Contract contingency in vertically related markets*

Emanuele BACCHIEGA[†1], Olivier BONROY[‡2,3], and Emmanuel PETRAKIS[§4]

[1]Dipartimento di Scienze Economiche, Alma Mater Studiorum - Università di Bologna, Italy.
[2]INRA, UMR 1215 GAEL, F-38000 Grenoble, France.
[3]Université Grenoble Alpes, UMR 1215 GAEL, F-38000 Grenoble, France.
[4]Department of Economics, University of Crete, Rethymnon 74100, Greece.

March 15, 2016

Preliminary, please do not cite without the authors' permission.

Abstract

In this paper we analyze the optimal behavior of an upstream monopolist that produces an input that is necessary to two downstream firms, which use it to produce variants of a vertically differentiated commodity. The upstream producer may enter an exclusive relationship with one downstream firm only or sign non-exclusive contracts with both, in which case it may decide whether to offer contingent or non-contingent contracts. Contingent contracts may contain terms that are finalized, within bargaining pair, in the occurrence the negotiation in the other pair irreversibly breaks down. Non-contingent contracts cannot contain such clauses. Once decided the contract characteristics, the actual contractual terms are set through secret negotiations between the upstream and downstream firm(s) through the generalized Nash bargaining solution. We show that when the upstream firm has a "high" bargaining power it prefers an exclusive contract with the high-quality producer. By contrast, for lower bargaining weights, it selects non-exclusive contracts. In particular, for "intermediate" bargaining power it prefers contingent contracts, while for "low" bargaining power it prefers non-contingent contracts.

Keywords: Vertical relationships, exclusive vs. non-exclusive relationships, contract contingency, two-part tariff, product differentiation.

JEL classification: D43, L13, L14.

*We wish to thank Tommaso Valletti and Christian Wey for very useful comments and discussion and the audiences at XXXV Jornadas de Economía industrial, Alicante 2015 and 5^{th} EIEF-Unibo-Bocconi IGIER Workshop on Industrial Organization, Rome 2015. The usual disclaimer applies.

† ✉emanuele.bacchiega@unibo.it

‡ ✉olivier.bonroy@grenoble.inra.fr

§ ✉petrakis@uoc.gr

1 Introduction

In this paper we delve into the optimal choices of an upstream monopolist that may sign a supply contract with two downstream producers. In particular, we analyze the role of the contingency of contracts to determine the decision by the upstream monopolist whether to enter an exclusive versus non-exclusive relationships with the downstream partners when bargaining power both upstream and downstream.[1] Specifically, we consider an industry where an upstream producer may enter in a contractual relationship with two downstream retailers for the supply of an input that is necessary for the production of the final good. The downstream firms are endowed with different technologies, more in detail, one of them may increase the quality of the input to vertically differentiate its product from that of the rival (see Gabszewicz and Thisse, 1979; Shaked and Sutton, 1983).[2] The upstream firm may commit to offer an exclusive supply contract to one of the downstream firms, or to offer non exclusive contract to both. In the case of non-exclusive contract, we consider both the cases of contingent and non-contingent contracts. Since we model the contracting stage between the upstream and downstream firms through the generalized Nash bargaining solution, the difference between contingent and non-contingent contracts turns out to be related to the outside options that are attributed to firms in the different cases, as in Milliou and Petrakis (2007). In particular, a non-contingent contract signed within a pair upstream firm-downstream firm cannot contain terms that are contingent on the (out-of-equilibrium) disagreement in the negotiation between the upstream firm and the *other* downstream firm. As a consequence, the outside option for the firms are determined by the equilibrium contractual terms only. By contrast, a contingent contract signed within a pair can contain specific terms that are finalized in the event of a breakdown in the negotiation in the *other* pair. In this case, the outside options for the firms fully internalize the implications of the negotiation failure in the other pair. As clearly pointed out by Milliou and Petrakis (2007), the assumption of non-contingent contracts "captures the idea that parties cannot commit to a *permanent* and *irrevocable* breakdown in their negotiations." By contrast, the assumption of contingent contract allows the firms to -possibly-definitively stop their negotiations without having reached an agreement. We carry out our analysis under the assumption that the upstream firm, if opting for non-exclusive contracts, may decide whether to make it contingent on not. Given this choice, the contractual terms are bargained over with the downstream firms.

When selecting which type of contract to offer (exclusive versus non-exclusive), the upstream firm faces the following basic trade off. With an exclusive contract competition down-

[1] According to (Bazerman and Gillespie, 1998, p. 155), "the terms of a contingent contract are not finalized until the uncertain event in question–the contingency–takes place."

[2] One could also imagine, for example, that the upstream firm holds a patent for a basic technology that is necessary to produce the final goods, in this case the contract between the upstream and downstream firms takes the form of a licensing contract, see e.g. Faulí-Oller *et al.* (2013) and the references therein contained.

stream is absent altogether, therefore, under non-linear contracts, the integrated outcome may be replicated. However, if the upstream firm commits to an exclusive contract, it has, in the industry under scrutiny, no outside option if the negotiation with the downstream partner ends in a failure. This entails that the apportioning of monopoly profits *only* depends on the bargaining weights of the upstream and downstream firms. In particular, the smaller the upstream bargaining power, the smaller the share of the profit this firm is able to extract. With non-exclusive contracts, instead, competition downstream erodes a part of the aggregate producer surplus, which cannot be restored under the assumption of secret offer of contracts (McAfee and Schwartz, 1994). Yet, with non-exclusive contracts, the upstream firm has *non negative outside options* that improve its position in the negotiations *for any bargaining weights distribution*. In this case the *characteristics* of the contract as far as its contingency is concerned, determine the *value* of the outside option.

