Industrial structure and the macroeconomy

modelling a macroeconomic system with oligopoly, endogenous market structure and heterogeneous social groups

Marco Mazzoli, Matteo Morini and Pietro Terna[1]

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ABSTRACT. This work introduces a new theoretical macroeconomic framework for an oligopolistic economy with heterogeneous agents, social mobility

[1]Marco Mazzoli, Department of Economics - DIEC - Genoa Unviersity, and MoFiR. Correspondence address: Department of Economics - DIEC, Università di Genova, Via Vivaldi 5 - 16126 Genova - Italy; e-mail: mazzoli@economia.unige.it.

Matteo Morini, Dipartimento di Scienze Economico-Sociali e Matematico-Statistiche, Scuola di Management ed Economia - Università di Torino, Corso Unione Sovietica n. 218 bis, 10134 Torino; e-mail: matteo.morini@unito.it.

Pietro Terna, Dipartimento di Scienze Economico-Sociali e Matematico-Statistiche, Scuola di Management ed Economia - Università di Torino, Corso Unione Sovietica n. 218 bis, 10134 Torino; e-mail: pietro.terna@unito.it. Although the paper is the product of the collaboratio of the three authors, Matteo Morini and Pietro Terna have written section 6, containing the information on the the numerical simulations (that may be performed at the link displayed in the pape, also written by Matteo Morini and Pietro Terna) and the graphics associated to the paper and Marco Mazzoli has written the remaining sections and the appendices of the paper.

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and wage rigidity, to generate macroeconomic fluctuations. Social mobility is modelled by micofounding the behaviour of consumers and oligopolistic firms in accordance with Aoki and Yoshikawa (2007) approach and by assuming that the agents, in spite of having the same preference function (modelled with a conventional CRRA utility function), are heterogeneous in their budget constraint. This allows to explicitly model conflict and social mobility: we assume that agents may change their social status in each period according to a stochastic process interacting with the labour market and the entry/exit process. The empirical part of this paper introduces some numerical simulations built on an "agent based" approach, where the business cycle is generated by the assumptions of the model. This theoretical framework may be employed for further research focused on entry/exit, business cycle and social mobility with heterogeneous agents.

Keywords: Cycles, Aggregate Income Distribution, Industrial Organization and Macroeconomics.

JEL Classification: E32, E25, L16,

1 Introduction

The birth and death of firms are observable phenomena related to the business cycle. The nature of this relation is complex and involves the simultaneous decisions of several kinds of agents. The purpose of this work is to introduce a new theoretical macroeconomic framework for an oligopolistic economy with heterogeneous agents and social mobility where the macroeconomic fluctuations can be determined not only by technology shocks, but also by the process of entry/exit of oligopolistic firms, potentially interacting with distributional shocks.

The standard New-Keynesian framework for monetary policy analysis departs from the Real Business Cycle literature by introducing monopolistic competition à la Dixit and Stiglitz (1977), where market structure was exogenous and the firms, producing differentiated goods, were modelled as a continuum in the space [0,1].[1] In this way each of the firms are assumed to be infinitesimal and this implies that entry/exit of such infinitesimal elements, cannot affect the production capacity. The monetary policy can only affect the "output gap", i.e. the gap between the actual price equilibrium and the benchmark case of flexible prices, although, in principle, price rigidity and price behavior should also be related to the market structure. Etro (2009) raises the issue of endogenously modelling market structure in macroeconomics, international economics, growth and economic policy, while Etro and Colciago (2010) introduce a complete model of business cycle with differentiated goods, endogenous market structure at the sectorial level, full employment, and different industrial sectors, where two separate benchmark cases of price (Bertrand) competition and quantity (Cournot) competition are extensively analyzed. They show that with no product differentiation and with a unique homogeneous good, mark ups only survive in the case

[1] See, for instance, the seminal works by Mc Callum and Nelson (1999), Galì (2002) and Walsh (2003), ch. 5.

of quantity (Cournot) competition, while they vanish in the case of price (Bertrand) competition, which degenerates into a conventional real business cycle model. In their model the interaction between business cycle and market structure goes as follows: an exogenous technology shock affects output and consumption, increases profits and, as a consequence, triggers entry. They do not explicitly refer to oligopoly (the word "oligopoly" never actually appears in their paper), and introduce instead a more general framework of "imperfect competition", that may include several sub-cases according to the pricing mechanism and/or to the value of the elasticity of substitution among commodities. In that context, the assumption of full employment and intrasectoral competition, in an economy whose production capacity and potential output is still only driven by technology shocks, amplifies the stochastic technology shocks, which generate changes in the firms' markups and profits and, only as a consequence, entry/exit and market structure endogeneity. Although Etro and Colciago provide an appealing explanation for a number of empirical stylized facts, such as countercyclical mark ups and pro-cyclical business creation, they do not discuss whether and how can the economic system move between the benckmark cases of Cournot and Betrand equilibria.

The model introduced here is simpler and analyzes an oligopolistic economy with an homogeneous good, where each individual agent cannot be worker and entrepreneur at the same time. Introducing in a macromodel the assumption of quantity (Cournot) competition might raise the problem of how are prices determined without referring to an auctioneer or, equivalently, what prevents the firms from implementing price undercutting and price competition. A possible way to deal with these problems is assuming a quantity precommitment à la Kreps and Scheinkman (1983), whose results can be extended under fairly general conditions (see in this regard Allen and Hellwig, 1986, Madden, 1998).

Introducing oligopoly and entry/exit in a macromodel with business cycle obviously means attaching a certain relevance to the role of information shocks and demand expectations in agents' decisions, which is, of course the object of extensive research. For instance, Lorenzoni (2009) introduces a model of business cycles driven by shocks to consumer expectations regarding aggregate productivity and where agents are hit by heterogeneous productivity shocks: they observe their own productivity and a noisy public signal regarding aggregate productivity and this "noise shocks," mimicks the features of aggregate demand shocks. News shocks (together with other shocks) are the focus of Jaimovich and Rebelo (2009) model, that generates both aggregate and sectoral comovement in response to both contemporaneous shocks and news shocks about fundamentals.

This model introduced here is not a standard DSGE, in the sense that it does not follows the modelling assumptions commonly employed in the DSGE literature, but it still is a microfounded, dynamic, stochastic and general equilibrium model and is meant to introduce an alternative modelling approach. The microfoundation is based on a peculiar notion of representative agent, which requires a few comments.

A potential source of problems in the conventional use of the representative agent lies in the aggregation of heterogeneous agents, as already pointed out many years ago by Forni and Lippi (1997), who show that many statistical features associated to the dynamic structure of a model (like Granger causality and cointegration), when derived from the micro theory do not, in general, survive aggregation. This means that the parameters of a macromodel do not usually bear a simple relationship to the corresponding parameters of the micromodel. Of course, this kind of problem cannot be solved without explicitly formalizing a statistical aggregation process and individuals' externalities and a first purpose of this model is to introduce a preliminary and simplified form of aggregation of heterogeneous agents with potential conflicts and externalities.

Another criticism to the representative-agent methodology was raised in an earlier contribition by Blinder (1986), who pointed out that microfounded models with a representative agent, by assuming that the observable choices of optimizing individuals are "internal solutions" may yield biased econometric estimates when the choices of a relevant portion of individuals are actually corner solutions: "For many goods, the primary reason for a downward sloping market demand curve may be that more people drop out of the market as the price rises, not that each individual consumer reduces his purchases" (Blinder, 1986, p. 76).

Finally, a last point raised here, is associated to the interpretation of the representative agent utility function: Logically speaking, what does the representative agent utility function represent? If we look at it with the criteria of "hard sciences" can it really be interpreted as a proper microfoundation of a macroeconomic system composed by a high number of heterogeneous individuals without formalizing any statistical law of aggregation that accounts for externalities and agents' rational interactions? Is it not instead a sort of "aggregate utility function", and if so, is it not rather a macroeconomic function? In other words, if the utility function of the representative agent is metaphorically meant to model all the consumers of an economy, is it not subject to the Lucas critique? Why can we not explicitly model agents' (rational) interactions by means of some statistical principles of aggregation? In this regard, Aoki and Yoshikawa (2007, p. 28) point out that "the standard approach in 'micro-founded' macroeconomics formulates complicated intertemporal optimization problems facing the representative agent. By so doing, it ignores interactions among nonidentical agents. Also, it does not examine a class of problems in which several types of agents simultaneously attempt to solve similar but slightly different optimization problems with slightly different sets of constraints. When these sets of constraints are not consistent, no truly optimal solution exists". Furthermore, for what concerns the role of microfoundation, "Roughly speaking, we de-emphasize the role of precise optimization of an individual unit while emphasizing the importance of proper aggregation for understanding the behavior of the macroeconomy. The experiences in disciplines outside economics such as physics, population genetics and combinatorial stochastic processes that deal with a large number of interacting entities amply demonstrate that details of specification of optimizing agents (units) frequently diminish as the number of agents become very large. Only certain key features of parameters such as correlations among agents matter in determining aggregate behavior". (Aoki and Yoshikawa (2007, pp.28-29).

Of course, one may object that even an “aggregate utility function” still allows to build the aggregate behavior on some rigorous, logical and consistent axiom of preference. Therefore, in this paper, the utility function of the representative agent, which is the basis for the derivation of the aggregate demand, will be employed as the basis for the microfoundation of the aggregate demand. However it is interpreted as an aggregate object and its budget constraint contains a principle of aggregation of heterogeneous agents, interactions, conflicts and externalities. In this sense, this paper follows Aoki and Yoshikawa approach, for what concerns some modelling tools employed to formalize the entry process of new firms and the presence of heterogeneous individuals with different (and sometimes conflicting) targets, but still builds the aggregate demand on a utility function and on a set of consistent axioms of preference and optimizing behavior.

All individuals can hold financial assets, but, in our simplifying formalization of the financial sector, the activity of "investing in shares", is rendered by the decision to undertake the (time consuming) monitoring activity on the firm’s decisions, which is equivalent to "being an entrepreneur", a "full time job" absorbing all the available time to an individual. Any other financial investments (including the investment in ”external finance”, as opposed to investing in the control of the firm), do not involve taking part into the firm’s decision process and may only be remunerated at the market rate. The entrepreneur controls the firm in the sense that she fully controls the cash flow allocation. All the individuals have a chance to become entrepreneurs, but for at least one period they are fully committed to their job. The stylized fact captured by this assumption is that, on the one hand, a prevailing activity exists for each individual, on the other hand (since we want to avoid the skizofrenic assumption that each individual is, at the same time, worker and entrepreneur and negotiates with herself the wage) workers and entrepreneurs might have diverging incentives and conflicts: In this way, a form of unconventional heterogeneity (in budget constraints and sources of income) is introduced among agents with the same preferences. The status of worker, entrepreneur or unemployed may stochastically change in each period, generating in this way distributional shocks on the aggregate demand. Entry/exit is associated to the transition process from being a worker to being entrepreneur or unemployed. In particular, entry/exit, by modifying the number of firms, affects the production capacity and may potentially interacts with monetary policy.

Finally, a last important detail that characterizes this model as a general equilibrium model is that the wage setting rule, the labor market equilibrium and the entry/exit decisions interact, since the workers are perceived by the incumbent firms as potential entrepreneurs and potential entrants. In other words, while explicitly formalizing the interactions between endogenous market structure and the macroeconomy, we assume that the workers may become entrepreneurs (or unemployed) and entrepreneurs may become unemployed in the future, at some entry or exit cost. Entry is obviously not simply a matter of substituting the firms which leave the market, although the interdependence between the sectorial rates of entry and exit is a well established empirical fact in the applied research on industry dynamics, as shown recently, among others by Manjón-Antolín, 2010, who investigates some empirical features of such interdependence. The issue of interaction between market structure and entry/exit decisions lead by the agents' expectation is not an exclusive concern of large industries and large firms. For instance, Dunne *et al.* (2009) empirically analyze the short run and long run dynamics of an oligopolistic sector and the role of entry costs and toughness of short-run price competition, by using micro data for the U.S. dentists and chiropractors industries, certainly not two sectors characterized by giant firms. The next sections describe respectively the consumers and aggregate demand, the labour market and firms, the interpretation and implications of the Cournot equilimrium, a summary of the more relevant equations and some very preliminary simulations associating entry/exit, social dynamics and income distribution to the business cycle.

2 Consumers, aggregate output and aggregate demand

The consumers choose their optimal consumption path by allocating their income and financial assets over time. We assume, for the sake of simplicity, that financial assets are risk free and include Government bonds and deposits (i.e. an aggregate approximately corresponding to M3), that deposits are remunerated and that the interest rate on risk free Government bonds is equal to the interest rate on deposits, since they are both assumed to be risk free financial assets. The monetary policy consists of interest rate setting. Exogenous changes in the money stock will not be considered in our analysis, but they could easily be formalized in this model as changes in the nominal amount of risk free financial assets, since M3 is a function of the money base. In the rest of the model we refer to the variable "A" that includes Government bonds and M3 as the generic "financial asset". The financial asset is risk free, since, for the sake of simplicity, in this model, the decision of investing in risky assets is equivalent to the decision of becoming and entrepreneur, i.e. allocating human capital into "being a new entrant": the suppliers of "external finance", as opposed to the individuals holding the control of the firm, do not take part into the firm's decision process.

We also assume that the deposits are issued by a perfectly competitive aggregate financial and banking sector, which perfectly diversifies its lending risk to industrial firms, so that we only consider a generic interest rate r, exogenously controlled by the policy makers.

Agents' heterogeneity might not change as regularly and predictably as age. In other words, qualitative differences in budget constraints can be a potential source of heterogeneity and a configuration with one huge firm and many workers might be different in many regards (and not just for commodity pricing) from a configuration with many ologopolists competing in many dimensions. Agents have the same utility function, while their main source of earnings may be given either by wages, or profits or transferals from the public sector to the unemployed individuals. These transferals are, in aggregate terms and, for the sake of simplicity, are assumed to be proportional to the income of the employed individuals (i.e. profits or wages).

