schedule defined in expression (4.3) are forthcoming:

THEOREM 9: *The equilibrium raw agricultural quantity is upward sloping in the final output price P.*

Proof:

A positive change in the output price P will shift the NARP function depicted in figure 1 upwards, and the equilibrium raw agricultural quantity x_0^a increases:

$$\left(\frac{\partial\, \mathrm{x}_0^{\mathrm{a}}\left(P, \mathbf{w^b}, \mathbf{w^d}, F, T_a\right)}{\partial P}\right) = \left(\frac{\left(\frac{\partial \mathrm{NARP}^{\mathrm{R}}(\cdot)}{\partial P}\right)}{\beta}\right) = \left(\frac{\left(\frac{\mathrm{S}^{\mathrm{R}}\left(P, \mathbf{w^b}, \mathrm{x}_0^{\mathrm{a}}(\cdot)\right)}{\mathrm{x}_0^{\mathrm{a}}(\cdot)}\right)}{\beta}\right) \geq 0. \tag{4.4}$$

THEOREM 10: *The equilibrium raw agricultural quantity is downward sloping in the processing production factor price w_j^b, j=1,…, B.*

Proof:

A positive change in a processing production factor price w_j^b will shift the NARP function depicted in figure 1 downwards, leading to a reduction in the equilibrium raw agricultural quantity x_0^a:

$$\left(\frac{\partial \mathrm{x}_0^{\mathrm{a}}\left(P,\mathbf{w^b},\mathbf{w^d},F,T_a\right)}{\partial w_j^b}\right)=\left(\frac{\left(\frac{\partial \mathrm{NARP}^{\mathrm{R}}(\cdot)}{\partial w_j^b}\right)}{\beta}\right)=-\left(\frac{\left(\frac{\mathrm{D}_{\mathrm{j}}^{\mathrm{R}}\left(P,\mathbf{w^b},\mathrm{x}_0^{\mathrm{a}}(\cdot)\right)}{\mathrm{x}_0^{\mathrm{a}}(\cdot)}\right)}{\beta}\right)\leq 0,\ \mathrm{j}=1,\ldots,B. \qquad (4.5)$$

THEOREM 11: *The equilibrium raw agricultural quantity is downward sloping in fixed cost F.*

Proof:

If the cooperative members decide to raise the collectively determined rent on productive capital *K*, fixed cost *F* also increases. The NARP function in figure 1 shifts downwards and the equilibrium raw agricultural quantity x_0^a decreases:

$$\left(\frac{\partial \mathrm{x}_0^{\mathrm{a}}\left(P,\mathbf{w^b},\mathbf{w^d},F,T_a\right)}{\partial F}\right)=\left(\frac{\left(\frac{\partial \mathrm{NARP}^{\mathrm{R}}(\cdot)}{\partial F}\right)}{\beta}\right)=-\left(\frac{\left(\frac{1}{\mathrm{x}_0^{\mathrm{a}}\left(P,\mathbf{w^b},\mathbf{w^d},F,T_a\right)}\right)}{\beta}\right)<0. \qquad (4.6)$$

THEOREM 12: *If the production factor x_j^d, j=1,…, D, is a normal production factor*[5] *in agriculture, the equilibrium raw agricultural quantity is downward sloping in the input price w_j^d, j=1,…, D.*

[5] The farming production factor x_j^d is a normal input factor when the aggregated marginal cost function of the cooperative farmers denoted by $\mathrm{MC_a}(\mathbf{w^d}, x_a^C; T_a)$, is increasing in the input price w_j^d.

Proof:

Let $C_a(\mathbf{w^d}, x_a{}^C; T_a)$ denote the aggregated cost function of the cooperative farmers. If the production factor $x_j{}^d$, $j = 1,\ldots, D$, is a normal input factor in agriculture, we have that:

$$\left(\frac{\partial^2 \mathrm{C_a}\left(\mathbf{w^d}, x_a^C; T_a\right)}{\partial x_a^C \partial w_j^d}\right) = \left(\frac{\partial \mathrm{MC_a}\left(\mathbf{w^d}, x_a^C; T_a\right)}{\partial w_j^d}\right) > 0,\ \mathrm{j} = 1,...,D. \tag{4.7}$$