We characterize the equilibria of a three stage game where the upstream firm first decides which type of non-linear contract (exclusive/non-exclusive, contingent/non-contingent) to be offered to the downstream firm(s), then simultaneously and secretly bargains over the contractual terms with all the firms it has offered a contract to. In the last stage the firms that have signed a supply contract simultaneously set their output prices. We show that, the characteristics of the offered contract(s) crucially depend upon the distribution of bargaining power between the upstream and downstream firms. In particular, when it has most of the bargaining power, the upstream firm prefers to offer an exclusive supply contract to the downstream firm that can produce the good of highest quality. The intuition is that when the upstream has most of the bargaining power, all else equal, this firm can extract a great share of the overall producer surplus. As a consequence, it prefers to avoid creating downstream competition, which reduces the level of aggregate profit. The choice to offer the contract to the firm producing the high-quality good is due to the fact that this good generates a higher consumer surplus to be extracted. As the relative bargaining power of the upstream firm decreases, all else equal, the share of surplus going to this firm shrinks, which makes the upstream firm willing to offer non-exclusive contracts, to improve its bargaining position through the creation of an outside option in each negotiation. This comes at the cost of reducing the total amount of producer surplus generated, because of the competition that is generated downstream. In this case, we show that for "intermediate" levels of bargaining power, the upstream firm prefers non-exclusive, contingent contracts. The reason is that, in this parameter range, the outside option granted to the upstream firm is larger than that with non-contingent contracts. Finally, when the bargaining power of the upstream is "low", the upstream firm prefers non-contingent contracts, which now insure the largest outside options. We also show that, from an aggregate welfare standpoint, exclusive contracts always reduce welfare relative to non-exclusive ones, although an exclusive contact is efficient, whereas non-exclusive contracts are not. Then we further explore the mechanics of our model by assuming that the downstream firms set quantities instead of prices. Under quantity competition the upstream monopolist never selects the non-exclusive non-contingent contract, because, in this case, the outside options that they imply are smaller than the ones with contingent contracts, confirming the intuition of the price competition scenario. Further, we claim that out results are not restricted to the case of vertical product differentiation, indeed they hold, qualitatively unaffected, for the utility function with substitute products and representative consumer as in Bowley (1924); Spence (1976); Dixit (1979).

Our paper connects to several strands of literature. First, it relates to the literature on vertical contracting, see, e.g. Horn and Wolinsky (1988); O'Brien and Shaffer (1992); McAfee and Schwartz (1994, 1995); Rey and Vergé (2004). Because of the presence of market power downstream, our paper is also in connection with the literature about the effects of buyer power (see, e.g. Inderst and Wey, 2003, 2007). More closely related to our analysis are the papers by Milliou and Petrakis (2007); Alipranti *et al.* (2014). In the first of these works the authors are interested in studying the merger incentives in markets characterized by successive oligopolies, whereas in the second they compare Cournot and Bertrand competition in vertically related markets. In both these papers, differently from ours, he focus is mainly on non-contingent contracts. Also, Miklós-Thal *et al.* (2011) develop a model where downstream retailers offer take-it-or-leave-it contracts to an upstream producer that may be contingent on exclusive relationships, and show that contingency may lead to the replication of monopoly outcomes. From another standpoint, Iozzi and Valletti (2014) delve into the role of negotiation breakdown observability in determining the outside option of an upstream supplier facing multiple downstream retailers, when negotiations are determined through the Nash bargaining solution. Finally, our paper also contributes to the discussion about the choice by a monopolist firm whether or not to offer a *pooling* or a *separating* menu in markets of vertical product differentiation, see Acharyya (1998); Bacchiega *et al.* (2013) and Chambolle and Villas-Boas (2015).

The remainder of the paper is organized as follows. Section 2 presents the model, Section performs the equilibrium analysis and presents the main results. Section 4 explores the robustness of our resuts. Finally, Section 5 provides a brief conclusion.

2 The model

2.1 Structure

Let us first sketch the basic structure of our model and the order of the moves.

Firms and Market Structure. Assume an upstream monopolist that produces at no cost a basic input and may sell it to two downstream firms. Downstream firms use the input as the only production factor to produce a final good, on a one-to-one basis. One of these firms has a proprietary technology that allows it to increase, again at no cost, the quality of the input. Label this product "high-quality good" (h) and its downstream producer "high-quality firm". The other firm has not such a technology, therefore may only sell the "basic" version of the good, label this product "low-quality good" (l) and its producer "low-quality firm". We assume that the upstream supplier decides whether to offer or not an exclusive supply contract to one of the downstream firms. When the exclusive contract is chosen the upstream monopolist commits to trade with one downstream firm only, therefore the market structure is a chain of monopolies. When, by contrast, the upstream supplier choses to sign non-exclusive contracts, it trades with both downstream firms and, consequently, two substitute goods are potentially available on the market. In both cases, we allow the parties to bargain over non-linear, two-part, supply contracts.

Demand. A continuum of consumers of unit mass is uniformly distributed with unit density over the interval $[0, 1]$, each of them considers purchasing one unit of the good(s) available for consumption. A generic consumer θ is characterized by the indirect utility function

$$U(\theta, u_i) = \begin{cases} \theta u_i - p_i & \text{when purchasing one unit of good } i, \\ 0 & \text{when abstaining from consumption.} \end{cases} \tag{1}$$

where u_i is the (given) quality level of good i and p_i is its price (by our assumptions, u_i only depends upon the downstream firm selling the good). Depending on the decision by the upstream monopolist to offer exclusive or non-exclusive contracts, one or two goods are available for consumption. The standard marginal consumer approach yields the demand(s) for the good(s), in the first case, the demand is

$$D_m(p_m) = 1 - \frac{p_m}{u_i}, \tag{2}$$

where the subscript m indicates "monopoly" and $i \in \{h, l\}$ depending on which firm the supply contract has been signed with. In this case, the consumer surplus is

$$CS_m(p_m) \equiv \int_{\frac{p_m}{u_i}}^{1} (\theta u_i - p_m) d\theta, \tag{3}$$

In the case of non-exclusive contracts, two goods are available, their demands are

$$D_h(p_h, p_l) = 1 - \frac{p_h - p_l}{u_h - u_l}, \quad D_l(p_h, p_l) = \frac{p_h - p_l}{u_h - u_l} - \frac{p_l}{ul}, \tag{4}$$

with $u_h > u_l > 0$ being the quality levels of the two goods. The surplus of the consumers is

$$CS(p_h, p_l) \equiv \int_{\frac{p_l}{u_l}}^{\frac{p_h - p_l}{u_h - u_l}} (\theta u_l - p_l) d\theta + \int_{\frac{p_h - p_l}{u_h - u_l}}^{1} (\theta u_h - p_h) d\theta. \tag{5}$$