We can think of the banking sector as an institution that instantaneously performs all the transactions among individuals, with no specific need for cash, provided that all budget constrains are satisfied. These transactions are assumed to be proportional to the aggregate income of all the individuals, who pay a commission on them, say ς. The cost of banks' intermediation is exogenous and assumed to be equal to the income of the perfectly competitive banking sector. The entrepreneurs can be incumbent, earning at time $t+i$ the incumbent profits π^{in}_{t+i} or new entrants, earning the new entrant profits π^{e}_{t+i} which, in general, diverge from π^{in}_{t+i} since the new entrants have to support the entry costs as shown in the further sections. The entrepreneurs hire the workers, pay them the wages ω_t for the period going from t to $t+1$. They pay themselves the same wage ω_t, and receive the residual profits, so that the remuneration for the entrepreneurial activity is given by ω_t plus π^{in}_{t+i} if the entrepreneur is an incumbent or ω_t plus π^{e}_{t+i} if she is a new entrant. When $\pi^{in}_{t+i} < 0$ and $\pi^{e}_{t+i} < 0$, respectively, the incumbent and the new entrant go bankrupt (which happens with a probability to be specified later), the entrepreneur and the workers become unemployed and, until they are hired again by a new firm, they receive the unemployment subsidy. τ is a tax, assumed to be proportional to the income, for the sake of simplicity, $(n_{t+i}\omega_{t+i} + n_{t+i}h^{in}_{t+i}\pi^{in}_{t+i} + n_{t+i}h^{e}_{t+i}\pi^{e}_{t+i})\tau$ is the transferal to unemployed at time $t+i$, $(n_{t+i}\omega_{t+i} + n_{t+i}h^{in}_{t+i}\pi^{in}_{t+i} + n_{t+i}h^{e}_{t+i}\pi^{e}_{t+i})\varsigma$ the transaction fees to the banking and financial system at time $t+i$. We assume that the magnitudes of τ and ς are constant and very small compared to the other variables. The behavior of the banking and financial system is not in the focus of this paper. The assumption made on the banking sector amounts to a "ceteris paribus" assumption. The banking system (not explicitly modelled here) is perfectly competitive and earns its revenues only from the transaction fees $(n_{t+i}\omega_{t+i} + n_{t+i}h^{in}_{t+i}\pi^{in}_{t+i} + n_{t+i}h^{e}_{t+i}\pi^{e}_{t+i})\varsigma$. These transaction fees to the banking and financial system also include the insurance fees to cover the bank risk for unpaid loans, in case the borrowing firms go bankrupt. The central bank handles the risk associated to unexpected shocks by acting as a lender of last resort to the whole banking system.

Being entrepreneur requires some skill that is acquired after working at least one period and is lost by not working and being unemployed for at least one period. Being a worker, on the other hand, does not requires any particular skill. Obviously, on the basis of our assumptions, we may simplify out the effects of tax transferrals and bank commissions and formalize the aggregate demand as follows

$$\begin{aligned} Q_{t+i} &= n_{t+i}(\omega_{t+i} + h^{e}_{t+i}\pi^{e}_{t+i} + h^{in}_{t+i}\pi^{in}_{t+i})(1-\tau-\varsigma) + \\ &\quad +(n_{t+i}\omega_{t+i} + n_{t+i}h^{in}_{t+i}\pi^{in}_{t+i} + n_{t+i}h^{e}_{t+i}\pi^{e}_{t+i})\tau + \\ &\quad +(n_{t+i}\omega_{t+i} + n_{t+i}h^{in}_{t+i}\pi^{in}_{t+i} + n_{t+i}h^{e}_{t+i}\pi^{e}_{t+i})\varsigma \qquad (1) \\ Q_{t+i} &= n_{t+i}(\omega_{t+i} + h^{e}_{t+i}\pi^{e}_{t+i} + h^{in}_{t+i}\pi^{in}_{t+i}) \end{aligned}$$

where Q_{t+i} is the aggregate nominal income at time "$t+i$", ω_{t+i} is the wage per worker before taxes, assuming that the labour contract is such that each worker receives the wage ω_{t+i} for the period going from "$t+i$" to "$t+i+1$", for a fixed amount of hours of labour, n_{t+i} the number of employed individuals at time t+i, h^{in}_{t+i}, (with $0 < h^{in}_{t+i} < 1$) the portion of incumbent entrepreneurs at time t+i, h^{e}_{t+i} (with $0 < h^{e}_{t+i} < 1$) the portion of new entrants at time t+i (with $h_{t+i} = h^{in}_{t+i} + h^{e}_{t+i}$ and $0 < h_{t+i} < 1$), therefore, the portion of workers over the total employed labour force is given by

$\beta = 1 - h^{in}_{t+i} - h^{e}_{t+i}$ [2].

We are now enabled to define the problem of the representative consumer with his budget constraint, while keeping at the same time in the model a specific notion of agents' heterogeneity.

Apart from the methodological interpretation of microfondation described in the introductory section, we consider a CRRA utility function, within a very simple and conventional intertemporal optimization problem (like, for instance, in Bagliano and Bertola, 2004). In addition, we assume here that the agents may rationally formulate commonly shared expectations on the relevant future variables of the model, although this detail will be better specified later. The aggregate intertemporal consumers' preferences at time t allow to define the problem as follows:

[2] If we want to express the the income in per capita terms, let us define:

$\xi = n/l$;

dividing both sides of the last row of the equation by l, we may also express :

$$q_{t+i} = \xi_{t+i}(\omega_{t+i} + \pi^{in}_{t+i}h^{in}_{t+i} + \pi^{e}_{t+i}h^{e}_{t+i})$$

If l is the number of individuals composing the labour force, then the per capita transferal to unemployed individuals is lower during recessions, when the income is lower and there are less firms and less employed workers.

$$\max U_t = E_t\left[\sum_{i=0}^{\infty}\left(\frac{1}{1+\rho}\right)^i u(C_{t+i})\right] \quad (2)$$

$$C_{t+i}, i = 0, ...\infty \quad (3)$$

for each $i = 0, 1, ...\infty$

subject to the following constraint:

$$E_t(a_{t+i+1}) = (1 + r_{t+i})E_{t+i-1}(a_{t+i}) + E_{t+i-1}(Q_{t+i}) - E_{t+i-1}(C_{t+i}) \quad (4)$$

and

$$C_{t+i} \geq 0$$

at every time $t + i$ from $i = 0, ..., \infty$

Where C_{t+i} are the real aggregate consumptions at a generical time $t + i$, ρ is the subjective rate of intertemporal preference for the consumers, a_{t+i} represents the financial assets in real terms on which the consumer can invest its wealth at time "$t + i$", Q_{t+i} is the real income at time "$t + i$" and r is the real interest rate on the financial asset and controlled by the central bank. Given interpretation of the utility function as an aggregate object, reflecting the preferences of an aggregation of consumers (and not a specific individual consumer), the constraint 4 reflects the accumulation process for the aggregation of heterogeneous consumers. In other words, at time "$t + i + 1$", the amount of wealth a_{t+i+1} will be held by some investor, no matter who is she and no matter from whom she bought it or whether she held it since the beginning as initial wealth.

The financial asset in which the agents may invest their wealth do not include shares: in this simplified model, investing in shares is a time consuming activity and implies undertaking the monitoring and organizational activity of being an entrepreneur.

The budget constraint 4 also holds for $i = 0, ..., \infty$. the transversality condition is the following

$$\lim_{j \longrightarrow \infty} a_{t+j} \left(\frac{1}{1 + r_{t+j}} \right)^j \geq 0 \tag{5}$$

If the marginal utility of consumption is always positive (i.e. not in the case of a quadratic utility function, but, for instance in a CRRA utility function) the above transversality condition 5 is always satisfied in terms of equality.

The financial wealth a_t and the human capital H_t are assumed to be valued at the beginning of period t, while $W_t = (1+r_t)(a_t+H_t)$ represents the overall wealth, which is valued at the end of period t, but before consumption C_t, that absorbs part of the available resources. We also assume that both profits and wages are paid at the end of the period, when consumption takes place. The human wealth valued at the beginning of time t is the following:

$$H_t = \frac{1}{(1 + r_t)} \sum_{i=0}^{\infty} \left(\frac{1}{1 + E_t(r_{t+i})} \right)^i E_t(Q_{t+i}) \tag{6}$$

and, as above

$$W_t = (1 + r_t)(a_t + H_t) \tag{7}$$

hence

$$E_t(W_{t+1}) = (1 + r_t) \left[E_t(a_{t+1}) + \frac{1}{(1 + r_t)} \sum_{i=0}^{\infty} \left(\frac{1}{1 + E_t(r_{t+i})} \right)^i E_t(Q_{t+1+i}) \right] \tag{8}$$

Substituting in 8 for the definition of a_{t+1} we get:

$$\begin{aligned} E(W_{t+1}) &= (1+r_t)\left[(1+r_t)a_t + Q_t - C_t + \right. \\ &\left. + \frac{1}{(1+r_t)} \sum_{i=0}^{\infty} \left(\frac{1}{1+E_t(r_{t+i})} \right)^i E_t(Q_{t+1+i}) \right] \end{aligned}$$

Hence

$$\begin{aligned} E(W_{t+1}) &= [(1+r_t)(a_t + H_t) - C_t] \\ &= (1+r_t)(W_t - C_t) \end{aligned}$$

and, generalizing

$$E(W_{t+i+1}) = (1 + r_{_{t+i}})(W_{t+i} - C_{t+i})$$

Where W_{t+i+1} is the state variable.

Let us assume now that the instantaneous utility be represented by the following function:

$$u_t = \frac{C_t^{1-\gamma}}{1-\gamma} \tag{9}$$

with $0 < \gamma < 1$

Therefore the consumer problem boils down into the following Bellman and Euler equations respectively:

$$V(W_t) = \max_{c_t} \left[\frac{C_t^{1-\gamma}}{1-\gamma} + \left(\frac{1}{1+\rho} \right) E(V(W_{t+1})) \right] \tag{10}$$

$$u'(C_t) = \frac{1+r_t}{1+\rho} E_t u'(C_{t+1}) \tag{11}$$

Applying the standard dynamic programming techniques yields the following consumption function (see appendix 1 for the algebraic details), which, in our model without fixed capital, also represents the aggregate expenditure:

$$D_t(r_t, W_t) = \left[1 - (1 + r_t)^{\frac{1-\gamma}{\gamma}} (1 + \rho)^{-\frac{1}{\gamma}}\right] (a_t + H_t) \tag{12}$$

We can rearrange the aggregate demand $D_t(\cdot)$ in order to account for income distribution and expected future variables. Let us start by defining

$$\Xi = \left[1 - (1 + r_t)^{\frac{1-\gamma}{\gamma}} (1 + \rho)^{-\frac{1}{\gamma}}\right]$$

Since ρ is constant and $0 < \gamma < 1$, then $\partial C(\cdot)/\partial r_t < 0$

Let ι_t be the inflation rate. Let us assume that (with no unexpected shocks) the best predictor for future inflation is the current inflation. Then the link between price level and inflation is $E(P_{t+n}) = P_t(1 + E(\iota_t))^n$.

Then, by multiplying and dividing by $E(P_{t+i})$ each term in the sum and by substituting $E(P_{t+i}) = P_t(1 + E(\iota_t))^i$ in the denominator, we get:

$$\begin{aligned}(a_t + H_t) &= \left(a_t + \frac{1}{1 + r_t} \sum_{i=0}^{\infty} \left(\frac{1}{(1 + E(r_{t+i}))}\right)^i E(Q_{t+i})\right) \\ &= \left(\frac{A_t}{P_t} + \frac{1}{P_t} \frac{1}{1 + r_t} \sum_{i=0}^{\infty} \left(\frac{1}{(1 + E(r_{t+i}))(1 + E(\iota_{t+i}))}\right)^i E(Y_{t+i})\right)\end{aligned}$$

where $E(Y_{t+i}) = E(P_{t+i})E(Q_{t+i})$

We are now enabled to define the aggregate demand:

$$D_t(\cdot) = \frac{\Xi(r_t)}{P_t} \left(A_t + \frac{1}{1 + r_t} \sum_{i=0}^{\infty} \left(\frac{1}{(1 + E(r_{t+i}))(1 + E(\iota_{t+i}))}\right)^i E(Y_{t+i})\right) \tag{13}$$

or, if we want to explicitly formalize income distribution

$$
\begin{aligned}
D_t(\cdot) &= (\Xi(r_t)/P_t)\cdot \\
&\cdot\{A_t + [1/(1+r_t)]\sum_{i=0}^{\infty}[(1+E\,(r_{t+i}))(1+E(\iota_{t+i}))]^{-i}\cdot \\
&\cdot E(P_{t+i})\cdot E(n_{t+i}(\omega_{t+i}+h^{e}_{t+i}\pi^{e}_{t+i}+h^{in}_{t+i}\pi^{in}_{t+i}))\}
\end{aligned} \tag{14}
$$

This is the aggregate demand, derived from the intertemporal optimization of the preferences of the individuals with infinite horizon. This function allows for agents' heterogeneity, distributional shocks, entry/exit shocks, expectational shocks and expected changes in the future monetary policy and in its timing. With no information shocks, we assume that the present expected value of each variable is the best predictor for its future income.

The monetary policy, by modifying r in its present and expected value, generates an overall effect on $D(\cdot)$, while the behavior of the expected inflation is more complex and depends on the way monetary policy interacts with the market structure.