Thus, an increase in the price of a normal production factor $x_j{}^d$, j = 1,…, D, in agriculture, will shift the inverse aggregated raw agricultural supply schedule depicted in figure 1 upwards, leading to a reduction in the equilibrium raw agricultural quantity $x_0{}^a$:

$$\left(\frac{\partial \mathrm{x_0^a}(\cdot)}{\partial w_j^d}\right) = -\left(\frac{\left(\dfrac{\partial^2 \mathrm{C_a}\left(\mathbf{w^d}, x_a^C; T_a\right)}{\partial w_j^d \partial x_a^C}\right)}{\left(\dfrac{\partial \mathrm{MC_a}(\cdot)}{\partial x_a^C} - \dfrac{\partial \mathrm{NARP^R}(\cdot)}{\partial x_a^C}\right)}\right) = -\left(\frac{\left(\dfrac{\partial \mathrm{MC_a}\left(\mathbf{w^d}, x_a^C; T_a\right)}{\partial w_j^d}\right)}{\beta}\right) < 0,\ \mathrm{j} = 1,...,D. \tag{4.8}$$

$$\mathrm{NARP^E}\left(P, \mathbf{w^b}, \mathbf{w^d}, F, T_a\right) = \mathrm{NARP^R}\left(P, \mathbf{w^b}, \mathrm{x}_0^a(\cdot), F\right). \tag{4.9}$$

Unless the inverse of the aggregated raw agricultural supply schedule intersects with the NARP function at the latter function's apex, the derivate of the stable-equilibrium NARP function $\mathrm{NARP^E}(P, \mathbf{w^b}, \mathbf{w^d}, F, T_a)$ defined in expression (4.9), with regards to the raw agricultural quantity will not be equal to zero. Accordingly, in the equilibrium pertaining to the unrestricting processing cooperative we generally have that:

$$\left(\frac{\partial \mathrm{NARP^E}\left(P, \mathbf{w^b}, \mathbf{w^d}, F, T_a\right)}{\partial P}\right) = \left(\frac{\mathrm{S^R}\left(P, \mathbf{w^b}, \mathrm{x_0^a}(\cdot)\right)}{\mathrm{x_0^a}(\cdot)}\right) + \left(\frac{\partial \mathrm{NARP^R}(\cdot)}{\partial x_a^C}\right)\left(\frac{\partial \mathrm{x_0^a}(\cdot)}{\partial P}\right) \Leftrightarrow \tag{4.10}$$

$$\left(\frac{\partial\,\mathrm{NARP}^{\mathrm{E}}\left(P,\mathbf{w^b},\mathbf{w^d},F,T_a\right)}{\partial P}\right)\neq\left(\frac{\mathrm{S}^{\mathrm{R}}\left(P,\mathbf{w^b},\mathrm{x}_0^{\mathrm{a}}(\cdot)\right)}{\mathrm{x}_0^{\mathrm{a}}(\cdot)}\right). \tag{4.11}$$

Likewise, differentiating the stable-equilibrium NARP function with regards to a processing production factor price w_j^b yields:

$$\left(\frac{\partial\,\mathrm{NARP}^{\mathrm{E}}\left(P,\mathbf{w^b},\mathbf{w^d},F,T_a\right)}{\partial w_j^b}\right)=-\left(\frac{\mathrm{D}_{\mathrm{j}}^{\mathrm{R}}\left(P,\mathbf{w^b},\mathrm{x}_0^{\mathrm{a}}(\cdot)\right)}{\mathrm{x}_0^{\mathrm{a}}(\cdot)}\right)+\left(\frac{\partial\,\mathrm{NARP}^{\mathrm{R}}(\cdot)}{\partial x_a^C}\right)\left(\frac{\partial\,\mathrm{x}_0^{\mathrm{a}}(\cdot)}{\partial w_j^b}\right)\Leftrightarrow \tag{4.12}$$

$$\left(\frac{\partial\,\mathrm{NARP}^{\mathrm{E}}\left(P,\mathbf{w^b},\mathbf{w^d},F,T_a\right)}{\partial w_j^b}\right)\neq-\left(\frac{\mathrm{D}_{\mathrm{j}}^{\mathrm{R}}\left(P,\mathbf{w^b},\mathrm{x}_0^{\mathrm{a}}(\cdot)\right)}{\mathrm{x}_0^{\mathrm{a}}(\cdot)}\right),\ j=1,\ldots,B. \tag{4.13}$$

From expressions (4.12) and (4.13) it is clear that unless the inverse aggregated raw agricultural supply schedule is either vertical or intersects with the NARP function at the latter function's maximum, the relative short-run choice functions of an unrestricting processing cooperative cannot be retrieved through the gradient of the stable-equilibrium NARP function in the price vector (P $\mathbf{w^b}$).