Timing. The interaction of the firms unravels around three stages. At the first, the upstream supplier decides whether or not to offer an exclusive contract to the downstream firms, in the case the contracts are non-exclusive, the monopolist selects whether to make them contingent or non-contingent. At the second stage, bargaining occurs between the upstream firm and the firm(s) it has decided to offer a contract to. Finally at the third stage, the downstream firms that have signed a supply contract with the upstream monopolist simultaneously set the market prices.[3]

3 Equilibrium analysis

We now develop the equilibrium analysis of our game. We start with the case of an exclusive contract and, then, move to that of non-exclusive contracts. In this second circumstance, we analyze the two cases of non-contingent and contingent contracts.

3.1 Exclusive contract

In the case of exclusive contracts there is only one good available in the final market, therefore the downstream firm is a monopolist in the sales of the final good. Let $T_m \equiv (w_m, t_m)$ be the two-part contract signed by the upstream and downstream firm, where w_m is the per-unit fee and t_m is the fixed fee. The profit of the upstream and downstream firms are, respectively:

$$\Pi_m(p_m, T_m) = D_m(\cdot) w_m + t_m, \quad \pi_m(p_m) = D_m(p_m, T_m)(p_m - w_m) - t_m. \tag{6}$$

The price stage is quickly dealt with. Indeed, the downstream firm maximizes its profit by setting $p_m = \frac{u_m + w_m}{2} \equiv \hat{p}_m(w_m)$. By substituting back into (6), we obtain that

$$\Pi_m(\hat{p}_m(w_m), T_m) = \frac{(u_i - w_m) w_m}{2u_m} + t_m \equiv \hat{\Pi}_m(T_m) \tag{7}$$

[3] Section 4 deals with the alternative assumption that firms compete in quantities.

and

$$\pi_m(\hat{p}_m(w_m), T_m) = \frac{(u_i - w_m)^2}{4u_i} - t_m \equiv \hat{\pi}_m(T_m). \tag{8}$$

We now tackle the bargaining stage of our model. We assume that the contractual terms are set through the generalized Nash bargaining solution, let $\mu \in]0,1[$ (res. $1-\mu$) be the bargaining weight of the upstream (res. downstream) firm. Since the upstream supplier is committed to offer an exclusive contract, in case of failure to reach an agreement neither the upstream nor the downstream firm can sell the product, therefore we set to zero the outside options for both firms. The Nash product is, therefore:

$$NP_m(T_m) = \hat{\Pi}_m(\cdot)^{\mu} \hat{\pi}_m(\cdot)^{1-\mu}. \tag{9}$$

The maximization of (9) with respect to w_m and t_m yields $w_m = 0$ and $t_m = \frac{u_i}{4}\mu$. As standard in this case, the two-part contract is set in such a way to maximize the joint profit of the chain by setting the input price equal to the upstream marginal production cost, and to apportion the total profit between the upstream and downstream firm through the fixed part of the tariff, according to the bargaining weights. As a consequence the profits accruing to the upstream firm are $\frac{u_i}{4}\mu$. Clearly, the actual value of the profit depends on the quality level of the good, which, in turn, is determined by the identity of the firm that has received the offer of the exclusive contract. It is however immediate to observe that the profit of the upstream producer (and that of the downstream firm too) is higher when the high-quality good is sold under the exclusive contract.

We summarize the preceding observations in the following.

Lemma 1. *If the upstream firm opts for an exclusive contract, it offers it to the downstream high-quality firm. The equilibrium contractual terms are $w_m^* = 0$ and $t_m^* = \frac{u_h}{4}\mu$. In this case the monopoly equilibrium price and demand are respectively $p_m^* = \frac{u_h}{2}$, $D_m^* = \frac{1}{2}$ and the profits to the upstream and downstream firms are $\Pi_m^* = \frac{u_h}{4}\mu$ and $\pi_m^* = \frac{u_4}{4}(1-\mu)$. The consumer surplus is $CS_m^* = \frac{u_h}{8}$.*

3.2 Non-Exclusive contracts

Let us now consider the case where the upstream firm decides to offer two supply contracts to the downstream firms. For the sake of brevity, we will often refer to the contract signed between the upstream firm and the high(low)-quality firm as "the high(low)-quality contract". We assume that the contracts are interim observable: the contracting occurs simultaneously and separately within each upstream-downstream pair, but, once the contracts have been signed, their terms become known to all the parties, as in, e.g. McAfee and Schwartz, 1995. It is well-known that, in such a situation, the vertical relationships between the upstream supplier and each of the downstream firms are affected by opportunism. Specifically, within each pair composed by the upstream supplier and the downstream firm i, an incentive exists to secretly reset the contractual terms at their own advantage and at the expense of the other downstream firm j $(i, j \in \{h, l\}, i \neq j)$. One of the consequences is that multiple equilibria may exist in this case. To deal with this issue and have a unique outcome, we invoke *pairwise proofness* in the equilibrium contracts (O'Brien and Shaffer, 1992; Milliou and Petrakis, 2007; Alipranti *et al.*, 2014).

In the following, we analyze the two separate cases of non-contingent and contingent contracts. A contract between the upstream firm and the downstream firm $i \in \{h, l\}$ is non-contingent if its terms cannot depend on the disagreement in the other pair upstream firm-downstream firm $j \in \{h, l\}$. By contrast, a contract is contingent when it depends on the disagreement in the other pair. Stated differently, under non-contingent contracts, the contractual terms that are executed between the upstream producer and the downstream firm i in the (out-of-equilibrium) occurrence of failure in the bargaining between the upstream firm and downstream firm j *are the same* as those carried out in the case of agreement in both negotiations. A contingent contract, by contrast, specifies different contractual terms that will be executed in the case of agreement or in the case of disagreement in the other pair. For a thorough discussion of contract contingency see Milliou and Petrakis (2007).