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3 Labour market and firms

First of all, we assume that when prices are null, there is no economic activity, no production and, as a consequence the market revenue function of each individual firm $\varkappa(P_t, \varphi_t)$ is null. All the firms are identical, use the same production technology to produce the same generic good in regime of oligopoly. Having defined $\varphi_{i,t}$ as the individual output produced by firm i at time t, the production technology of each individual firm i is described by a Cobb-Douglas production function with labour only:

$$
\varphi_{i,t} = \lambda_{i,t}\Lambda L^{\alpha}_{i,t} \tag{15}
$$

Where L_t is the (nonnegative and discrete) number of workers employed in the firm at time t for the period from t to $t+1$, Λ is the usual parameter describing the state of technology, λ_t is a random variable capturing the technology shocks distributed as a normal $N(1,\sigma_\lambda)$ with constant variance σ_λ. With no technology shocks, λ_t is just a constant. We assume an *ex ante* labour contract establishing a fixed number of hours to be worked. Therefore, hiring a worker implies hiring a fixed number of working hours for the firm. We also assume that starting a new firm involves entrepreneurial and organizational skills and exogenous sunk costs of entry F. The model can be easily extended by including capital, but for the sake of simplicity we assume that only labour is employed in the production process.

Entry/exit, by increasing/reducing the number of existing firms, affects the production capacity. The workers that remain unemployed for at least one period loose their skill to become entrepreneurs, but they can still become workers at no cost if they are hired by a firm. Only those who have been workers for at least one period have the skill to become entrepreneurs. At any generic point t in time, a portion $(h_t^e + h_t^{in})$ of the employed labour force n_t is composed by entrepreneurs (where h_t^{in} represents the incumbents and h_t^e the new entrants). In this oligopolistic economy there are exogenous fixed costs of entry F, which can be though of as organizational costs and setting up costs. Setting up the organization of the firm is assumed to be time consuming. Therefore, to enter the market at time t, the new entrant has to bear these costs at time $t-1$, raise an amount F of financial funds and repay the amount $(1+r_{t-1})F$ between time t and $t+1$. In other words, the potential entrants decide at time $t-1$ wether to be in or out in the next period and decide wether or not to bet the amount of money $(1+r_{t-1})F$ to get the expected future income of a new entrant (weighted with the probability of survival). Once the new entrants have entered the market, at time t, they enter a two stage à la Kreps and Scheinkman (1983) (with the extended assumptions introduced by Madden, 1998). Both stages conceptually take place at time t. The new entrants discount the espected bankruptcy probability at the moment when they decide to enter, at time $t-1$ for time t. Bygones are bygone, and once the new entrants are in the market, they play their game and compete like any other oligopolistic firm: they have assessed and accounted for the failure risk while deciding to enter the market and, at time t they just play their game like anyone else.

Differently from Etro and Colciago (2010), the entry costs are not given by the stock price of a generic incumbent firm: they are exogenous instead and consist of all the costs that need to be supported in order to start the economic activity and make the new firm known to the consumers, who would otherwise buy the good only from the existing and known firms[3].

Since the workers may decide to become entrepreneurs, the wage and entry decisions are related and both depend on an incentive compatibility constraint: the labour market does not necessarily clear, due to a particular kind of wage rigidity, explained below. With unemployment, the firms are wage setters and with full employment the workers are wage setters. the wages applying for the next period (from time t to time $t+1$) are set between time $t-1$ and time t. This wage setting process involves the existing firms at time $t-1$ and the employed workers and takes place before the potential entrants decide whether or not to enter the market. The oligopolistic entrepreneurs have incentive to keep the wages low, but not so low to trigger entry. Individuals are subject to idiosyncratic informational shocks. Without these shocks, all the individuals would have the same expectations and if the wages were set at a level where the expected profits for new entrants (weighted with the expected probability of survival) is lower than the expected future wages,

[3]The existence of different social groups at a generical initial time t_0 can be thought of as being determined by a random initial distribution of wealth, allowing a subset $h_{in,0}$ of the labour force l at the initial time *0*, to cover,once for all, the initial exogenous sunk costs, at the initial instant of the whole economic process.

then nobody would enter the market. On the other hand, if the wages were set so low that their expected future value would be lower than the expected future value of the profits of an entrant weighted with the probability of bankruptcy, then the workers would prefer to bear the risk of entering the market as entrepreneurs, large scale entry would take place until profits vanish out and the entrepreneurs would only earn the wage they pay themselves and get zero profits. This means that when the entrepreneurs are wage setters (i.e. when there is unemployment), the wages correspond to a level that discourage entry and entry is only due to idiosyncratic informational shocks in the workers' expectations. We call the wage that, ex ante, does not trigger entry, the "incentive compatible wage" and denote it ω^*.

Let us first consider the case where the firms are wage setters and there is unemployment. Between time $t-1$ and time t the firms designate a common representative, who publically announce the incentive compatible wage ω^*.[4]

We assume that the unemployment subsidy τ has a small magnitude compared to the other parameters (and only exists to allow the survival of

[4] We can imagine that no incumbent firm has incentive to deviate from this "publically announced" incentive compatible wage by making the following assumption: since there is rivalry among the oligopolistic firms, the publically announced wage is a form of coordination. If there were no coordination, each firm would have incentive to "steal" the workers to the rivals by offering them a marginally higher wage. Therefore, as soon as anyone deviates from the publically announced wage, the firms' coordination breaks down and all the firms, in order not to be pushed out of the market, raise the wages up to the point where the entrepreneur get no extra profits and onlly remunerate herself with the same wage as his workers. Therefore we assume that, with uneployment, there is a coordination agreement consisting of Nash equilibrium in the repeated game (with indefinite oìhorizon, although not with infinte horizon) among the incumbents where the firm offering a marginally lower salary than the other incumbents is the object of an endless joint retaliation by all the other incumbent firms. This can be interpreted as a case of *folk theorem* among the incumbents.

unemployed individuals): with positive unemployment, all the incumbent firms have exactly the same incentive not to offer any wage that is greater or equal to the "incentive compatible wage". Every employed worker who is offered (between time $t-1$ and t) the "incentive compatible wage" for the next period (from t to $t+1$), knows that, by rejecting that offer, she would be substituted by an unemployed worker and become unemployed for the next period.

In the case of full employment, the nature of wage setting is radically different. The incumbents do not have any credible way to induce their workers to accept a "no entry wage", no matter how this is defined, because there is no longer any credible threat of offering the same contract to unemployed workers. As a consequence, there is no longer any incentive for the incumbents in coordinating themselves and offering a common wage in the process of wage setting. On the other hand, the rivalry among firms still exists and each rival can push a competitor out of the market by "stealing" its workers and offering them marginally higher wages. The only way to prevent this, for each incumbent firm is offering wages that eliminate the extra profits, so that the entrepreneurs are only remunerated by the wage they pay themselves. Let us call this wage ω^0.Any lower wage offered by a firm to its workers would expose the entreprener to the risk of being pushed out of the market by her competitors, who could potentially steal her workers by offering them ω^0. In this case, $\omega^0$is obviously a Nash equilibrium in a stage game among the oligopolistic incumbent firms.

Let us define the "incentive compatible wage" ω^*. Once ω^* is announced, but still between time $t-1$ and time t, some existing workers may receive an idiosyncratic informational shock that generates entry. Unexpected monetary policy taking place after the announcement of ω^* but before time t may also affect entry by affecting the interest rate (i.e. the cost of money to borrow the resources to cover the entry costs) and, as a consequence, the entry costs themselves, as explained below. Entry/exit, by affecting the number of existing firms, also affects production capacity and aggregate employment.

The risk free interest rate r_t, for the sake of simplicity is assumed to be exogenous and under the control of the monetary authorities. Since, as explained above, the revenue of the perfectly competitive banking and financial system is determined by the transaction fees and not by any interest margin, r_t is also the interest rate that banks charge on their loans. In case of bankruptcy, the entrepreneur would loose her job and not have the right to start a new firm next period, but would keep all of her risk free financial assets A_t for herself. In aggregate terms it is irrelevant who is actually holding the financial assets A_t. Since any new firm has limited liability, any new entrant will borrow from the banking and financial system in order to cover the cost of entry.

The expected remuneration of the entrepreneur is given by the profits (π_t^e if it is a new entrant, π_t^{in} if it is an incumbent) plus the wage ω_t^* that the entrepreneur pays to herself.

The expected remuneration of the new entrant at time $t-1$ for time t, $(E_{t-1}(\pi_t^e) + \omega_t^*)\,(1 - \tau - \varsigma))$, is different from the expected remuneration of the incumbent $(E_{t-1}(\pi_t^{in}) + \omega_t^*)\,(1 - \tau - \varsigma))$.

Since entry and exit are determined by stochastic shocks, ceteris paribus, with no modifications in the entry costs and monetary policy, the survival of new entrants depend on their ability to substitute the firms that abandon the market, or by new equilibria configurations in the (oligopolistic) market for goods. The firms that abandon the market can be either the new entrants of the previous period or (less likely) incumbents. Unless one assumes a deterministic process of formation of heterogeneous expectation (which is beyond the purposes of this paper), it is reasonable to assume that all the agents (incumbents, new entrants, workers and unemployed) share a common knowledge on the nature (i.e. analytical form) of the statistical distributions of price expectations $E_t\,(P_{t+1})$. This is also consistent with the outcome of the Bertrand game with quantity precommitments among the oligopolistic firms. *Ex ante* each individual does not know whether his expectation diverges from the average market expectation and is affected by an individual idiosyncratic shock. This is only known *ex post*, once the entry decisions are taken, the firms have hired their workers and, therefore, have precommitted themselves to a certain individual production capacity and sell their output on the market.

The firms' behavior is modelled as a Betrand game with quantity precommitments à la Kreps and Scheinkmann (1983), following the modifications introduced by Madden (1998), which extends the results of the Kreps and Scheinkmann two-stage game to the case of uniformly elastic demand curve.

The firms are price setters and the price of the homogeneous good is set at the second stage of the entry game. Appendix 2 shows the details on the existence and unicity of the Cournot-Nash equilibrium by invoking the results provided by Madden (1998), which exactly correspond to the assumptions of our model. In particular, we assume that the proportional rationig rule apply (which is one of the benchmark cases of Madden 1998 and the case of Allen and Hellwig, 1986) and that at time t, after the quantities are set (given the quantity precommitment given by the number of employed workers and by the labor contracts specifying the amount of hours to be worked and given the number of firms, which is observable and common knowkedge and knowing the aggregate demand), the incumbents announce the market clearing price, which is also the Cournot-Nash equilibrium.

Let us define the individual output of the oligopolistig firm at time $t+1$ as φ_{t+1}. In equilibrium φ_t is a fraction of the aggregate Demand $D_t(\cdot)$ and, as we said, the value of this fraction (like the actual variables P_{t+1}, φ_{t+1}, L^*_{t+1}) depends on the outcome of the two-stage game among the oligopolistic firms.

The expected profits for the incumbent are:

$$E_t(\pi^{in}_{t+1}) = \left[E_t\left(P_{t+1}\right) E_t(\varphi_{i,t+1}) - \omega_{t+1} E_t(L^*_{i,t+1})\right](1-\tau-\varsigma) =$$
$$= \left[E_t\left(P_{t+1}\right) E_t(\lambda_{i,t+1}\Lambda L^{*\alpha}_{i,t+1}) - \omega_{t+1} E_t(L^*_{i,t+1})\right](1-\tau-\varsigma)$$

and her expected price is:

$E_t\left(P_{t+1}\right) = [E_t(\pi^{in}_{t+1})\left(1-\tau-\varsigma\right)^{-1} + \omega_{t+1}E_t(L^*_{i,t+1})]E_t(\lambda_{i,t+1}\Lambda L^{*\alpha}_{i,t+1})^{-1} =$

$= E_t(\pi^{in}_{t+1})\left(1-\tau-\varsigma\right)^{-1} E_t(\lambda_{i,t+1}\Lambda L^{*\alpha}_{i,t+1})^{-1} + \omega_{t+1}L^{*1-\alpha}_{i,t+1}E_t(\lambda_{i,t+1}\Lambda)^{-1} =$

=(Net incumbent's profit per unit of output)$_{t+1}$ **+ wages**$_{t+1}$**/(real labour productivity)**$_{t+1}$;

where all the variables must be interpreted as referred to the individual firm.

The expected profits for the new entrant are:

$E_t(\pi^e_{t+1}) = [E_t\left(P_{t+1}\right)E_t(\lambda_{i,t+1}\Lambda L^{*\alpha}_{j,t+1}) - \omega_{t+1}E_t(L^*_{j,t+1}) - (1+r_t)F]\left(1-\tau-\varsigma\right)$

and its expected price:

$E_t\left(P_{t+1}\right) = [E_t(\pi^e_{t+1})\left(1-\tau-\varsigma\right)^{-1} + \omega_{t+1}E_t(L^*_{j,t+1}) + (1+r_t)F]E_t(\lambda_{j,t+1}\Lambda L^{*\alpha}_{j,t+1})^{-1} =$

$= E_t(\pi^e_{t+1})\left(1-\tau-\varsigma\right)^{-1} E_t(\lambda_{j,t+1}\Lambda L^{*\alpha}_{j,t+1})^{-1} + \omega_{t+1}L^{*1-\alpha}_{j,t+1}E_t(\lambda_{j,t+1}\Lambda)^{-1} +$
$(1+r_t)F \cdot E_t(\lambda_{j,t+1}\Lambda L^{*\alpha}_{j,t+1})^{-1} =$

=(Net entrant's profit per unit of output)$_{t+1}$ **+ wages**$_{t+1}$**/(real labour productivity)**$_{t+1}$ **+ [(1+r**$_t$**)F/(new entrant's output)**$_{t+1}$**]**

If we assume that the expected values for h^{in} and h^e are equal to the current values (i.e. $E_t(h^{in}_{t+1}) = h^{in}_t$ and $E_t(h^e_{t+1}) = h^e_t$), then, by aggregating, we get:

$E_t\left(P_{t+1}\right) = \Theta^e_t(E_t(\pi^{in}_{t+1}), E_t(\pi^e_{t+1}), h^e_t, h^{in}_t) + \omega_{t+1}[E_t(L^*_{i,t+1})/E_t(\lambda_{i,t+1}\Lambda L^{*\alpha}_{i,t+1})] =$

$= \omega_{t+1}[E_t(L^{*(1-\alpha)}_{i,t+1})/E_t(\lambda_{i,t+1}\Lambda)] + \Theta^e_t(E_t(\pi^{in}_{t+1}), E_t(\pi^e_{t+1}), h^e_t, h^{in}_t)$

Where Θ^e_t represents the average expected profits and is a function of the incumbents and new entrants expected profits and of their respective weights. Its *ex post* outcome, π^{AVG}_t, is the first moment of a stochastic distribution i.e. the distribution associated to the mixed strategy Cournot-Nash equilibrium among the oligopolistic firms, whose existence is shown in Appendix 2. π^{AVG}_t has a distribution determined by the "proportional rationing" mechanism that we assume to take place in the oligopolistic industrial sector of this model (see Appendix 2).