Just like the restricted NARP function defined in expression (3.7) and the unrestricted NARP function defined in expression (3.12), the stable-equilibrium NARP function in expression (4.9) is nondecreasing in output price P, nonincreasing in processing production factor prices $\mathbf{w^b}$, nonincreasing in fixed cost F, and nonincreasing in the input prices w_j^D, j = 1,…, D, of normal farming production factors. But convexity, continuity, and positive linear homogeneity in the extended vector (P $\mathbf{w^b}$ F) cannot be proven for the stable-equilibrium NARP function defined in expression (4.9). Thus, unambiguous qualitative comparative static results for the relative choice functions of an unrestricting processing cooperative are not forthcoming on the basis of the stable-equilibrium NARP function when the inverse aggregated raw agricultural supply function exhibits a general positive slope.

5. Conclusion

In this article I have shown that the gradient of the unrestricted NARP function in the price vector (P $\mathbf{w^b}$) contains the unrestricted relative *NARP*-maximising choice functions of a processing cooperative with a peak coordinator that is able to restrict members' supplies of the raw agricultural product. Unambiguous qualitative comparative static results for the relative choice functions are derived on the basis of the unrestricted NARP function proven to be convex in the vector (P $\mathbf{w^b}$ F). The large amount of economic information embedded in the curvature properties of the unrestricted NARP function has not been stated explicitly in the economic cooperative theory, and resembles the curvature properties of the profit per worker function related to Labour-Managed firms analysed exhaustively by Hugh Neary (1988), Nava Kahana (1989), and Elmar Wolfstetter (1992).

When the aggregated raw agricultural supply schedule is not completely vertical, but instead exhibits a general positive slope, unambiguous qualitative comparative static results for the relative choice functions of an unrestricting processing cooperative are not forthcoming on the basis of the stable-equilibrium NARP function. Most European agribusiness processing cooperatives are unrestricting in the sense that the daily management cannot control members' supplies of the raw agricultural product. Therefore, the lack of unambiguous qualitative comparative static results for the relative choice functions of unrestricting processing cooperatives poses a great challenge. LeVay (1983) describes other plausible objectives that the daily management of a processing cooperative may pursue on behalf of its members. Unambiguous qualitative comparative static results similar to those derived for the restricting processing cooperative are derived in Senhaji (2008b) for these objectives in the short term as well as in the long run. There is a close resemblance between the plausible adjustments of processing cooperatives on the one hand, and the possible adjustments pertaining to supplying- and purchasing cooperatives on the other. Unambiguous qualitative comparative static results for the suggested adjustments pertaining to supplying- and purchasing cooperatives that are described in LeVay (1983), can be derived on the basis of the average purchasing cost function as illustrated in Senhaji (2008c).

Appendix

Proofs of statements (*III*) are presented in this appendix. The first statement III.1 reflects how the restricting processing cooperative contributes its maximum effort in order to meet fixed cost. The peak coordinator takes the cooperative firm out of business once the variable processing costs cannot be met so that *ARP* turns negative.

Statement III.2 says that the unrestricted NARP function is nondecreasing in the final output price P. When $P_2 \geq P_1$, we have that

$$\mathrm{NARP}^{\mathrm{U}}\left(P_2,\mathbf{w^b},F\right)=\left(P_2\left(\frac{\mathrm{S}^{\mathrm{U}}\left(P_2,\mathbf{w^b},F\right)}{\mathrm{x}_{\mathrm{a}}^{\mathrm{U}}\left(P_2,\mathbf{w^b},F\right)}\right)-\sum_{j=1}^{B}w_j^b\left(\frac{\mathrm{D}_j^{\mathrm{U}}\left(P_2,\mathbf{w^b},F\right)}{\mathrm{x}_{\mathrm{a}}^{\mathrm{U}}\left(P_2,\mathbf{w^b},F\right)}\right)-\frac{F}{\mathrm{x}_{\mathrm{a}}^{\mathrm{U}}\left(P_2,\mathbf{w^b},F\right)}\right)\geq$$