The price stage is unaffected by contract (non-)contingency and will be quickly dealt with. Given the demand system in (4) and the supply contracts $T_i \equiv (w_i, t_i)$ signed between the upstream firm and downstream firm $i \in \{h, l\}$ the profits of the downstream firms are

$$\pi_i(p_h, p_l, T_i) = D_i(\cdot)(p_i - w_i) - t_i, \quad i \in \{h, l\}. \tag{10}$$

By solving the system defined by the first-order conditions $\frac{\partial \pi_i(\cdot)}{\partial p_i} = 0$ and observing that second-order ones are fulfilled as long as $u_h > u_l > 0$ it is easy to obtain the optimal prices at the last stage of the game, which are the following

$$\hat{p}_h(w_h, w_l) \equiv \frac{u_h[2(u_h - u_l + w_h) + w_l]}{4u_h - u_l}, \quad \hat{p}_l(w_h, w_l) \equiv \frac{u_l(u_h - u_l + w_h) + 2u_h w_l}{4u_h - u_l}. \tag{11}$$

At these prices, the demands are

$$\hat{D}_h(w_h, w_l) \equiv \frac{2u_h^2 - u_h(2u_l + 2w_h - w_l) + u_l w_h}{4u_h^2 - 5u_h u_l + u_l^2}, \quad \hat{D}_l(w_h, w_l) \equiv \frac{u_h[u_h(u_l - 2w_l) - u_l(u_l - w_h - w_l)]}{u_l(u_h - u_l)(4u_h - u_l)} \tag{12}$$

As in the case of exclusive contract, plugging the prices (11) back into the profits of the downstream firms returns

$$\hat{\pi}_h(T_h, w_l) \equiv \frac{\left[2u_h^2 + u_h(w_l - 2(u_l + w_h)) + u_l w_h\right]^2}{(u_h - u_l)(4u_h - u_l)^2} - t_h, \tag{13}$$

$$\hat{\pi}_l(T_l, w_h) \equiv \frac{u_h\left[u_h(u_l - 2w_l) + u_l(w_h + w_l - u_l)\right]^2}{u_l(u_h - u_l)(4u_h - u_l)^2} - t_l. \tag{14}$$

Similarly to the monopoly case, the profit of the upstream firm is defined as $\Pi(p_h, p_l, T_h, T_l) \equiv D_h(\cdot)w_h + D_l(\cdot)w_l + t_h + t_l$, which, at the optimal prices writes

$$\hat{\Pi}(T_h, T_l) \equiv \frac{u_l\left[w_h^2(u_l - 2u_h) + 2u_h w_h(u_h - u_l)\right] + u_h u_l w_l(u_h - u_l + 2w_h) + u_h w_l^2(u_l - 2u_h)}{u_l(u_h - u_l)(4u_h - u_l)} + t_h + t_l. \tag{15}$$

3.2.1 Bargaining stage

We are now in a position to tackle the bargaining stage. The case of non-contingent contracts will be treated first.

Non-contingent contracts. As hinted above, in the case of non contingent contracts the pair –say– upstream firm-downstream firm h cannot include in the contract terms that would be executed only in the (out-of-equilibrium) case of negotiation failure for the other pair upstream firm-downstream firm l. As clearly explained in (Milliou and Petrakis, 2007, p.970), this implicitly amounts to assuming that "the bargaining parties cannot commit to a *permanent* and *irrevocable* breakdown in their negotiations." This entails that the outside option for the upstream firm when bargaining with firm i *depends on the terms of the equilibrium contract signed with firm* j, $i, j \in \{h, l\}, i \neq j$. Let $T_i^N \equiv (w_i^N, t_i^N), i \in \{h, l\}$ the equilibrium non-contingent contract signed within the pair upstream firm-downstream firm i. In the bargaining with –say– firm h, the outside option of the upstream firm is the profit it would earn in the case of failure in the negotiation with firm h itself. Should this occur, the upstream firm still expects to sign the contract T_l^N with firm l, but, in this case, the downstream firm is alone on the final market and will consequently behave as a monopolist. The outside option for the upstream firm is, therefore, $\hat{\Pi}_m(T_l^N)$, whereas the outside option for the downstream firm h is zero, and the same applies for the bargaining with firm l.[4] Accordingly, the Nash

[4] Inderst and Wey (2003); de Fontenay and Gans (2005) develop an explicit strategic bargaining game to model the idea that the negotiation between parties can come to a breakdown.

products are:

$$NP_h^N(T_h, T_l^N) = \left[\hat{\Pi}(T_h, T_l^N) - \hat{\Pi}_m(T_l^N)\right]^\mu \hat{\pi}_h(T_h, w_l^N)^{1-\mu}, \quad (16)$$

$$NP_l^N(T_h^N, T_l) = \left[\hat{\Pi}(T_h^N, T_l) - \hat{\Pi}_m(T_h^N)\right]^\mu \hat{\pi}_l(T_l, w_h^N)^{1-\mu}. \quad (17)$$

Standard maximization techniques allow to find the equilibrium non-contingent contracts, which are as follows.[5]

$$T_h^N = (w_h^N, t_h^N) = \left(\frac{u_l}{4}, \frac{8\mu u_h^3 - 4(1+\mu)u_h^2 u_l + 2(1-\mu)u_h u_l^2 - (1-\mu)u_l^3}{32u_h^2}\right), \quad (18)$$

and

$$T_l^N = (w_l^N, t_l^N) = \left(\frac{u_l^2}{4u_h}, \frac{u_l[2\mu u_h - (3-\mu)u_l]}{32u_h}\right). \quad (19)$$

Substitution back into prices, demands and profits yields the following.