As explained below, ω^*_{t+1}, is set in advance between time t and $t+1$.

With no information shocks (and *before* the information shocks are known, *ex post*, to each individual), we assume that the expected values $E_t(\varphi_{t+1})$, $E_t(L^*_{t+1})$ are equal to their actual values at time t, since changes in policy variables are given by exogenous changes stochastic shocks in r_t. The *ex post* outcome is:

$$
\begin{aligned}
E_t(P_{t+1}) &= \{[E_t(\pi^{in}_{t+1})\frac{h^{in}_t}{h^e_t + h^{in}_t} + E_t(\pi^e_{t+1})\frac{h^e_t}{h^e_t + h^{in}_t}]\,(1-\tau-\varsigma)^{-1} + \quad (16)\\
+\omega_{t+1}n_{t+1} &= \quad (17)\\
P_{t+1} &= \omega_{t+1}E_t(n_{t+1}) + \pi^{AVG}_{t+1}(\pi^{in}_{t+1}, \pi^e_{t+1}, h^e_{t+1}, h^{in}_{t+1})
\end{aligned}
$$

Of course, π^{AVG}_{t+1}can be interpreted as a stochastic mark up over the costs

Obviously, the probability at time t for the generic new entrant to stay in the market is:

$$
\begin{aligned}
\Pr(\pi^e_t &\geq 0) = \Pr[E_{t-1}\,(P_t)\,E(\varphi_t) - \omega_t E_{t-1}(L^*_t) - \\
-(1+r_{t-1})F]\,(1-\tau-\varsigma) &\geq 0 \quad (18)
\end{aligned}
$$

The definition of $(E_{t-1}(\pi^e_t) + \omega^*_t)\,(1-\tau-\varsigma))$ and 18 show that a reduction in the interest rate r_{t-1} between time $t-1$ and time t decided by the monetary authorities affects the wage setting and, as a consequence, 18 and $(E_{t-1}(\pi^e_t) + \omega^*_t)\,(1-\tau-\varsigma))$. Of course, 18 and $(E_{t-1}(\pi^e_t) + \omega^*_t)\,(1-\tau-\varsigma))$ are also affected by the outcome of wage bargaining (and, as shown later, on the equilibrium among the oligopolistic firms). In other words, there are several causation effects and each of them is the outcome of simultaneous decisions of agents with different (and possibly conflicting) incentives, targets and sources of information. One way to model this kind of complexity while keeping, at the same time, the assumption of individuals' rationality (i.e. by keeping the assumption that the individuals do not make systematic mistakes and do not display a persistent bias in their process of information spreading) is the following: we assume that **when policy changes occur, the rational individuals operating in the market do not make , on average, systematic mistakes, although the frequency of mistakes increase, due to the increase in the complexity of the process of information processing that takes place when changes in monetary policy interact with the entry/exit process and with the expectations formulated by the individuals. We render this specific feature of the model by assuming that for given $E_t(\pi_{t+1}^{in})$ and $E_t(\pi_{t+1}^{e})$ (i.e. for the first moments of the frequency distributions of profits expectations formulated by the market) their variances increase with changes in the monetary policy because changes in the monetary policy (i.e. changes in r) by modifying the entry costs, increase the uncertainty, i.e. the variance of the expected profits, for a given expectation.** This modelling feature is particularly important for any numerical simulation based on this model, since, as discussed later, it will boil down into an increase in entry (exit) associated to expansionary (restrictionary) monetary policy.

The *ex ante* probability of bankruptcy of the generic new entrant at time t is defined as follows :

$$1 - \Pr(\pi_t^e \geq 0)$$

or

$$\begin{aligned}&\Pr(\pi_t^e < 0) = 1 - \Pr(\pi_t^e \geq 0) = \\ &= \Pr\left\{\left[E_{t-1}\left(P_t\right)E_{t-1}(\varphi_t) - \omega_t E_{t-1}(L_t^*) - (1+r_{t-1})F\right] \cdot (1-\tau-\varsigma) < 0\right\}\end{aligned}$$

The *ex ante* probability for the generic incumbent to stay in the market at time t *is the following*:

$$\Pr(\pi_t^{in} \geq 0) = \Pr\left\{\left[E_{t-1}\left(P_t\right)E(\varphi_t) - \omega_t E_{t-1}(L_t^*)\right](1-\tau-\varsigma) \geq 0\right\}$$

$E_{t-1}\left(P_t\right)$ is the expected price, in case of entry, $E_{t-1}(\varphi_t)$ the expected quantity sold by the new entrant and ω_t^* the "no entry wage", set between period $t-1$ and t. Obviously

$$\Pr(\pi_t^{in} < 0) = 1 - \Pr(\pi_t^{in} \geq 0)$$

When a firm goes bankrupt both the entrepreneur and the workers lose their job and get unemployed. Therefore an entrepreneur who goes bankrupt at time t, is unemployed at time $t+1$ and can only hope to be hired as a worker at time $t+2$. With this assumption, we do not need to impose any "ad hoc" bankruptcy costs.

At time t the new entrant survive with probability $\Pr(\pi_t^e \geq 0)$ and go bankrupt, get unemployed and get the unemployment subsidy $(n_t\omega_t + h_t^{in}\pi_t^{in} + h_t^e\pi_t^e)\tau\left(l - n_t + h_t\right)^{-1}$ with probability $[1 - \Pr(\pi_t^e \geq 0)]$. At time t, the new entrant will have 2 possible outcomes, or "future paths". At time *t+1,* if successful, she will be an incumbent and survive with probability $\Pr(\pi_{t+1}^{in} \geq 0)$ or fail with probability $[1 - \Pr(\pi_{t+1}^{in} \geq 0)]$; still at time *t+1*, the unsuccessful new entrant will be unemployed and have a certain probability of still being unemployed and another probability of being hired as a worker, and so on. In other words, at time *t=1* there will be 2 possible outcomes (or "future paths") for the new entrant, at time *t=2* there will be 4 possible "future paths", at time *t=3*, there will be 8 possible "future paths", at time *t=n* there will be 2^n possible "future paths". Similarly, the worker who decides not to enter the market as an entrepreneur, with probability $\Pr(\pi_t^{in} \geq 0)$ will earn the wage ω_t and (in the event that her firm goes bankrupt) loose the job with probability $[1 - \Pr(\pi_t^{in} \geq 0)]$ and earn the unemployment subsidy $(n_t\omega_t + h_t^e\pi_t^e + h_t^{in}\pi_t^{in})\tau\left(l - n_t\right)^{-1}$. However, if we move on in time, for instance, at time $t+2$, the surviving entrant will get with probability $\Pr\left\{\pi_{t+1}^{in} \geq 0\right\}$ the profit of the incumbent π_{t+1}^{in} and with probability $\left[1-\Pr(\pi_{t+1}^{in} \geq 0)\right]$ the unemployment subsidy. Valuating the expectation of future profits for the new entrant means valuating a tree of outcomes where from *t+1* onwards, in each period the firm can survive (with a certain probability) or going bankrupt (with the complementary probability). Having gone bankrupt in period *t+1* can be followed by the event of being hired as a worker by a new firm or remaining unemployed, and so on. In other word, the rational forward-looking decision maker that makes plans at time *t=1,2,3..n* (i.e. for all the future periods from t onwards), faces 2^k different "future paths" for every $k-$periods interval in its future. For instance, when *t=3*, i.e. 3 periods ahead from the moment where the decision is taken, there will be $2^3 = 8$ possible "future paths", each of them with a given sequence of conditional probabilities.[5]

The further away the expectations formulated at time "t" the higher the number of combinations of possible "future paths" that characterizes the future of the decision maker. This boils down into a degree of on-going uncertainty which is increasing in the length of future time expectations and in the number of possible outcomes on which expectations are formulated, since at every future time, each agent can be in one out of several states that depend on the decisions simultaneously taken by all the other agents. Therefore, the variance of such expectations, in general, are higher the further away in the future is the forecast.[6] What we need to assume here, for the

[5]So, for instance, the probability that the firm of the new entrant will survive after the entry and for 2 periods ahead after the entry is given by $\Pr(\pi_t^e \geq 0) \cdot \Pr(\pi_{t+1}^{in} \geq 0) \cdot \Pr(\pi_{t+2}^{in} \geq 0)$. The probability that a new entrant will survive 3 periods and then go bankrupt is given by $\Pr(\pi_t^e \geq 0) \cdot \Pr(\pi_{t+1}^{in} \geq 0) \cdot \Pr(\pi_{t+2}^{in} \geq 0) \cdot \left[1-\Pr(\pi_{t+3}^{in} \geq 0)\right]$.

[6]The influence of externality in individuals' choices and in expectation formulation is certainly a very relevant issue. In this regard, the rational beliefs assumption (Kurz, 1994a, 1994b) could be an interesting approach that could be profitably applied to this

determination of the incentive-compatibility constraint in wage setting are just two approximations:

We approximate and define then the expected future stream of income $J_{t+1}$from time $t+1$ onwards for the successful entrant at time t as follows:

$$J_{t+1} = J_{t+1}(\overbrace{E(h^{in}_{t+1})}^{-}, \overbrace{E(\overset{-}{h}{}^{e}_{t+i+1}(\overbrace{r_{t+i}}^{-}))}^{+}, \overbrace{\Pr(\pi^e_t \geq 0)}^{+}, \overset{-}{r_t}) \tag{19}$$

with $i = 2, 3 \ldots \infty$

J_{t+1} negatively depends on r_t because, as one may see from 18 and 3, the lower r_t, the higher the profits of the new entrants and their probability of survival. However, once the generic new entrant has survived, it becomes, from the next period, an incumbent and, as a future incumbent, her future profits from time $t+2$ onwards is higher the lower the number of future new entrants at time $t+2, t+3, t+n$, respectively $h^e_{t+2}, h^e_{t+3}, h^e_{t+n}$

As a consequence, J_{t+1} negatively depends on r_t and positively depends on r_{t+1}, r_{t+i} . Therefore, generalizing, J_{t+i} negatively depends on r_{t+i-1} and positively depends on the expected difference $E(r_{t+i} - r_{t+i-1})$. Without loss of generality, we may simplify 19 as follows:

$$J_{t+1} = J_{t+1}(\overbrace{\Pr(\pi^e_t \geq 0)}^{+}, \overset{-}{r_t}, \overbrace{E(r_{t+i} - r_{t+i-1})}^{+})$$

Changes in the interest rate reflect policy decisions of the central bankers, hence $E(r_{t+i} - r_{t+i-1})$ describes the expectations of changes in monetary policy.

This also means that only temporary reductions in the interest rate increases entry, while a permanent reduction in the interest rate might affect

model too. However, questioning the nature of expectations and the assumption of rational expectations is beyond the purpose of this draft.

(as shown below) the labour market equilibrium, but not necessarily entry. Obviously J_{t+1} positively depends on the probability of survival of the new entrants $\Pr(\pi_t^e \geq 0)$ and negatively depends on the relative number of competitors, whether they are incumbent (h^{in}) or new entrants (h^e), in each and every future period of time.

In a similar way we may approximate in the following way the expected future stream of income from time $t+1$ onwards for the worker employed by an incumbent surviving at the beginning of time $t+1$

$$\Gamma_{t+1} = \Gamma_{t+1}(\overbrace{\Pr(u_{t+i} = 0)}^{+}, \overbrace{\Pr(\pi_{t+1}^{in} \geq 0)}^{+}, E(w_{t+1}^{*})) \tag{20}$$

Since the firms are price setters with quantity precommitments, what matter for the workers is the expected rivalry among firms, the probability of survival of the firm they work for and whether or not the economy will be in full employment. We can think of the probability of full employment (or zero unemployment) at time j, defined as $pr(u_j = 0)$ as a positive function of expansionary policies, since they would increase the demand and market size. Symmetrically to that happens for J_{t+1}, the individuals can expect that the future demand and market size (from 14) will be larger for lower values of the future interest rates, i.e. the larger $E(r_{t+i} - r_{t+i-1})$.

$$\Gamma_{t+1} = \Gamma_{t+1}(\overbrace{E(r_{t+i} - r_{t+i-1})}^{-}, \overbrace{\Pr(\pi_{t+1}^{in} \geq 0)}^{+}, E(w_{t+1}^{*}))$$

The definition of Γ_{t+1} , the expected future stream of incomes of the worker at time t, takes into account the future probability of the worker to loose her job (due to the simultaneous decisions and interactions of other agents) and get the unemployment subsidy. In a similar way, let us define Υ_{t+2} as the expected stream of income from time $t+2$ onwards of an individual unemployed at time $t+1$.