$$\left(P_2\left(\frac{\mathrm{S}^{\mathrm{U}}\left(P_1,\mathbf{w^b},F\right)}{\mathrm{x}_{\mathrm{a}}^{\mathrm{U}}\left(P_1,\mathbf{w^b},F\right)}\right)-\sum_{j=1}^{B}w_j^b\left(\frac{\mathrm{D}_j^{\mathrm{U}}\left(P_1,\mathbf{w^b},F\right)}{\mathrm{x}_{\mathrm{a}}^{\mathrm{U}}\left(P_1,\mathbf{w^b},F\right)}\right)-\frac{F}{\mathrm{x}_{\mathrm{a}}^{\mathrm{U}}\left(P_1,\mathbf{w^b},F\right)}\right)\geq \tag{A.1}$$

$$\left(P_1\left(\frac{\mathrm{S}^{\mathrm{U}}\left(P_1,\mathbf{w^b},F\right)}{\mathrm{x}_{\mathrm{a}}^{\mathrm{U}}\left(P_1,\mathbf{w^b},F\right)}\right)-\sum_{j=1}^{B}w_j^b\left(\frac{\mathrm{D}_j^{\mathrm{U}}\left(P_1,\mathbf{w^b}\right)}{\mathrm{x}_{\mathrm{a}}^{\mathrm{U}}\left(P_1,\mathbf{w^b},F\right)}\right)-\frac{F}{\mathrm{x}_{\mathrm{a}}^{\mathrm{U}}\left(P_1,\mathbf{w^b},F\right)}\right)=\mathrm{NARP}^{\mathrm{U}}\left(P_1,\mathbf{w^b},F\right)\Leftrightarrow$$

$$\mathrm{NARP}^{\mathrm{U}}\left(P_2,\mathbf{w^b},F\right)\geq \mathrm{NARP}^{\mathrm{U}}\left(P_1,\mathbf{w^b},F\right)\Leftrightarrow \tag{A.2}$$

$$\mathrm{NARP}^{\mathrm{U}}\left(P_2,\mathbf{w^b},F\right)-\mathrm{NARP}^{\mathrm{U}}\left(P_1,\mathbf{w^b},F\right)=\Delta NARP^{U} \;\geq\; 0. \tag{A.3}$$

Statement III.3 says that the unrestricted NARP function is nonincreasing in any of the *B* processing input prices. When $\mathbf{w_2}^{\mathbf{b}} \geq \mathbf{w_1}^{\mathbf{b}}$, we have that

$$\mathrm{NARP^U}\left(P,\mathbf{w}_2^{\mathbf{b}},F\right)=\left(P\left(\frac{\mathrm{S^U}\left(P,\mathbf{w}_2^{\mathbf{b}},F\right)}{\mathrm{x_a^U}\left(P,\mathbf{w}_2^{\mathbf{b}},F\right)}\right)-\sum_{j=1}^{B}w_{2j}^{b}\left(\frac{\mathrm{D}_j^{\mathrm{U}}\left(P,\mathbf{w}_2^{\mathbf{b}},F\right)}{\mathrm{x_a^U}\left(P,\mathbf{w}_2^{\mathbf{b}},F\right)}\right)-\frac{F}{\mathrm{x_a^U}\left(P,\mathbf{w}_2^{\mathbf{b}},F\right)}\right)\leq$$

$$\left(P\left(\frac{\mathrm{S^U}\left(P,\mathbf{w}_2^{\mathbf{b}},F\right)}{\mathrm{x_a^U}\left(P,\mathbf{w}_2^{\mathbf{b}},F\right)}\right)-\sum_{j=1}^{B}w_{1j}^{b}\left(\frac{\mathrm{D}_j^{\mathrm{U}}\left(P,\mathbf{w}_2^{\mathbf{b}},F\right)}{\mathrm{x_a^U}\left(P,\mathbf{w}_2^{\mathbf{b}},F\right)}\right)-\frac{F}{\mathrm{x_a^U}\left(P,\mathbf{w}_2^{\mathbf{b}},F\right)}\right)\leq$$