Lemma 2. *Under non-contingent contracts, the equilibrium contractual terms are* (18) *and* (19). *The equilibrium output prices are equal to* $p_h^N = \frac{2u_h - u_l}{4}$, $p_l^N = \frac{u_l}{4}$, *which determine the equilibrium demands* $D_h^N = \frac{1}{2}$, $D_l^N = \frac{1}{4}$. *The equilibrium profits of the upstream firm are* $\Pi^N = \frac{8\mu u_h^3 - 2\mu u_h^2 u_l + (1-\mu)u_h u_l^2 - (1-\mu)u_l^3}{32u_h^2}$, *and those of the downstream firms are* $\pi_h^N = \frac{(1-\mu)(2u_h-u_l)^2(2u_h+u_l)}{32u_h^2}$ *and* $\pi_l^N = \frac{(1-\mu)u_l(2u_h+u_l)}{32u_h}$. *Finally, the equilibrium consumer surplus is* $CS^N = \frac{u_h}{8} + \frac{5}{32}u_l$.

Direct inspection of the values of equilibrium prices, demands and profits reveals that they are all non negative, provided that $\mu \in]0, 1[$.

The upstream prices are larger than the marginal production cost because of the endeavor by the upstream firm to dampen price competition, yet, because of the secret offer of contracts, the *commitment effect* (McAfee and Schwartz, 1994) prevents the contractual terms from maximizing the aggregate industry profits. Furthermore, they do not depend on the bargaining weights, because it is known that with two-part contracts, wholesale prices are used to maximize the joint surplus of each upstream-downstream firm pair, which clearly does not depend on the sharing parameter μ. The fixed parts of the tariffs, instead may be positive or negative, depending on the value of μ, as reported in the following.

[5] Maximize each Nash product $N_i^N(\cdot)$ w.r.t. t_i first, then plug the solution back into the Nash product i itself and maximize it w.r.t. w_i. Finally, solve the system of the four conditions so obtained to have the equilibrium contractual terms. Second-order conditions are locally satisfied, which, together with the uniqueness of the maximizers, insures the uniqueness of the solution. The detailed (and cumbersome) calculations are available upon request.

Lemma 3. *Under non-contingent contracts*

$$t_h^N \gtreqless 0 \quad \Leftrightarrow \mu \quad \gtreqless \frac{u_l(4u_h^2 - 2u_h u_l + u_l^2)}{(2u_h - u_l)^2(2u_h + u_l)} \in]0, 1[\tag{20}$$

$$t_l^N \gtreqless 0 \quad \Leftrightarrow \mu \quad \gtreqless \frac{3u_l}{2u_h + u_l} \in]0, 1[. \tag{21}$$

When the bargaining weight of the upstream firm is "large" (μ is "high"), the share of the aggregate profit within each pair accruing to this firm through the fixed fee is "large" (and positive). As μ decreases, the bargaining weight of the upstream firm decreases and that of the downstream symmetrically increases, which makes the fixed fees shrink. Eventually, when μ is "small" the transfers t_i^N become *negative*: the upstream firm partially compensates, through the fixed fees, the downstream ones.[6] As a last remark, it is easy to ascertain that, for $0 < \mu < \frac{u_l}{3u_h + u_l}$, the upstream firm is actually subsidizing the low-quality firm, in fact the revenue collected through the per-unit fee is smaller than the (negative) transfer: $w_l^N D_l^N < |t_l^N|$.

Exclusive vs. non-exclusive, non-contingent contracts. Let us now compare the market outcomes obtained under exclusive and non-exclusive, non-contingent contracts. First, it is instructive that the per-unit fee is larger in the case of non-exclusive contracts. This is as expected, because with an exclusive contract there is no need to relax the downstream price competition by an increase of the marginal production costs of the downstream firms. As a consequence, with an exclusive contract the upstream price is set equal to the upstream marginal production cost, namely zero, and, consequently, the integrated outcome obtains. This notwithstanding, the price of the high-quality good is lower in the case of non-exclusive contracts, due to the competition in the final market. Indeed, when faced with the option whether or not to offer non-exclusive contracts, the upstream firm faces a trade-off between increasing the sales of its input by serving the low-quality firm as well, and creating a fiercer competition downstream due to the presence of two substitute products. Non-exclusive contracts have a beneficial effect on the profits of the upstream firms for two reasons. First, they make the overall sales of the input to increase ($\frac{3}{4} = D_h^N + D_l^N > D_m^* = \frac{1}{2}$), which allows the upstream firm to collect the fees from the two downstream firms instead of from the high-quality firm only. Second, non-exclusive contracts create an outside option for the upstream producer, which improves, for any μ, the upstream bargaining position relative to the downstream high-quality firm. On the other hand, however, with non-exclusive contracts, the downstream competition is increased, which erodes the total producer surplus, in fact

[6] Recall that the upstream firm always receives positive payments through the wholesale prices.

$\Pi^N + \pi_h^N + \pi_l^N = \frac{u_h}{4} - \frac{u_l}{16} < \frac{u_h}{4} = \Pi_m^* + \pi_m^*$).[7] To complete the analysis of the forces at stake, it is worth mentioning that the consumer surplus is always larger in the case of non-exclusive contracts and the same holds for total welfare, standardly defined as the sum of the profit of the firms and of the surplus of consumers. This is not surprising, in fact non-exclusive contracts increase the downstream competition, which lowers prices and ultimately increases the aggregate level of consumption. We state

Lemma 4. *Under non-contingent contracts, the total welfare and consumer surplus are maximized with non-exclusive contracts, whereas producer surplus is maximized with an exclusive contract.*

Let us now consider the choice of the upstream firm between exclusive and non-contingent, non-exclusive contracts.

Proposition 1. *Let* $\tilde{\mu} \equiv \frac{u_h u_l - u_l^2}{(2u_h - u_l)(u_h + u_l)} \in]0, 1[$. *Under non-contingent contracts the upstream producer offers non-exclusive contract when* $\mu \in]0, \tilde{\mu}]$*, it offers an exclusive contracts to the high-quality firm when* $\mu \in [\tilde{\mu}, 1[$.