Υ_{t+2} Positively depends on the probability of being hired as a worker by a firm the next period, and negatively on the number of unemployed individuals.

We are now enabled to write the incentive compatibility constraint for setting the wage under unemployment. In this case the wage is set by the oligopolistic firms in such a way to discourage entry, therefore it has to satisfy the incentive compatibility constraint saying that the expected future discounted stream of income from time $t+1$ onwards for the worker employed by an incumbent surviving at the beginning of time $t+1$ has to be greater or equal to the expected future discounted stream of income from time $t+1$ onwards for the new entrant.

$$\Pr(\pi_t^e \geq 0)(1+\rho)^{-1}\{[E_{t-1}(\pi_t^e) + \omega_t](1-\tau-\varsigma)+$$
$$+J_t(\cdot)\} + [\Pr(\pi_t^e < 0)](1+\rho)^{-1}\cdot$$
$$\cdot E_{t-1}[(n_t(\omega_t + h_t^e\pi_t^e + h_t^{in}\pi_t^{in}))\tau(l-n_t)^{-1}+$$
$$+\Upsilon_{t+1}] \leq (1+\rho)^{-1}\cdot\Pr(\pi_t^{in} \geq 0)\cdot$$
$$\cdot[\omega_t(1-\tau-\varsigma) + \Gamma_t(\cdot)] + (1+\rho)^{-1}\cdot$$
$$\cdot\{\Pr(\pi_t^{in} < 0)\cdot E_{t-1}[n_t(\omega_t + h_t^e\pi_t^e + h_t^{in}\pi_t^{in})\tau(l-n_t)^{-1} + \Upsilon_{t+1}]\}$$

The term $\Pr(\pi_t^e \geq 0)(1+\rho)^{-1}\{[E_{t-1}(\pi_t^e) + \omega_t](1-\tau-\varsigma) + J_t(\cdot)\}$ is the expected future stream of income for the successful entrant, weighted with the probability of surviving in the first period. The term $[\Pr(\pi_t^e < 0)](1+\rho)^{-1}\cdot E_{t-1}[(n_t(\omega_t + h_t^e\pi_t^e + h_t^{in}\pi_t^{in}))\tau(l-n_t)^{-1} + \Upsilon_{t+1}]$ is the expected future stream of income for the unsuccessful entrant, weighted with the probability of going bankrupt in the first period. The term $(1+\rho)^{-1}\cdot\Pr(\pi_t^{in} \geq 0)\cdot[\omega_t(1-\tau-\varsigma) + \Gamma_t(\cdot)]$ is the expected stream of future income for the worker who decides to remain worker and whose firm survives. The term $\{\Pr(\pi_t^{in} < 0)\cdot E_{t-1}[n_t(\omega_t + h_t^e\pi_t^e + h_t^{in}\pi_t^{in})\tau(l-n_t)^{-1} + \Upsilon_{t+1}]\}$ is the expected stream of future income for the worker who decides to remain worker and whose firm goes bankrupt. As we said, for τ very small, the term $E_{t-1}[n_t(\omega_t + h_t^e\pi_t^e + h_t^{in}\pi_t^{in}))\tau(l-n_t)^{-1}]$ will be very small and negligible. Let us define it "*subsidies*". The term Υ_{t+1} will also be very small. Therefore, with no full employment, the wages are set by the firms according to the following incentive compatibility constraint:

$$\begin{aligned}
\omega_t^* \geq & \frac{\Pr(\pi_t^e \geq 0) \cdot E_{t-1}(\pi_t^e)}{[\Pr(\pi_t^{in} \geq 0) - \Pr(\pi_t^e \geq 0)] \cdot (1-\tau-\varsigma)} + \\
& + \frac{\Pr(\pi_t^e \geq 0) \cdot J_{t+1}(\cdot)}{[\Pr(\pi_t^{in} \geq 0) - \Pr(\pi_t^e \geq 0)] \cdot (1-\tau-\varsigma)} + \\
& + \frac{-\Gamma_{t+1}(\cdot) \Pr(\pi_t^{in} \geq 0)}{[\Pr(\pi_t^{in} \geq 0) - \Pr(\pi_t^e \geq 0)] \cdot (1-\tau-\varsigma)} + \\
& + \frac{\Upsilon_{t+1}[\Pr(\pi_t^e < 0) - \Pr(\pi_t^{in} < 0)]}{[\Pr(\pi_t^{in} \geq 0) - \Pr(\pi_t^e \geq 0)] \cdot (1-\tau-\varsigma)} + \\
& + \frac{[\Pr(\pi_t^e < 0) - \Pr(\pi_t^{in} < 0)] subsidies}{[\Pr(\pi_t^{in} \geq 0) - \Pr(\pi_t^e \geq 0)] \cdot (1-\tau-\varsigma)}
\end{aligned} \tag{21}$$

The denominators are always positive, since $\Pr(\pi_t^{in} \geq 0)$ is always greater than $\Pr(\pi_t^e \geq 0)$. As explained above, we assume that the magnitudes of Υ_{t+1} and "$subsidies$" are small compared to the other variables, therefore the terms $\frac{\Upsilon_{t+1}[\Pr(\pi_t^e<0)-\Pr(\pi_t^{in}<0)]}{[\Pr(\pi_t^{in}\geq 0)-\Pr(\pi_t^e\geq 0)]\cdot(1-\tau-\varsigma)}$ and $\frac{[\Pr(\pi_t^e<0)-\Pr(\pi_t^{in}<0)]subsidies}{[\Pr(\pi_t^{in}\geq 0)-\Pr(\pi_t^e\geq 0)]\cdot(1-\tau-\varsigma)}$ in 21 can be neglected. ω_t^* is increasing in the expected profits of the new entrants and decreasing in the expected profits of the incumbents. This implies that ω_t^* is (negatively) affected by the lagged interest rate r_{t-1}, which means that monetary policy (i.e. exogenous changes in r_{t-1}) only affects wage setting (and the conditions for entry and aggregate supply) with one period lag, while it instantaneously affects the aggregate demand. This modeling feature (implied by the theoretical assumptions in 13 and 14) reflects a stylized empirical fact in literature: a certain delay in the effects of monetary policy.

If the incentive-compatibility constraint 21 on wage setting were not respected all the workers would have incentive to leave their jobs and start a new firm. Next period there would be full employment and all the firms would have lost their bargaining power on the labour market and the wages, as we said, would be set at a level where expected profits would be zero. The situation that creates stronger incentives to enter the market at time $t+1$ is a temporaneous decrease in the interest r_t , followed by expected increases in the interest rate at times $t+1$, $t+2$, ..., $t+n$.

When the economy reaches the full employment, the bargaining power is on the side of the workers, and the wages are set at a "zero-profit" level.

Considering 21 as a binding equality and neglecting the last two fractions of 21 (which are both positive and whose magnitudes are very small compared to the rest of the inequality), and given the information set Ω_{t-1} of all the observable variables at time t, ω_t^* is a function of the following variables:

$$\omega_t^* = \omega_t^*(\overbrace{E_{t-1}\,(\pi_t^e)}^{+}, \overbrace{[\Pr(\pi_t^{in} \geq 0) - \Pr(\pi_t^e \geq 0)]}^{-}, \overbrace{E(r_{t+i} - r_{t+i-1})}^{+}, \overbrace{r_{t-1}}^{-}) \quad (22)$$

The terms $E_{t-1}\,(\pi_t^e)$ and $[\Pr(\pi_t^{in} \geq 0) - \Pr(\pi_t^e \geq 0)]$ are affected by any shock or exogenous change (for instance, technology shocks affecting F) in the equilibrium among the oligopolistic firms in the goods market and, of course, are (negatively) affected by r_{t-1}.

Defining the right-hand side of inequality 21 as Φ_t, we can introduce an object that turns out to be useful in aggregating the behaviour of heterogeneous agents: **the probability of entry,** $\Pr(entry)_t$, which may be interpreted as **the sum of the stochastic idiosincratic shocks generating any entry decision, i.e. the integral (over the whole population of workers $n_t(1-h_t)$ at time t) of the perceived probability that the wage is set at a lower level than the expected present discounted value of the future profits as an entrepreneur in case of entry.**

$$\Pr(entry)_t = \int_0^{n_t(1-h_t)} (\Pr(w_t < E_{t-1,i}(\Phi_t))_i di \quad (23)$$

This is due to idiosyncratic information shocks in the right-hand side of the equilibrium equality $w_t = E_{t-1,i}(\Phi_t)$. In this sense, ideally,

$\int_{0}^{n_t(1-h_t)} (\Pr(w_t < E_{t-1,i}(\Phi_t))_i di$ is the integral of all the idiosyncratic information shocks on $E_{t-1,i}(\Phi_t)$for any individual worker i at time t, which include the information shocks on the expected profits (determined by the outcome of the game among the oligopolists) as well as the information shocks on the expected future monetary policy. The integral represents the portion of "optimistic" workers counting on a successful entry. As we said, expansionary changes in the monetary policy, for given average market expectations $E_t(\pi^e_{t+1})$ and $E_t(\pi^{in}_{t+1})$, increase the variance of the distributions of these two variables: there is a higher frequency of prediction mistakes. We can then define

$$\Pr(entry)_t = \psi_t(\Delta \bar{r}_{t-1})$$

Since we have already defined the probability of bankruptcy of a new entrant and an incumbent (which are $(1-\Pr(\pi^{in}_t \geq 0))$ and $(1-\Pr(\pi^{in}_t \geq 0))$ respectively) the probability of bankruptcy of a generic firm $\Pr(exit)_t$ may be expressed as follows:

$$\Pr(exit)_t = \frac{h^e_t(\Pr(\pi^e_t < 0)) + h^{in}_t(\Pr(\pi^{in}_t < 0))}{h_t}$$

Given the definitions of $\Pr(\pi^e_t < 0)$ and $\Pr(\pi^{in}_t < 0)$, for $\Pr(exit)_t$ is increasing in the interest rate r_{t-1} and is also affected by any change and/or stochastic shock affecting the outcome of the oligopolistic firms in the goods market. Let use define ε^{δ}_t a generic stochastic shock affecting the new entrants profits, where ε^{δ}_t is distributed as $N(0, \varepsilon^{\delta}_t)$.

$$\Pr(exit)_t = \delta(\bar{r}_{t-1}, \varepsilon_{\delta,t}) \tag{24}$$

therefore, in empirical terms, the amount of firms leaving the market at time t, that we define δ_t , is given by $n_t \cdot \delta(r_{t-1}, \varepsilon_{\delta,t})$ and is affected by all the shocks affecting $h_t^e, \Pr(\pi_t^e < 0), h_t^{in}, \Pr(\pi_t^{in} < 0)$, summarized in the random variable $\varepsilon_{\delta,t}$.

In the case of full employment, for the reasons explained before, there are no extra-profits, all the entrepreneur are incumbent, but since they do not enjoy any market power, the remuneration of each entrepreneur is given by the wage she pays to herself. Therefore, in this case the wage is derived by the condition $E_{t-1}(\pi_t^{in}) = 0$, which implies

$$\omega_t^{fu} = \frac{E_{t-1}\left(P_t \mid \Omega_{t-1}\right) E_{t-1}(c_t \mid \Omega_{t-1})}{E_{t-1}(L_t^* \mid \Omega_{t-1})} \tag{25}$$

or, in aggregate terms:

$$\omega_t^{fu} = \frac{E_{t-1}\left(P_t \mid \Omega_{t-1}\right) E_{t-1}(Q_t \mid \Omega_{t-1})}{E_{t-1}\left[n_t(1-h_t) \mid \Omega_{t-1}\right]} \tag{26}$$

Therefore the determination of the wages has a point of discontinuity triggered by the level of full employment. In fact:

$$\omega_t = \begin{cases} \omega_t^* & if \quad n_t < l \\ \omega_t^{fu} & if \quad n_t = l \end{cases} \tag{27}$$

The situation of full employment is subject to a number of shocks and may be interpreted as a temporary equilibrium.

3.1 Modelling entry/exit in the macroeconomic equilibrium

Following Aoki and Yoshikawa (2007), we start by introducing an interpretation of how do interacting agents behave at a microeconomic level. Suppose that agents have binary choices or there are two types of agents. The two choices can be represented by two states (say state 0 and state 1). If we have n agents, the state of n agents may be represented as follows:

$$s = (s_1, s_2, ..., s_n)$$

where the choice by agent i is denoted by $s_i = 1$ or $s_i = 0$ and so on. A set of all the possible values of s is called "state space" S. This vector contains a complete description of who has chosen what. The purpose of this assumption is to describe the dynamic process of how do agents revise their choices in time, due to incentives, externalities, costs and unexpected news. Since we are interested in time evolution the states, we consider a stochastic process in discrete time (differently from Aoki and Yoshikawa, who consider jump Markov process in continuous time).

The process we are interested in concerns the workers who become entrepreneurs and the entrepreneurs who go bankrupt (i.e., given the number of employed individuals n_t at time t, how many of them increase or decrease the portion of existing firms h_t).

On the other hand, the increase or decrease in n_t is a mere consequence of the process of entry/exit of new firms and may be easily modelled, since we have assumed that in each period t the number of workers employed by each firm is determined between $t-1$ and t, when the number of firms operating next period t is known and all the firms set their quantity precommitment by setting the labour contracts.