$$\left(P\left(\frac{\mathrm{S^U}\left(P,\mathbf{w}_1^{\mathbf{b}},F\right)}{\mathrm{x_a^U}\left(P,\mathbf{w}_1^{\mathbf{b}},F\right)}\right)-\sum_{j=1}^{B}w_{1j}^{b}\left(\frac{\mathrm{D}_j^{\mathrm{U}}\left(P,\mathbf{w}_1^{\mathbf{b}},F\right)}{\mathrm{x_a^U}\left(P,\mathbf{w}_1^{\mathbf{b}},F\right)}\right)-\frac{F}{\mathrm{x_a^U}\left(P,\mathbf{w}_1^{\mathbf{b}},F\right)}\right)=\mathrm{NARP^U}\left(P,\mathbf{w}_1^{\mathbf{b}},F\right)\Leftrightarrow$$

(A.4)

$$\mathrm{NARP^U}\left(P,\mathbf{w}_2^{\mathbf{b}},F\right)\leq\mathrm{NARP^U}\left(P,\mathbf{w}_1^{\mathbf{b}},F\right)\Leftrightarrow \tag{A.5}$$

$$\mathrm{NARP^U}\left(P,\mathbf{w}_2^{\mathbf{b}},F\right)-\mathrm{NARP^U}\left(P,\mathbf{w}_1^{\mathbf{b}},F\right)=\Delta\,\mathrm{NARP^U}\leq 0. \tag{A.6}$$

Statement III.4 says that the unrestricted NARP function is positively linearly homogeneous in the vector (P $\mathbf{w}^{\mathbf{b}}$ F). Let the superscript «T» denote the transpose operator. Notice that

$$\mathrm{NARP^U}\left(tP,t\mathbf{w}^{\mathbf{b}},tF\right)=\max_{y,\mathbf{x}^{\mathbf{b}},x_a^C}\left\{\frac{tPy-t\mathbf{w}^{\mathbf{b}\mathrm{T}}\mathbf{x}^{\mathbf{b}}-tF}{x_a^C}\mid y\leq \mathrm{f}\left(\mathbf{x}^{\mathbf{b}},x_a^C\right)\right\}=$$

(A.7)

$$t\max_{y,\mathbf{x}^{\mathbf{b}},x_a^C}\left\{\frac{Py-\mathbf{w}^{\mathbf{b}\mathrm{T}}\mathbf{x}^{\mathbf{b}}-F}{x_a^C}\mid y\leq \mathrm{f}\left(\mathbf{x}^{\mathbf{b}},x_a^C\right)\right\}=t\,\mathrm{NARP^U}\left(P,\mathbf{w}^{\mathbf{b}},F\right).$$

In order to prove statement III.5 saying that the unrestricted NARP function is convex in the price vector (P $\mathbf{w}^{\mathbf{b}}$), we first introduce three different price vectors $\mathbf{P_1}$, $\mathbf{P_2}$, and $\mathbf{P_3}$ containing (B+1) strictly positive final output- and processing input factor prices. The three corresponding optimal vectors of relative supplies equal to [$\mathrm{S^U}(\mathbf{P_i}, F)/\mathrm{x_a^U}(\mathbf{P_i}, F)$] and relative processing input demands [$\mathrm{D_j^U}(\mathbf{P_i}, F)$ / $\mathrm{x_a^U}(\mathbf{P_i}, F)$] with negative signs, j = 1,…,B, and i =

1,…,3, are given by the relative quantity vectors $\mathbf{y_1}^*$, $\mathbf{y_2}^*$, and $\mathbf{y_3}^*$. Let α ε (0,1), and define $\mathbf{P_3}$ as the weighted average of the former two price vectors: $\mathbf{P_3} = \alpha\, \mathbf{P_1} + (1\text{-}\alpha)\, \mathbf{P_2}$. Again the superscript «T» denotes the transpose operator. Based on these definitions, I prove that $\mathrm{NARP}^{\mathrm{U}}(P, \mathbf{w^b}, F)$ is convex in $(P\ \mathbf{w^b})$ by stating first of all that