Proof. Follows from direct comparison of Π_m^* and Π^N. □

When μ is "low" the bargaining weight of the downstream firms is high, which translates, all else equal, into "low" fixed fees that are bargained over at the second stage, and consequently, into a low profit extraction by the upstream firm. In this case, the best option for the upstream producer is to offer non-exclusive contracts to the downstream firms. By doing so, the upstream producer increases the demand for its input (and charges positive per-unit fees), and improves its bargaining power thanks to the creation of outside options, in particular relative the high-quality firm. It is instructive to observe that this may come at a (implicit) cost, in fact, for $\mu < \frac{u_l(4u_h^2 - 2u_h u_l + u_l^2)}{(2u_h - u_l)^2(2u_h + u_l)}$, the fixed fees of non-exclusive, contingent contracts are *negative*. This means that, through the fixed fees, the upstream firm pays back a part of the profit reaped with the wholesale prices to the downstream partners. The outside options, however, create a positive lower bound to the net profit the upstream firm may obtain at the negotiation stage, which is, instead, equal to zero with the exclusive contract. By contrast, when μ is "high", *ceteris paribus*, the upstream firm can extract a large share of the downstream profits, therefore it prefers an exclusive contract which replicates the outcome of the vertically integrated structure, without incurring in the aggregate profit erosion due to competition in the final market.

[7] This clearly implies that, for the high-quality downstream firm, an exclusive contract brings about higher profits than non-exclusive ones.

Contingent contracts. Contingent contracts capture the idea that the bargaining pairs can come to a permanent and irrevocable breakdown in their negotiations (see, e.g. Inderst and Wey (2003); de Fontenay and Gans (2005)). Hence, a contingent contact between the upstream firm and the downstream firm i contains specific terms that will be executed in the case the negotiation between the upstream firm and the downstream firm j fails $(i, j \in \{h, l\}, i \neq j)$. In our case, as in Milliou and Petrakis (2007), if the negotiation between the upstream firm and the downstream firm j fails, the downstream firm i is now monopolist in the final good market. In this occurrence, the payoff for the upstream firm is as under exclusive contracts, namely $\frac{u_i}{4}\mu$ (see Section 3.1), which is therefore the outside option of the upstream firm in the bargaining with the downstream firm i . Accordingly, the Nash products in this case are the following.

$$NP_h^C(T_h, T_l^C) = \left[\hat{\Pi}(T_h, T_l^C) - \frac{u_l}{4}\mu\right]^{\mu} \hat{\pi}_h(T_h, w_l^C)^{1-\mu}, \tag{22}$$

$$NP_l^C(T_h^C, T_l) = \left[\hat{\Pi}(T_h^C, T_l) - \frac{u_h}{4}\mu\right]^{\mu} \hat{\pi}_l(T_l, w_h^C)^{1-\mu}. \tag{23}$$

Unlike the case of non-contingent contracts, concavity of the two functions (22) and (23) at the critical points identified by the FOCs is not always guaranteed. However, a sufficient condition to have concavity at the unique solution of the system of the FOCs is that $\mu < \frac{3}{4}$.[8] To develop a complete analysis of the case of contingent contracts, therefore, we will deal separately with the two parameter regions $\mu \in]0, \frac{3}{4}[$ and $\mu \in [\frac{3}{4}, 1[$.

Assume, first, that $\mu \in]0, \frac{3}{4}[$; maximizing $NP_i^C(\cdot)$ with respect to w_i and $t_i, i \in \{h, l\}$, and solving the set of equations so obtained yields the following equilibrium contracts.[9]

$$T_h^C = (w_h^C, t_h^C) = \left(\frac{u_l}{4}, \frac{4(2\mu - \mu^2)u_h - (3 + \mu)u_l)}{16(2 - \mu)}\right), \tag{24}$$

and

$$T_l^C = (w_l^C, t_l^C) = \left(\frac{u_l^2}{4u_h}, \frac{u_l[(-1 + 6\mu - 4\mu^2)u_h - (2 - \mu)u_l]}{16(2 - \mu)u_h}\right). \tag{25}$$

It is a matter of simple calculations to ascertain that, at the optimal contracts (24) and (25) we have $lim_{\mu \to \frac{3}{4}} \hat{\pi}_l(T_l^C, w_h^C) = 0$. This is intuitive: as the bargaining weight of the upstream increases, the optimal transfers increase as well, until the profit of the low-quality firm is nil.

Now assume that $\mu \in [\frac{3}{4}, 1[$. In this region (22) is concave whereas (23) no longer is. In order to treat the case of non-exclusive contracts in this region, we proceed as follows. First, we observe that in this region the low-quality firm cannot enjoy positive profits, nonetheless,

[8] The binding constraint is that on the concavity of $NP_l^C(\cdot)$. In fact, for $\mu \in [\frac{3}{4}, 1[$, the profit of the low-quality downstream producer at the solution of the set of the FOCs becomes negative, thereby violating the participation constraint for that firm.

[9] The maximization follows the steps outlined in footnote 5.

to make it sign the contract, it must not incur in losses as well. As a consequence, the optimal contract must be such that the low-quality firm reaps zero profits. Second, the low-quality contract must still maximize the bilateral profits of the upstream and low-quality downstream firms which, by virtue of the foregoing observation, completely accrue to the upstream firm. To sum up, the optimal contracts are obtained by solving the following set of conditions.

$$\max_{w_h,t_h} NP_h^C(T_h,T_l^C), \quad \hat{\pi}_l(T_l,w_h^C) \overset{t_l}{=} 0, \quad \max_{w_l}[\hat{\Pi}(T_h^C,T_l)+\hat{\pi}_l(T_l,w_h^C)]. \tag{26}$$

The equilibrium contract are easily obtained and are the following,

$$T_h^C=(w_h^C,t_h^C)=\left(\frac{u_l}{4},\frac{4u_h\mu+u_l(-3+(3-4\mu)\mu))}{16}\right), \tag{27}$$

and

$$T_l^C=(w_l^C,t_l^C)=\left(\frac{u_l^2}{4u_h},\frac{u_l(u_h-u_l))}{16u_h}\right). \tag{28}$$