The agents are assumed to make a binary choice between two states (in this case being a worker and being an entrepreneur) which can be interpreted as one agent changing his mind (and his state). Following the notation of Aoki and Yoshikawa, we have, between time $t-1$ and t, when entry takes place the following transition rates:

$$q(n_t h_t, n_t h_t + 1) = [n_t(1-h_t)]\, \eta_1(h_t) \tag{28}$$

$$q(n_t h_t, n_t h_t - 1) = n_t h_t \eta_2(h_t) \tag{29}$$

Equation 28 represents the transition rate of an increase in the number of workers who were not entrepreneurs and decide to enter the market (with $0 < h_t < 1$). The transition rate refers to a notion of feasibility (not probability in itself) of the choice to enter the market and depends on the number $n_t(1-h_t)$ of employed people who are not entrepreneurs. On the other hand, $\eta_1(h_t)$ is a function that takes into account externality: for this reason it is a decreasing function of h_t because the decision to enter the market is discouraged by a high number of existing entrepreneurs. The higher h_t, the smaller $\eta_1(h_t)$. In the benchmark case where the economy reaches full employment, the workers will be remunerated exactly like the entrepreneurs, and there will be no incentive and no room for new entries. The second transition rate equation can be simplified in the following way. Let us assume that the random shock on δ_t is linear and additive:

$$\delta_t = E_{t-1}(\delta_t(r_{t-1})) + \varepsilon_{\delta,t}$$

Then $\delta_t(\cdot)n_t h_t$ is the outflow of existing firms out of the market and $\varepsilon_{\delta,t}$ reflects any idiosyncratic shock leading to exit[7].

Following again Aoki and Yoshikawa (2007), we need to determine the "master equation", or Chapman-Kolmogorov equation as the equation describing the time evolution of the probability distribution of states[8]. First of

[7] This assumption is qualitatively different from the assumption of exogenous bankruptcy rate made by Etro and Colciago (2010).

[8] For our purposes we only need to use it here in a simplified way, to identify the stationarity or equilibrium probabilities of states, without considering other solution tools and techniques suggested by Aoki and Yoshikawa, like the probability generating function or the Taylor expansion or the cumulant generating function. Differently from Aoki and Yoshikawa (2007), we define the Chapman-Kolmogorov equation in discrete time and not

all, using again the notation by Aoki and Yoshikawa (2007), we define the *equilibrium probabilities of states* as follows:

$$\Pr(s(s_1, s_2, ..., s_n))_{t+1} - \Pr(s(s_1, s_2, ..., s_n))_t =$$

$$= \sum_{s'} q(s', s) \cdot \Pr(s', t) - \Pr(s, t) \sum_{s} q(s, s')$$

Where the sum is taken over all states $s' \neq s$ and $q(s', s)$ is the transition rate from state s' to s. Intuitively speaking:

$\Delta \Pr(\cdot)/\Delta t =$ (inflow of probability fluxes into s) - (outflow of probability fluxes out of s). Here, of course, Δt is only a unit time interval.

In our case we can define the net inflow of probability of "being entrepreneur" $\Delta^h \Pr(\cdot)t$ as follows:

$$\begin{aligned} \Delta^h \Pr(\cdot) &= \sum_{(n-h)} q(n_t h_t, n_t h_t + 1) \cdot \Pr(entry)_{t+1} - n_t h_t \delta_t(\cdot) \\ &= [n_t(1 - h_t)]\, \eta_1(h_t) \cdot \psi_{t+1}(\Delta \bar{r}_t) - n_t h_t \delta_t(\cdot) \end{aligned}$$

i.e.

$$\Delta^h \Pr(\cdot) = [n_t(1 - h_t)]\, \eta_1(h_t) \cdot \psi_{t+1}(\Delta \bar{r}_t) - n_t h_t \delta_t(\cdot) \tag{30}$$

i.e. the inflow probability of firms increases with the level of employment n_t, with the probability of entry at time $t+1$ and decreases with h_t. Then, since $[n_t(1 - h_t)]\, \eta_1(h_t) \cdot \psi_{t+1}(\Delta \bar{r}_t)$ generates the new born firms (i.e. the entrants) at time t+1, we have:

in continuous time.

$$\begin{aligned}
\Delta^h \Pr(\cdot) &= n_{t+1}h^e_{t+1} - n_t\delta_t(\cdot)h_t && (31)\\
n_{t+1}h_{t+1} - n_t h_t &= [n_t(1-h_t)]\,\eta_1(h_t)\cdot\psi_{t+1}(\bar{\Delta r_t}) - n_t\delta_t(\cdot)h_t && (32)\\
n_{t+1}h_{t+1} &= n_t[\eta_1(h_t)\cdot\psi_{t+1}(\bar{\Delta r_t}) + h_t(1+\eta_1(h_t)\cdot\psi_{t+1}(\bar{\Delta r_t}) + && (33)\\
&\quad -\delta_t(\cdot))]
\end{aligned}$$

For the sake of numerical simulations, by setting a suitable value for $\eta_1(h_t)$, one may use equations 32 and 33 to calculate the number of firms for time $t+1$ or their change from time t to $t+1$.

Ex ante, if the wage are set by the oligopolistic firms according to the incentive compatibility constraint 21 and if all the individuals had perfectly identical expectations (i.e. if there were no idiosyncratic informational shocks), then $\Pr(entry)_{t+1}$ will be null. In these conditions, entry is given by idyosincratic shocks.The ex post deviations would be those caused by all the possible stochastic shocks affecting the right-hand side of inequality 21**.**

If the incentive-compatible wage setting rule is not violated, if no random shock occurs, then in equilibrium no worker would have incentive for entry.

Manipulating 32, we may express the dynamics in terms of growth rate

$$\frac{n_{t+1}h_{t+1} - n_t h_t}{n_t h_t} = \left(\frac{1-h_t}{h_t}\eta_1(h_t)\cdot\psi_{t+1}(\bar{\Delta r_t}) - \delta_t(\cdot)\right) \tag{34}$$

Obviously, in a stationary equilibrium (and only in a stationary equilibrium), when $\Delta^h \Pr(\cdot) = 0$, we have:

$$\psi_{t+1}(\bar{\Delta r_t}) = \frac{h_t}{1-h_t}\cdot\frac{\delta_t(\cdot)}{\eta_1(h_t)} \tag{35}$$

From the production function 15 of the generic firm i, given the optimal output $\varphi_{i,t}$ decided by the firm i at the initial stage of the Cournot-Nash game (once entry and exit have been decided) we get the amount of labour employed by each firm:

$$L_{i,t}=\left(\frac{E_{t-1}(\varphi_{i,t})}{\Lambda}\right)^{1/\alpha}$$

Therefore, the amount of labour employed by all the firms is:

$$L_t^*=\sum_{i=1}^{n_t h_t} L_{i,t}=\sum_{i=1}^{n_t h_t}\left(\frac{E_{t-1}(\varphi_{i,t})}{\Lambda}\right)^{1/\alpha} \tag{36}$$

$\Delta^h \Pr(\cdot)$ in 30, or 32, or 33, together with 36, determine the dynamics of employment, since an increase in the number of firms would determine, in general, a higher level of output and, as a consequence, given the production function 15, a higher level of employment.

$$n_{t+1}=n_t\left\{1+h_t\left[\sum_{i=1}^{n_t h_t}\left(\frac{E_{t-1}(\varphi_{i,t})}{\Lambda}\right)^{1/\alpha}\right]\left(\frac{1-h_t}{h_t}\eta_1(h_t)\cdot\psi_{t+1}(\Delta\bar{r}_t)-\delta_t(\cdot)\right)\right\} \tag{37}$$

Once the wage for time t is set (between $t-1$ and t), the entry decision are taken: both the new entrants and the incumbents decide the number of workers to hire for the next period (i.e. from t to $t+1$), on the basis of her profit expectations. In this way, since the labour contract *ex ante* establishes a fixed number of hours to be worked, each oligopolistic firm pre-commit itself to a certain output.

Given our assumptions on the unit elastic demand function, each firm would not have incentive to increase the number of hours to be worked in each period, because an increase in the output would not increase the revenues; furthermore, in this way a firm would trigger a retaliation from the other oligopolistic firms (see Appendix 2).

Between time $t-1$ and t but before time t, all the new entrant and the existing firms plan and arrange the labour contracts (which start at time t), hire workers and organize entry, which actually takes place at time t. Then, at time "t" the firms (both incumbents and new entrants, i.e.

the former workers) are binded with contracts to their workers and to the lenders who lent them the money to cover the fixed entry cost $(1 + r_t)F$, set the amount of output to produce, i.e. precommit themselves to the quantity to be produced and sold from t to $t + 1$. Madden (1998) extends the well known Kreps-Scheinkman result (Bertrand competition with quantity precommitment yields Cournot-Nash equilibrium in oligopoly) to fairly general conditions. This is briefly discussed in the next section.

4 Interpretation and implications of the Cournot equilibrium

This section shows that, under fairly general assumptions (which include as a subset the assumptions of our model), with quantity precommitments for all the firms (no matter if they are incumbents or new entrants), our model displays a Cournot-Nash equilibrium even with price competition. This is due to an extension provided by Madden (1998) of the famous Kreps-Scheinkman (1983) result. The results by Madden (1998) that are relevant for this work are shown in Appendix 2.

Between time $t-1$ and t the wages that apply between time t and $t+1$are set by the firms and, immediately after that, the process of entry begins, between time $t - 1$ and time t, but still before time t. Once entry has taken place, the two-stage game among the oligopolists take place, at time t. The incumbents and the new entrants hire the workers by setting one-year labour contracts that also specify the amount of hours to be supplied by each worker for the coming period. The labour cost is a sunk cost for both the incumbents and the new entrants. Given the production function, this also implicitly determines the quantity precommitment. Since, as we said, all incomes (profits, labour and unemployment subsidies) for time t are received by all the individuals at the end of time t, labour costs are first accounted as debt of the firms toward the workers, at the beginning of time t, and then paid out to workers at the end of time t. However, they occur (since they are recorded as debt) at the beginning of time t.

The assumptions of this model, with the addition of **Assumption D2** (see Appendix 2), meet the requirements of Madden (1998) theorem shown in appendix 2, therefore the oligopolistic firms of our model, having defined capacity and output at stage 1 of the game, have a cournot payoff $\pi_i^c(\varphi^1, ..., \varphi^h)$ at stage 2, which is sequential to stage 1 and takes place at time t and is known at time t.

Furthermore, Madden shows that if the demand is uniformly elastic (and our constant elasticity function is a sub case) and asymptotic (with the vertical and horizontal axes as asymptotes) and the firms' costs can be represented by a convex and strictly increasing function and if the assumptions on quantity determination and rationing rules reported in Appendix 2 hold, then there exists at least one pure strategy Cournot-Nash equilibrium. In our case this equilibrium is also unique, since at stage 2 of the Cournot-Nash game, the costs are symmetric (see Madden, 1998, theorem 3, p. 205). We assume that the matching among firms and consumers happen according to the proportional rationing rule, a benchmark case that allows for a unique price Cournot-Nash equilibrium.

Wages, as we said, are pre-determined and sunk. The amount of labour employed at time t by the generic firm i, $L_{i,t}$, ise determined on the basis of *ex ante* expectations. We call it the optimal *ex ante* amount of labour, associated to the optimal *ex ante* individual firm output $E_{t-1}(\varphi_{i,t})$ at time t.

All firms (no matter if they are incumbents of new entrants) share the same marginal cost function.

The price at time t is determined by the following price equation:

$$P_t = \omega_t + \pi_t^{AVG}(\pi_t^{in}, \pi_t^{e}, h_t^{e}, h_t^{in}) \tag{38}$$

5 Summarizing the theoretical model for numerical simulations

All the previous sections contain an explanation of the theoretical model, which boils down into the aggregate demand 14, pricing equation 16, wage determination (from equations 22, 26, 27) as well as the employment dynamics (such as determined in (36 and 37).

The aggregate demand may be written as a function of aggregate income (13, the first equation below) or by explicitly accounting for the distributional shocks, like in 14, the second equation below:

$$D(P_t, W_t) = \frac{\Xi(r_t)}{P_t}\left(A_t + \frac{1}{1+r_t}\sum_{i=0}^{\infty}\left(\frac{1}{(1+E\left(r_{t+i}\right))(1+E(\iota_{t+i}))}\right)^i E(Q_{t+i})\right)$$

or

$$\begin{aligned} D(P_t, W_t) &= (\Xi(r_t)/P_t)\cdot \\ &\cdot\{A_t + [1/(1+r_t)]\sum_{i=0}^{\infty}[(1+E\left(r_{t+i}\right))(1+E(\iota_{t+i}))]^{-i}\cdot \\ &\cdot E(n_{t+i}(\omega_{t+i} + h^e_{t+i}\pi^e_{t+i} + h^{in}_{t+i}\pi^{in}_{t+i}))\} \end{aligned}$$

The second formulation of the demand equation shows that entry/exit generates distributional shocks affecting the aggregate demand. The price level is determined by plugging equation 16. As we said, P_t is interpreted as the general price level, while idiosyncratic price shocks to the consumers are generated by the rationing mechanism in the Cournot-Nash equilibrium among the oligopolistic firms. The inflation rate ι_t is defined as $\frac{P_t - P_{t-1}}{P_{t-1}}$

For the sake of possible empirical analyses, having assumed that

$\varphi_{i,t} = E_{t-1}(\varphi_{i,t}) + \varepsilon^{\varphi}_{i,t}$, then the aggregate production is simply $\varphi^*_t = \sum_{i=1}^{n_t h_t}\varphi_{i,t}$

And, from the individual firm's production function: $\varphi_{i,t} = \lambda_{i,t}\Lambda L_{i,t}^{\alpha}$

The exogenous and contemporaneous variables appearing in the equation are the liquid assets A_t (positively affecting the level of consumption) and the interest rate r_t (negatively affecting consumption).

The values of the future forward-looking variables $E\left(r_{t+i}\right)$ and $E(\iota_{t+i})$ are assumed to be induced by their current value, i.e. their current value is the best predictor and the best expectation for its future values). Of course they are subject to information stochastic shocks and other kinds of stochastic shocks. In particular, the different prices that consumers might face, due to the various rationing mechanisms in stage 2 of the game among the oligopolisitc firms can be interpreted as idiosyncratic price shocks for the consumers.