$$\mathrm{NARP}^{\mathrm{U}}\left(\mathbf{P_1},F\right)=\left(\mathbf{P}_1^{\mathrm{T}}\mathbf{y}_1^*-\frac{F}{\mathrm{x}_{\mathrm{a}}^{\mathrm{U}}\left(\mathbf{P_1},F\right)}\right)\geq\left(\mathbf{P}_1^{\mathrm{T}}\mathbf{y}_3^*-\frac{F}{\mathrm{x}_{\mathrm{a}}^{\mathrm{U}}\left(\mathbf{P}_3,F\right)}\right),\text{ and} \tag{A.8}$$

$$\mathrm{NARP}^{\mathrm{U}}\left(\mathbf{P_2},F\right)=\left(\mathbf{P}_2^{\mathrm{T}}\mathbf{y}_2^*-\frac{F}{\mathrm{x}_{\mathrm{a}}^{\mathrm{U}}\left(\mathbf{P}_2,F\right)}\right)\geq\left(\mathbf{P}_2^{\mathrm{T}}\mathbf{y}_3^*-\frac{F}{\mathrm{x}_{\mathrm{a}}^{\mathrm{U}}\left(\mathbf{P}_3,F\right)}\right). \tag{A.9}$$

Multiply both sides of inequalities (A.8) and (A.9) by α and (1- α) respectively, and add the two appearing inequalities in order to get

$$\alpha\,\mathrm{NARP}^{\mathrm{U}}\left(\mathbf{P}_1,F\right)+\left(1-\alpha\right)\mathrm{NARP}^{\mathrm{U}}\left(\mathbf{P}_2,F\right)\geq\left(\alpha\left(\mathbf{P}_1^{\mathrm{T}}\mathbf{y}_\mathbf{3}^*\right)+\left(1-\alpha\right)\left(\mathbf{P}_2^{\mathrm{T}}\mathbf{y}_3^*\right)-\frac{F}{\mathrm{x}_{\mathrm{a}}^{\mathrm{U}}\left(\mathbf{P_3},F\right)}\right)=$$
$$\left(\left(\alpha\mathbf{P}_1^{\mathrm{T}}+\left(1-\alpha\right)\mathbf{P}_2^{\mathrm{T}}\right)\mathbf{y}_\mathbf{3}^*-\frac{F}{\mathrm{x}_{\mathrm{a}}^{\mathrm{U}}\left(\mathbf{P_3},F\right)}\right)=\left(\mathbf{P}_3^{\mathrm{T}}\mathbf{y}_3^*-\frac{F}{\mathrm{x}_{\mathrm{a}}^{\mathrm{U}}\left(\mathbf{P_3},F\right)}\right)\equiv\mathrm{NARP}^{\mathrm{U}}\left(\mathbf{P_3},F\right)\Leftrightarrow \tag{A.10}$$

$$\alpha\,\mathrm{NARP}^{\mathrm{U}}\left(\mathbf{P}_1,F\right)+\left(1-\alpha\right)\mathrm{NARP}^{\mathrm{U}}\left(\mathbf{P}_2,F\right)\geq\mathrm{NARP}^{\mathrm{U}}\left(\alpha\mathbf{P_1}+\left(1-\alpha\right)\mathbf{P}_2,F\right) \tag{A.11}$$

From expression (A.11) we see that the unrestricted NARP function is convex in the price vector $(P\ \mathbf{w^b})$. A similar procedure can be undertaken in order to prove that the unrestricted NARP function is convex in the extended vector $(P\ \mathbf{w^b}\ F)$. From the convexity property of the unrestricted NARP function in the extended vector $(P\ \mathbf{w^b}\ F)$, it follows that the unrestricted NARP function is also continuous in $(P\ \mathbf{w^b}\ F)$.