Substitution back into prices, demands and profits allows us to state

Lemma 5. *Under contingent contracts*

(i) In the region $\mu \in]0,\frac{3}{4}[$*, the equilibrium contractual terms are (24) and (25), which yield equilibrium output prices* $p_h^C=\frac{2u_h-u_l}{4}, p_l^C=\frac{u_l}{4}$*, and demands* $D_h^C=\frac{1}{2}$*,* $D_l^C=\frac{1}{4}$*. The equilibrium profits of the upstream firm are* $\Pi^C=\frac{\mu[4u_h-u_l+4(1-\mu)(u_h+u_l)]}{16(2-\mu)}$ *and those of the downstream firms* $\pi_h^C=\frac{(1-\mu)[4u_h(2-\mu)-5u_l]}{16(2-\mu)}$ *and* $\pi_l^C=\frac{u_l(1-\mu)(3-4\mu)}{16(2-\mu)}$*.*

(ii) In the region $\mu \in [\frac{3}{4},1[$*, the equilibrium contractual terms are (27) and (28), which yield equilibrium output prices* $p_h^C=\frac{2u_h-u_l}{4}, p_l^C=\frac{u_l}{4}$*, and demands* $D_h^C=\frac{1}{2}$*,* $D_l^C=\frac{1}{4}$*. The equilibrium profits of the upstream firm are* $\Pi^C=\frac{\mu[4u_h+u_l(3-4\mu)]}{16}$ *and those of the downstream firms* $\pi_h^C=\frac{(1-\mu)[4u_h-u_l(1+4\mu)]}{16}$ *and* $\pi_l^C=0$*.*

(iii) The consumer surplus is $CS^C=\frac{u_h}{8}+\frac{5}{32}u_l$*.*

The values of prices and profits reported in the preceding Lemma are positive. As a first remark, it is worth mentioning that the variable part of the tariff in the contingent contracts (24, 25 and 27, 28) are the same as that in the case of non-contingent contracts (see 18 and 19). This is not surprising, because, with two-part tariff contract, the variable part of the tariff is used to maximize the joint surplus in each pair net of the values of the outside options, and the fixed part apportions the net surplus of each pair according to the bargaining weights. In the present paper (as in Milliou and Petrakis, 2007), both under contingent and non-contingent contracts, the value of the outside options in the bargaining within pair i does not depend on w_i, $i \in \{h,l\}$, which entails that the first-order conditions relative to

w_i for the maximization of the net surplus are the same under the two types of contracts. Furthermore, since the upstream prices coincide in the two cases, the downstream prices are the same as well. This, in turn, entails that equilibrium demands, and consumer surplus coincide too (compare Lemmata 2 and 5). By contrast, the profits accruing to the firms do not coincide, because the fixed fees of the equilibrium contracts are different under contingent and non-contingent contracts, reflecting the differences in the outside options.

As in the previous case, the fixed parts of the tariffs may be negative when $\mu \in [0, \frac{3}{4}]$, as reported in the following Lemma.

Lemma 6. *Under contingent contracts*

(i) In the region $\mu \in]0, \frac{3}{4}[$,

$$t_h^C \gtreqless 0 \quad \Leftrightarrow \mu \quad \gtreqless \frac{8u_h - u_l - \sqrt{64u_h^2 - 64u_h u_l + u_l^2}}{8u_h} \in]0, \frac{3}{4}[, \tag{29}$$

$$t_l^C \gtreqless 0 \quad \Leftrightarrow \mu \quad \gtreqless \frac{6u_h + u_l - \sqrt{20u_h^2 - 20u_h u_l + u_l^2}}{8u_h} \in]0, \frac{3}{4}[. \tag{30}$$

(ii) In the region $\mu \in [\frac{3}{4}, 1[$, $t_h^C > 0$ *and* $t_l^C > 0$.

The explanation of part (i) of the previous Lemma 6 parallels that provided in the case of non-contingent contracts.

Finally, it is worth noting that for $\mu \in [\frac{3}{4}, 1[$, the fixed part of both tariffs is always positive and higher than under non-contingent contracts.

Exclusive vs. non-exclusive, contingent contracts. As explained above, the upstream prices under contingent contracts coincide with those of non-contingent contracts, and, consequently, the downstream prices, demands aggregate profits and consumer surplus coincide as well. It is immediate, therefore, to state

Lemma 7. *Under contingent contracts, the total welfare and consumer surplus are maximized with non-exclusive contracts, whereas producer surplus is maximized with an exclusive contract.*

Let us now look at the choice if the upstream firm between an exclusive non-exclusive, contingent contracts.

Proposition 2. *Under contingent contracts the upstream firm offers non-exclusive contracts when* $\mu \in]0, \frac{3}{4}[$, *it offers an exclusive contract to the high-quality firm when* $\mu \in [\frac{3}{4}, 1[$.

Proof. Follows from direct comparison of Π_m^* and Π^C. $\square$

As under non-contingent contracts, when μ is "large", the high-quality firm offers an exclusive contract to the high-quality firm. Intuitively, when the bargaining power of the upstream is "high" the outside option in the negotiation with the low-quality firm ($\frac{u_h}{4}\mu$) becomes large, in turn pushing up the value of the (positive) fixed fee t_l^C to an extent that, given the wholesale prices, all the profit of downstream low-quality firm is not sufficient to cover it. By contrast, for all $\mu \in]0, \frac{3}{4}[$, the upstream firm prefers non-exclusive, contingent contracts to an exclusive one. The intuition is again that by offering non-exclusive contracts the upstream firm accepts a decrease in the aggregate producer surplus relative to the exclusivity case, but improves its bargaining positions by creating non-negative outside options. This allows the upstream producer to extract a larger share of the aggregate profit relative to the exclusive contract case.

3.3 Contract choice

We are now in a position to put together the results of Propositions 1 and 2. We state

Proposition 3. *The upstream producer offers*

(i) non exclusive, non-contingent contracts for $\mu \in]0, \tilde{\tilde{\mu}}]$,

(ii) non exclusive, contingent contracts for $\mu \in [\tilde{\tilde{\mu}}, \frac{3}{4}[$,

(iii) an exclusive contract to the high-quality downstream firm for $\mu \in [\frac{3}{4}, 1[$.