P_t is determined by the pricing equation 38:

$$P_t = \omega_t + \pi_t^{AVG}(\pi_t^{in}, \pi_t^{e}, h_t^{e}, h_t^{in})$$

Wages are predetermined at time t by equations 27, **??** and 26

$$\omega_t = \begin{cases} \omega_t^* & if \quad n<l \\ \omega_t^{fu} & if \quad n=l \end{cases}$$

where :

$$\omega_t^* = \omega_t^*(\overbrace{E_{t-1}\left(\pi_t^{e}\right)}^{+}, \overbrace{[\Pr(\pi_t^{in} \geq 0) - \Pr(\pi_t^{e} \geq 0)]}^{-}, \overbrace{E(r_{t+i} - r_{t+i-1})}^{+}, \overbrace{r_{t-1}}^{-}$$

and

$$\omega_t^{fu} = \frac{E_{t-1}\left(P_t\right) E_{t-1}(Q_t)}{E_{t-1}[n_t(1-h_t)]}$$

The dynamics of the firms number is given by equations 32 and 33 Where, for $E_{t-1}\left(P_t\right)$ and $E_{t-1}(Q_t)$, we may introduce the assumption that, with no unexpected shocks, the current variable is the best predictor for the future variable.

Having included the equations for the aggregate demand, wage and price determination, and entry h_t^e, what we need to determine here is some law of motion of the employed individuals.

The dynamics of employment, is determined respectively by 31, 32, 33

$$\Delta^h \Pr(\cdot) = n_{t+1}h_{t+1}^e - n_t\delta_t(\cdot)h_t$$
$$n_{t+1}h_{t+1} - n_t h_t = [n_t(1-h_t)]\,\eta_1(h_t)\cdot \Pr(entry)_{t+1} - n_t\delta_t(\cdot)h_t$$

$$n_{t+1}h_{t+1} = n_t[\eta_1(h_t)\cdot \Pr(entry)_{t+1} + h_t(1+\eta_1(h_t)\cdot \Pr(entry)_{t+1} - \delta_t(\cdot))]$$

$$n_{t+1} = n_t\left\{1 + h_t\left[\sum_{i=1}^{n_t h_t}\left(\frac{E_{t-1}(\varphi_{i,t})}{\Lambda}\right)^{1/\alpha}\right]\left(\frac{1-h_t}{h_t}\eta_1(h_t)\cdot\psi_{t+1}(\Delta\bar{r}_t) - \delta_t(\cdot)\right)\right\}$$

The equations reported in this section (excluding the last two) may provide a basis for numerical simulations or empirical analysis.

6 A few [very preliminary] simulations

A very preliminary agent-based version of the model is available at `http://goo.gl/Szfxud` [9], courtesy Pietro Terna. The model can be interactively run, as a Java applet, from any Java-enabled browser, and has been written, as a fully functional prototype, in NetLogo[10].

Including heterogeneous agents within the model is easily achieved by resorting to the agent-based paradigm: besides, conflict and social mobility

[9] Long URL version: `http://eco83.econ.unito.it/terna/firms_laborforce/firms_laborforce_0.3.2.html`

[10] NetLogo is a modeling environment authored by Uri Wilensky and developed at the Center for Connected Learning and Computer-Based Modeling the the Northwestern University: `http://ccl.northwestern.edu/netlogo/`

```
to go
  tick
  produce
  if production = 0 [output-print "BIG ECONOMIC CRASH"
                     stop]
  evaluateDemand
  evaluatePrice
  changeWages
  evaluateProfit
  updateFirmSize
  createNewFirms
  moveInterestRate

  if ticks >= 1000 [stop]


end
```

Figure 1:

can be explicitly modelled, and changes in social statuses “happen” according to a stochastic process. The agents simply need to interact within the labour market and are subject to the ongoing entry/exit process.

This very preliminary version of the paper temporarily includes some simplified assumptions.

In the above picture, the different steps of the simulation are represented, in the original NetLogo code.

The above list of instructions displays the time schedule of the model: during each time step (called "tick") all the functions are executed in sequential order; there is a 1,000 ticks limit, after which the model stops running. The functions have been assigned names closely recalling the actions performed within each of them.

Some parameters are exogenously set at the beginning of the simulation process. For instance, the mumber of firms is set to 5, the population consists of 150 individuals. Other parameters will be modified in different simulation runs, such as the probability to entry, which is a function of the interest rate changes. The probability to entry generates, in its turn the probability to hire hire new workers, which is exogenously set in each simulation run.

Some of the above mentioned procedures, are worth being explicitly shown, as NetLogo code, in order to exemplify the inner workings of the ABM. In order to evaluate their own profits, each agent executes the following snippet of code:

```
to evaluateProfit
  ask firms [set profit
    price * labProductivity * count links with [end1 = myself]
    - (wageLevel + fixedAssetCost * count links with [end1 = myself]
    - (unemployedAid * count laborForce with [employed = false])
    / count firms
end
```

The wage setting mechanism is simplified in the current version of the agent-based simulation. The logics driving wage drops to the (given) "unemployment level", which are consequential to significant unemployment spells, is coded as follows. A variable, countThresholdCrossing, is used to measure how long (in ticks) the unemployment rate is stuck below a given threshold.

```
if countThresholdCrossing+ = relatedRepetitions
   [
    if wageLevel != unemploymentWageLevel
      [output-type ''cycle ''
       output-type ticks
       output-print '' set wages DOWN'' ]
    set wageLevel unemploymentWageLevel
    set countThresholdCrossing+ 0
    ]
```

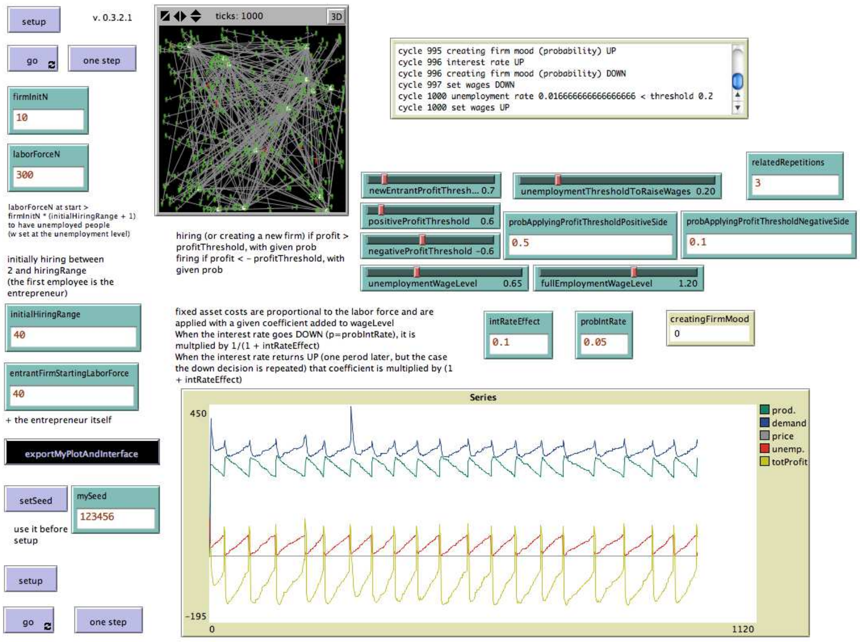

Figure 2:

The graphics of the very preliminary simulations show that the model can generate cyclical fluctuations in the economy, as an effect of the entry/exit mechanism associated to the social mobility and informational shocks.

In the following picture we may oberve the graphical user interface of the model.

Two different simulation runs are compared in the following pictures. In the first one the entrepreneurs are very reactive to changes in the economic conditions and very keen on hiring and firing workers (and so are the parameters referring to the probability of hiring and firing workers, as shown in the picture above). In the second one the entrepreneurs display low reaction to changes in the economic conditions and tend to hire and fire workers with much lower probability.

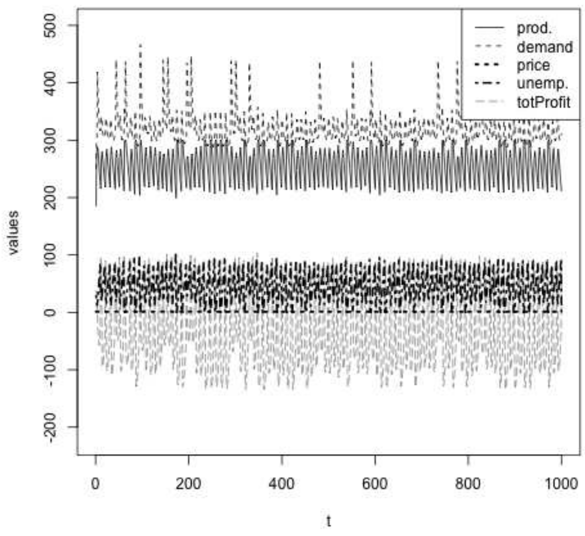

Figure 3:

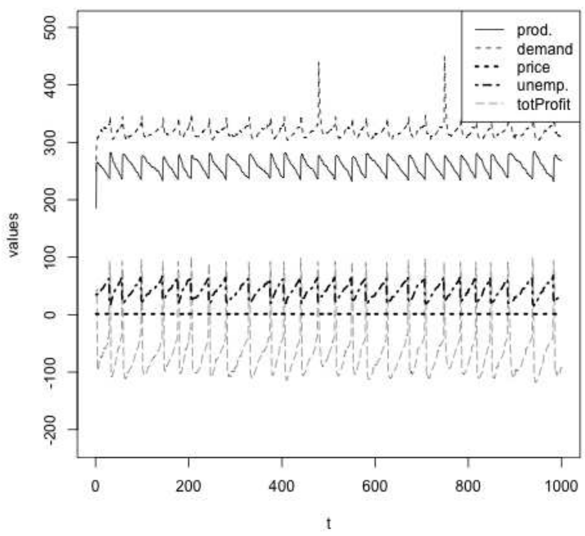

Figure 4:

The first case displays much higher volatility and noise, the second case displays a very simplified business cycle. By producing numerical simulations with several changes in parameter, we found that small changes in parameters values generates quantitatively significant changes. The model seems to approach complexity in the sense suggested by Bragin (2012).

7 Concluding remarks

We have introduced here a theoretical macroeconomic framework for an oligopolistic economy with heterogeneous agents and wage rigidity where the macroeconomic fluctuations can be determined not only by technology shocks, but also by the process of entry/exit of oligopolistic firms, potentially interacting with distributional shocks. In this framework, microfoundation is interpreted in a peculiar way, where agents have the same preferences, modelled with a conventional CRRA utility function, are heterogeneous in their budget constraint and may change their social status in each period according to a stochastic process which interacts with labour market and with the process of entry/exit.

The graphics of the very preliminary simulations show that the model can generate cyclical fluctuations in the economy, as an effect of the entry/exit mechanism associated to the social mobility and informational shocks.

This theoretical framework may be employed for further research focused on the process of entry/exit and its potential interactions with monetary policy, which may trigger higher entry or exit of oligopolistic firms and be associated to macroeconomic fluctuations.

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Appendix 1 - Microfoundation of consumption and aggregate demand

Derivation of the aggregate expenditure function

Let us recall the consumer problem:

$$\begin{aligned} \max U_t &= E_t \left[\sum_{i=0}^{\infty} \left(\frac{1}{1+\rho} \right)^i u(C_{t+i}) \right] \\ C_{t+i}, i &= 0, ...\infty \end{aligned}$$

for each $i = 0, 1, ...\infty$ where $\left(\frac{1}{1+\rho}\right)$ is the subjective discount factor for the consumers

subject to the following constraint in real terms:

$$E(a_{t+i+1}) = (1 + r_{t+i})E(a_{t+i}) + E(y_{t+i}) - C_{t+i}$$

and

$$C_{t+i} \geq 0$$

Having chosen the following analytical form for consumers' preferences:

$$u_t = \frac{C_t^{1-\gamma}}{1-\gamma}$$

Then we can define the following Bellman equation:

$$V(W_t) = \max_{c_t} \left[\frac{C_t^{1-\gamma}}{1-\gamma} + \left(\frac{1}{1+\rho} \right) E(V(W_{t+1})) \right] \tag{39}$$

Subject to

$$E(W_{t+1}) = (1+r_t)(W_t - C_t) \tag{40}$$

Where W_{t+i+1} is the state variable.

Now we assume (and later verify) that the value function has the same analytical form of the utility function, i.e.

$$V(W_t) = K \frac{W_t^{1-\gamma}}{1-\gamma} \tag{41}$$

Where K is a positive constant whose exact value will be shown later. By using the definition of $V(W_t)$41, the Bellman equation can be rewritten as follows:

$$K \frac{W_t^{1-\gamma}}{1-\gamma} = \max_{C_t} \left[\frac{C_t^{1-\gamma}}{1-\gamma} + \frac{1}{1+\rho} E\left(K \frac{W_t^{1-\gamma}}{1-\gamma} \right) \right] \tag{42}$$

hence, using the constraint 40 and deriving with respect to c_t, we get the F.O.C:

$$C_t^{-\gamma} = \frac{1+r_t}{1+\rho} K \left[(1+r_t)(W_t - C_t) \right]^{-\gamma}$$

and solving for c_t we get the consumption (demand) function:

$$C_t = \frac{1}{1 + (1+r_t)^{\frac{1-\gamma}{\gamma}} (1+\rho)^{-\frac{1}{\gamma}} K^{\frac{1}{\gamma}}} W_t$$

where K is the constant to be determined.