Finally, statement III.6 follows directly from the Viner-Wong envelope theorem. One way of proving this statement is to construct Silberberg's (1972) primal dual function for the present unrestricted *NARP*-maximising model that is identical to

$$z\left(P,\mathbf{w^b},\mathbf{x^b},x_a^C,F\right)=\mathrm{NARP}^{\mathrm{U}}\left(P,\mathbf{w^b},F\right)-\mathrm{NARP}\left(P,\mathbf{w^b},\mathbf{x^b},x_a^C,F\right). \tag{A.12}$$

The primal-dual function in equation (A.12) is defined as a function of all the prices and quantities that appear in the present *NARP*-model. The first function on the right side of expression (A.12) is the indirect objective function derived above that calculates the maximum *NARP* achievable for a given extended vector (P $\mathbf{w^b}$ F). The second NARP function on the right side of expression (A.12) computes any other *NARP*-value for a non-optimal quantity vector. Notice that the latter function is linear in all variables in the extended vector (P $\mathbf{w^b}$ F). When the latter NARP function is evaluated at the optimal quantity vector for a given extended vector, both functions on the right side of expression (A.12) compute the same maximum *NARP*-value, and the primal-dual function attains its global minimum value of zero. The first-order conditions for the primal-dual model in expression (A.12) are

$$\left(\frac{\partial\,\mathrm{z}\left(P,\mathbf{w^b},\mathbf{x^b},x_a^C,F\right)}{\partial x_j^b}\right)=-\left(\frac{\partial\,\mathrm{NARP}(\cdot)}{\partial x_j^b}\right)=-\frac{\left(P\,\mathrm{f}^{'}_{x_j^b}(\cdot)-w_j^b\right)}{x_a^C}=0,\ \mathrm{j}=1,\ldots,B. \tag{A.13}$$

$$\left(\frac{\partial\,\mathrm{z}(\cdot)}{\partial x_a^C}\right)=-\left(\frac{\partial\,\mathrm{NARP}(\cdot)}{\partial x_a^C}\right)=-\frac{\left(P\,\mathrm{f}^{'}_{x_a^C}(\cdot)-\mathrm{NARP}(\cdot)\right)}{x_a^C}=-\left(\frac{\mathrm{NMRP}(\cdot)-\mathrm{NARP}(\cdot)}{x_a^C}\right)=0\,. \tag{A.14}$$

$$\left(\frac{\partial\,\mathrm{z}\left(P,\mathbf{w^b},\mathbf{x^b},x_a^C,F\right)}{\partial w_j^b}\right)=\left(\frac{\partial\,\mathrm{NARP}^{\mathrm{U}}(\cdot)}{\partial w_j^b}\right)-\left(\frac{\partial\,\mathrm{NARP}(\cdot)}{\partial w_j^b}\right)=0,\ \mathrm{j}=1,\ldots,B,\Leftrightarrow \tag{A.15}$$

$$\left(\frac{\partial\,\mathrm{NARP}^{\mathrm{U}}\left(P,\mathbf{w^b},F\right)}{\partial w_j^b}\right)=-\left(\frac{x_j^{bU}}{x_a^U}\right)=-\left(\frac{\mathrm{D}_j^{\mathrm{U}}\left(P,\mathbf{w^b},F\right)}{\mathrm{x}_{\mathrm{a}}^{\mathrm{U}}\left(P,\mathbf{w^b},F\right)}\right),\ \mathrm{j}=1,\ldots,B. \tag{A.16}$$

$$\left(\frac{\partial\,\mathrm{z}\left(P,\mathbf{w^b},\mathbf{x^b},x_a^C,F\right)}{\partial P}\right)=\left(\frac{\partial\,\mathrm{NARP}^{\mathrm{U}}(\cdot)}{\partial P}\right)-\left(\frac{\partial\,\mathrm{NARP}(\cdot)}{\partial P}\right)=0\Leftrightarrow \tag{A.17}$$

$$\left(\frac{\partial\,\mathrm{NARP}^{\mathrm{U}}\left(P,\mathbf{w^b},F\right)}{\partial P}\right)=\left(\frac{y}{x_a^U}\right)=\left(\frac{\mathrm{S}^{\mathrm{U}}\left(P,\mathbf{w^b},F\right)}{\mathrm{x}_{\mathrm{a}}^{\mathrm{U}}\left(P,\mathbf{w^b},F\right)}\right). \tag{A.18}$$

$$\left(\frac{\partial\,\mathrm{z}\left(P,\mathbf{w^b},\mathbf{x^b},x_a^C,F\right)}{\partial F}\right)=\left(\frac{\partial\,\mathrm{NARP}^{\mathrm{U}}(\cdot)}{\partial F}\right)-\left(\frac{\partial\,\mathrm{NARP}(\cdot)}{\partial F}\right)=0\Leftrightarrow \tag{A.19}$$

$$\left(\frac{\partial\,\mathrm{NARP}^{\mathrm{U}}(\cdot)}{\partial F}\right)=-\left(\frac{1}{x_a^U}\right)=\left(-\frac{1}{\mathrm{x}_{\mathrm{a}}^{\mathrm{U}}\left(P,\mathbf{w^b},F\right)}\right) \tag{A.20}$$

Expressions (A.16)-(A.20) are restatements of the Viner-Wong envelope theorem. The first-order conditions in expressions (A.13) and (A.14) secure that the general NARP function is evaluated at the optimal relative quantity vector [(y^*/x_a^{U*}),…, (x_B^{b*}/x_a^{U*})]. Above, I concluded that the primal-dual function attains the global minimum value of zero when the general NARP function is evaluated at the optimal quantity vector for a given price vector (P $\mathbf{w^b}$). The production factor vector $\mathbf{x}$ is equal to ($\mathbf{x^b}$, x_a^C). It follows that the Hessian matrix for the primal-dual model is identical to

$$\mathbf{H}^{\text{P-D}}_{\substack{(2B+2)x \\ (2B+2)}} = \begin{pmatrix} z''_{PP} & \left[\mathbf{z}''_{P\mathbf{w^b}}\right] & \left[\mathbf{z}''_{P\mathbf{x}}\right] \\ \left[\mathbf{z}''_{\mathbf{w^b}P}\right] & \left[\mathbf{z}''_{\mathbf{w^b}\mathbf{w^b}}\right] & \left[\mathbf{z}''_{\mathbf{w^b}\mathbf{x}}\right] \\ \left[\mathbf{z}''_{\mathbf{x}P}\right] & \left[\mathbf{z}''_{\mathbf{x}\mathbf{w^b}}\right] & \left[\mathbf{z}''_{\mathbf{xx}}\right] \end{pmatrix} \Leftrightarrow \tag{A.21}$$

$$\mathbf{H}^{\text{P-D}}_{\substack{(2B+2)x \\ (2B+2)}} = \begin{pmatrix} NARP^{U}_{PP} & \left[\text{NARP}^{\text{U}}_{P\,\mathbf{w^b}}\right] & \left[-\text{NARP}_{P\,\mathbf{x}}\right] \\ \left[\text{NARP}^{\text{U}}_{\mathbf{w^b}P}\right] & \left[\text{NARP}^{\text{U}}_{\mathbf{w^b}\mathbf{w^b}}\right] & \left[-\text{NARP}_{\mathbf{w^b}\mathbf{x}}\right] \\ \left[-\text{NARP}_{\mathbf{x}\,P}\right] & \left[-\text{NARP}_{\mathbf{x}\,\mathbf{w^b}}\right] & \left[-\text{NARP}_{\mathbf{x}\,\mathbf{x}}\right] \end{pmatrix}. \tag{A.22}$$

The second-order conditions for the primal-dual model require that the Hessian matrix in expression (A.22) be positive semidefinite. In order for that to be the case, all the three matrices on the diagonal must necessarily also be positive semidefinite. Starting in the bottom right corner we find the first matrix on the diagonal given by $\left[-\text{NARP}_{\mathbf{xx}}\right]$. This matrix is positive semidefinite if the production function is locally strictly concave in the input vector **x** in equilibrium. I assume that to be the case here. The second matrix on the diagonal given by $\left[\mathbf{z}''_{\mathbf{w^b}\mathbf{w^b}}\right]$ is identical to $\left[\mathbf{NARP}^{\text{U}}_{\mathbf{w^b}\mathbf{w^b}}\right]$ since all prices enter linearly into the general NARP function in equation (A.12). This matrix contains the second-order derivatives of the unrestricted NARP function with regards to the processing input price vector $\mathbf{w^b}$. This matrix is positive semidefinite since the unrestricted NARP function is proven above to be convex in the processing input price vector $\mathbf{w^b}$. Finally, we find a positive scalar in the top-left corner of the Hessian matrix that equals the second-order derivative of the unrestricted NARP function with regards to the final output price *P*.

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