To complete the solution, we still use the Bellman equation 42, substitute the consumption function in it and we set:

$$M \equiv (1+r_t)^{\frac{1-\gamma}{\gamma}} (1+\rho)^{-\frac{1}{\gamma}}$$

just to simplify the notation. Then we get:

$$\begin{aligned} K\frac{W_t^{1-\gamma}}{1-\gamma} &= \frac{1}{1-\gamma}\overbrace{\left(\frac{W_t}{1+MK^{\frac{1}{\gamma}}}\right)}^{c_t}{}^{1-\gamma} + \\ &+\frac{1}{1+\rho}\frac{K_t}{1-\gamma}\underbrace{\left[(1+r_t)\frac{MK^{\frac{1}{\gamma}}}{1+MK^{\frac{1}{\gamma}}}W_t\right]^{1-\gamma}}_{W_{t+1}} \end{aligned} \tag{43}$$

The value of K satisfying 43 can be obtained by equating the coefficients of $W_t^{1-\gamma}$ in the two sides of the equation and solving for K:

$$K = \left(\frac{1}{1-M}\right)^{\gamma}$$

Under the condition $M < 1$ the consumption (expenditure) function is fully specified:

$$V(W_t) = \left(\frac{1}{1-(1+r_t)^{\frac{1-\gamma}{\gamma}}(1+\rho)^{-\frac{1}{\gamma}}}\right)^{\gamma}\frac{W_t^{1-\gamma}}{1-\gamma}$$

and

$$C(W_t) = \left[1-(1+r_t)^{\frac{1-\gamma}{\gamma}}(1+\rho)^{-\frac{1}{\gamma}}\right]W_t$$

i.e.

$$C(W_t) = \left[1-(1+r_t)^{\frac{1-\gamma}{\gamma}}(1+\rho)^{-\frac{1}{\gamma}}\right](a_t+H_t)$$

Looking at 13, having defined then:

$$\Psi_t = \Xi(r_t)\left(A_t + \frac{1}{1+r_t}\sum_{i=0}^{\infty}\left(\frac{1}{(1+E\left(r_{t+i}\right))(1+E(\iota_{t+i}))}\right)^i E(Q_{t+i})\right)$$

which allows to define the aggregate demand:

$$P_t = \Psi_t / D(W_t) \tag{44}$$

and its inverse

$$D(W_t) = \Psi_t / P_t \tag{45}$$

we have obtained then a unit elastic aggregate demand function.

If we need to have the consumption function in per capita terms, we need to rearrange 12 and get:

$$d(W_t) = \frac{\Xi(r_t)}{P_t}\left(A_t + \frac{1}{1+r_t}\sum_{i=0}^{\infty}\left(\frac{1}{(1+E\left(r_{t+i}\right))(1+E(\iota_{t+i}))}\right)^i E(q_{t+i})\right) \tag{46}$$

and, to simplify the notation:

$$\phi_t = \Xi(r_t)\left(A_t + \frac{1}{1+r_t}\sum_{i=0}^{\infty}\left(\frac{1}{(1+E\left(r_{t+i}\right))(1+E(\iota_{t+i}))}\right)^i E(q_{t+i})\right)$$

or

$$\begin{aligned}\phi_t &= \Xi(r_t)\{A_t + (1+r_t)^{-1}\sum_{i=0}^{\infty}[(1+E(r_{t+i}))(1+E(\iota_{t+i}))]^{-i} \cdot \\ &\quad \cdot E(\omega_{t+i}\xi_{t+i} + \pi_{t+i}^{in}\xi_{t+i}h_{t+i}^{in} + \pi_{t+i}^{e}\xi_{t+i}h_{t+i}^{e})\}\end{aligned}$$

and obtain a constant elastic demand function defined in per capita terms:

$$P_t = \phi_t / d(W_t) \tag{47}$$

or its inverse

$$d(W_t) = \phi_t / P_t \tag{48}$$

Finally, if we want to explicitly formalize the income distribution between labour and capital, we get:

$$\begin{aligned}d(P_t, W_t) &= \frac{\Xi(r_t)}{P_t}\{A_t + (1+r_t)^{-1}\sum_{i=0}^{\infty}[(1+E(r_{t+i}))(1+E(\iota_{t+i}))]^{-i} \cdot \\ &\quad \cdot E(\omega_{t+i}\xi_{t+i} + \pi_{t+i}^{in}\xi_{t+i}h_{t+i}^{in} + \pi_{t+i}^{e}\xi_{t+i}h_{t+i}^{e})\}\end{aligned} \tag{49}$$

Appendix 2 - Entry and output determination: the existence of a Cournt-Nash equilibrium

Madden (1998) shows that if the demand curve is uniformly elastic (and our case of unit elastic demand curve is a special case) and if all costs are sunk prior to the output being brought to market (like in our case for both entry costs and labour costs), then the model has the exact Cournot reduced form, i.e. a Cournot equilibrium exists in the second stage of a Bertrand competition game with quantity precommitments. In particular, Madden shows that under both the proportional rationing and the efficient rationing case, "the expected payoffs of the oligopolistic firms in a Bertrand-Edgeworth equilibrium are those induced by the market cleaning prices of a Cournot equilibrium if the given (first stage) aggregate quantity is at a point on the demand curve where the demand is elastic" (Madden, 1998, p.200). Since the demand curve of our model is always unit elastic, Madden's results hold. These results are rather general, since they hold for arbitrary number of firms with arbitrary costs functions and an arbitrary rationing scheme between (and including) the efficient and proportional extremes.

The optimal amount of labour $L_t^*(\varphi_t^i)$ of the generic firm $''i''$ is a monotonic function of the homogenous good φ_t^i produced at time t by firm $''i''$. At time t there are h firms, with firm i producing a quantity of homogeneous good φ^i at cost $c_i(\varphi^i), i = 1, 2, ..., n$, where $c_i : R_+ \to R_+$ is firm i's cost function and $c_i(0) = 0, i = 1, ..., n$.

Since all costs occur at the beginning of stage 1 (i.e., at the end of time t-1 and just before time t), they are all sunk and since the labour contract establishes the amount of hours to be worked by each worker, as a consequence, there is no need to distinguish between capacity and output decision. A few assumptions guarantee the existence of the aggregate equilibrium in the goods market. These assumptions correspond to those contained in Madden (1998), showing that with uniformly elastic demand function, the Kreps-Schenkman two-stage quantity-price game reduces to the Cournot model with any rationing mechanism between the efficient and proportional extremes and if all costs are sunk at the first stage.

Assumption D1

a) The aggregate demand function $D : R_{++} \to R_{++}$ **is** C^2 **with** $D'(P) < 0$ **everywhere,** $\lim\limits_{P \to 0} D(P) = +\infty$ **and** $\lim\limits_{P \to \infty} D(P) = 0$**.**

b) Having defined the market revenue function for firm $"i"$ **in**

terms of price as $\varkappa_t^i(p_i, \varphi^i)$, where $\varkappa^i : R_{++} \rightarrow R_{++}$, there exists $a \geq 0$ such that the market revenue function $\varkappa_t^i : R_{++} \rightarrow R_{++}$ is strictly increasing on $(0, a)$ and non-increasing on (a, ∞).

Madden also introduces the equivalent assumption, applying to the inverse demand (in his paper, Assumption 2**)**, that we won't consider here, since it is equivalent and unnecessary.

Part (a) of *Assumption D1* ensures that the market demand curve is well-behaved, downward sloping and therefore asymptotic to the axes (like our demand function 44). Part (b) says that market demand is inelastic at prices $p < a$ and elastic at $p > a$. The case $a = 0$ indicates *uniformly elastic demand*, while $a > 0$ admits *eventually inelastic demand*. In our case we have, of course, $a = 0$, therefore we always have uniformly elastic demand function. As Madden points out, "a well-known special case of the uniform elastic demand specification is provided by the constant elasticity demand functions", that apply to our case. Therefore, in our case, the revenues are constant and, therefore, non-increasing.

In the Cournot model firms choose output levels φ^i simultaneously, producing an aggregate output $D = \sum_{i=1}^{n} \varphi^i$.

Furthermore, we define the Cournot payoff functions π_i^c for the generic firm i as:

$$\pi_i^c(\varphi^1, ..., \varphi^h) = \begin{cases} \varkappa(p_i, \varphi^i) - c^i(\varphi^i) & if \ c^i > 0 \\ 0 & if \ c^i = 0 \end{cases} \tag{50}$$

Where $\varkappa(\varphi^i)$ is the revenue of the individual firm (assumed to be null if prices are null and there is no production), $c^i(\varphi^i)$ is the cost function. The main result by Madden (1998) is the following. In the Kreps-Scheinkman model firms choose output levels simultaneously at stage 1. Then, with sunk production costs and production levels that are common knowledge, the firms simultaneously choose the prices at stage 2. In Kreps and Scheinkman (1983), and in Osborne and Pitchik (1986) and Vives (1986) the demand at stage 2 is rationed among the firms according to the efficient (or surplus-maximizing) rule; the following one is the demand faced by firm i applying the production vector φ^i if the announced prices for stage 2 are p;

$$\Delta_{iE}(\varphi, p) = \max\left\{0, \left[D(p_i). \sum_{p_k<p_i} \varphi^k\right] \frac{\varphi^i}{\sum\limits_{p_k=p_i} \varphi^k}\right\} \tag{51}$$

With this rationing rule, the firms charging less than firm i serve those consumers with the highest valuation of the good and the term in square bracket is shared among the firms charging p_i, in proportion to their production level. At an opposite extreme is the proportional (or Beckmann, 1967) rule, used by Allen and Hellwig (1986):

$$\Delta_{iP}(\varphi, p) = \max\left\{0, \left[1 - .\frac{\varphi^k}{\sum\limits_{p_k<p_i} D(p_k)}\right] D(p_i) \cdot \frac{\varphi^i}{\sum\limits_{p_k=p_i} \varphi^k}\right\} \tag{52}$$

In this case the consumers served by lower priced firms are chosen randomly; $\varphi^k/D(p_k)$ is the fraction of consumers served by k. Then we can define assumption D2 (corresponding to assumption 3 in Madden, 1998):

Assumption D2

The rationed demand function at stage 2 of the Kreps-Scheinkman game for firm $i, i = 1, ...h$ is $\Delta_i : R^n_+ \times R^n_+ \to R_+$ and satisfies

i) $\Delta_{iE}(\varphi, p) \leq \Delta_i(\varphi, p) \leq \Delta_{ip}(\varphi, p), (\varphi, p) \in R^n_+ \times R^n_{++}$

ii) Δ_i **only depends on these** p_i **for which** $\varphi^i > 0$

As Madden points out, Allen and Hellwig (1986) imply that under proportional rationing and under his assumptions concerning the demand (which correspond to the assumptions of this paper), the unique payoffs in a Bertrand-Edgeworth equilibrium (i.e. the second stage of the game among the oligopolistic firms, still at time t, after the new entrants have entered the market and after the quantities have been set, in stage one at time t) aggregate quantity is at a point in the demand curve where the demand is elastic. In our model, given the assumptions on our aggregate demand, this is always the case.

Theorem 1 (Madden, 1998). **Suppose that assumptions D1 and D2 hold and that the quantity $D(P) = \sum \varphi^i$ is given at stage 1 of the Kreps-Scheinkman game, if demand is elastic at $D(P)$, then the expected revenue in any Nash equilibrium of the stage 2 following $D(P)$ is $\pi_i^c(\varphi^1, ..., \varphi^h)$ (See Theorem 1 in Madden, 1998, p. 204 for the proof)**. When $D(P) > 0$, Madden shows that Theorem 1 is proved by the lemmas 1 and 2

Lemma 1 (Madden, 1998). **Suppose that assumptions D1 and D2 hold and that the quantity $D(P) = \sum \varphi^i$ is given at stage 1 of the Kreps-Scheinkman game, then in the following stage 2 of the game:**

a) the pure strategy $p_i = D^{-1}(Q)$ guarantees firm i a revenue of $\varphi^i D^{-1}(Q)$;

b) any price $p_i < D^{-1}(Q)$ is strictly dominated for firm i, if $\varphi^i > 0$.

Lemma 2 (Madden, 1998). **Suppose that assumptions D1 and D2 hold and that the quantity $D(P) = \sum \varphi^i$ is given at stage 1 of the Kreps-Scheinkman game. Suppose the demand is elastic at Q, then the pure strategies $p_i = D^{-1}(Q), i = 1, ..., n$** are a Nash equilibrium of the stage 2 subgame following Q.

The meaning of *Lemma 2* is that a Nash deviation in which a firm raises price from the suggested equilibrium cannot be beneficial since it earns the firm at most the same share of the market revenue at the higher price, which is lower because of the elasticity.

Furthermore, Madden (1998), defines the correspondence between $\pi_i^c(\varphi^1, ..., \varphi^h)$ and the "exact Cournot reduced form", meaning that if the quantities are chosen at stage 1 of the Kreps-Scheinkman game, then the second stage subgame Nash equilibrium that follows, always induce expected payoffs equal to the Cournot payoffs. Following Madden, we can characterize the equilibrium as follows:

Theorem 2 (Madden, 1998). **If assumptions D1 and D2 hold and the quantity $D(P) = \sum^i \varphi^i$ is given at stage 1 of the Kreps-Scheinkman game, if demand is elastic at $D(P)$, then the Kreps-Scheinmnam model has the exact Cournot reduced form (See theorem 2 in Madden, 1998, p. 204 for the proof)**.

Madden last theorem (theorem 3 in his paper) shows that if the demand is uniformly elastic (like in our model) and assumptions D1 and D2 hold, there exists at least one pure strategy Cournot-Nash equilibrium and if the firm costs are symmetric (like in our model) the Cournot-Nash equilibrium is unique.

Finally, if the costs are symmetric (which is our case for the firms at time t, once entry has taken place), the Cournot-Nash equilibrium is unique.

Theorem 1 and the exact Cournot reduced form resulting from theorem 2 do not apply to the case of an inelastic demand curve, but this is not the case of our model because we have a unit elastic demand curve.