Proof. Follows directly from Propositions 1 and 2 and the observation that $\Pi^N > \Pi^C$ for all $\mu < \tilde{\tilde{\mu}} \equiv \frac{2u_l(u_h - u_l)}{10u_h^2 + u_h u_l - u_l^2}$, with $\tilde{\tilde{\mu}} < \tilde{\mu} < \frac{3}{4}, \forall u_h > u_l > 0$. $\square$

To understand points (*i*) and (*ii*) of the foregoing Proposition, it is useful to look at the behavior of the outside options of the upstream firm under the two non-exclusive contract types. For $\mu \in]0, \frac{3}{4}[$, under contingent contracts the outside option is the share of the (integrated) monopoly profit obtained with a per unit input price equal to zero ($w_i^m = 0$). Under non-contingent contracts, by contrast, the unit prices are never equal to zero, due to the commitment effect, indeed $w_i^N > 0, i \in \{h, l\}$. Since w_i^N does not depend on μ, it remains positive even when $\mu \to 0$, thus a decrease in this parameter reduces the fixed part of the tariff that contributes to define the outside option of the upstream firm, but *does not* decrease the part of the non-contingent outside option due to the variable fees. This ultimately entails that the outside option of the upstream firm is larger under non-contingent contracts than under non contingent contracts for μ "close enough" to 0. In particular, for the low-quality contract, $\frac{u_l}{4}\mu < \frac{(u_l - w_l^N)w_l^N}{2u_l} + t_l^N \Leftrightarrow \mu < \frac{u_l(u_h - u_l)}{u_h(6u_h - u_l)}$, and for the high-quality

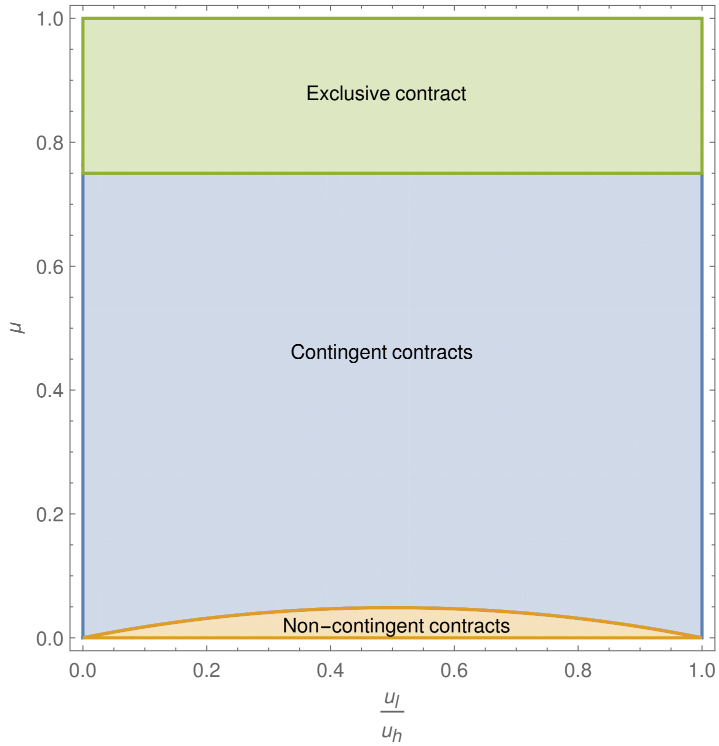


Figure 1: Equilibrium contract partition under price competition.

contract $\frac{u_h}{4}\mu < \frac{(u_h - w_h^N)w_h^N}{2u_h} + t_h^N \Leftrightarrow \mu < \frac{u_l(u_h - u_l)}{4u_h^2 + 2u_h u_l - u_l^2}$. As we assume that the choice of the non-exclusive contract type does not affect the bargaining weight μ, the choice of the type of contract by the upstream firm affects its bargaining position only through the value of its outside options. Furthermore, because the optimal variable fees are the same under the two contract types (which entails that market prices and consequently demands are also the same), choosing the contract type that guarantees the highest profit for the upstream firm amounts to selecting the one which contemplates the most favorable fixed fees. Yet, it is easy to ascertain that, for all $\mu \leq \tilde{\tilde{\mu}}$, t_i^N and $t_i^C, i \in \{h, l\}$ are *negative*: the upstream firm is *subventioning* the downstream ones. Thus, the optimal contract type for the upstream firm for "low" μ is the one that contemplates the least total transfers towards the downstream firms. It is in fact a matter of calculations to show that

$$|t_h^N + t_l^N| < |t_h^C + t_l^C| \Leftrightarrow \mu < \tilde{\tilde{\mu}}. \tag{31}$$

This explains why, when μ is "low" (case*(i)*), the upstream firm prefers non-contingent contracts, whereas, for larger values of μ ($[\tilde{\tilde{\mu}}, \frac{3}{4}[$, case *(ii)*), contingent contracts are preferred. Eventually, when μ is very high (case *(iii)*), the bargaining power of the upstream firm is high and, *ceteris paribus*, it can extract a large share of the aggregate producer surplus. Thus, it prefers an exclusive contract to avoid the profit dissipation due to downstream competition. Figure 1 depicts the contract type equilibrium partition in the space of the relative product qualities and upstream bargaining weight. The horizontal axis reports the relative product quality levels $\frac{u_l}{u_h} \in]0,1[$, where 0 is the maximum differentiation level and 1 stands for homogeneous products, the vertical axis reports the upstream firm bargaining weight $\mu \in]0,1[$.

4 Extensions

4.1 Quantity competition

In this Section, we explore the choice of the type of contract by the upstream monopolist under the alternative assumption of quantity competition. It is known that quantity setting reduces the competitive pressure on firm in one-tier industries (see e.g. Singh and Vives, 1984), but this result is not robust when considering multi-layer industries (Alipranti *et al.*, 2014). The main insight of this type of extension is that, under quantity competition, non-exclusive, non-contingent contracts are never selected by the upstream firm. This type of contract is always dominated by an exclusive contract with the high-quality producer. Yet, when it has a "low" bargaining weight, the upstream firm may want to offer non-exclusive, contingent contracts to the downstream firms. In this case the intuition is similar to that obtained under price competition. When it has most of the bargaining weight (μ is high), the monopolist can extract most of the surplus from downstream, hence it prefers to hinder product market competition at the cost of relinquishing the outside options. By contrast, when it has a low bargaining weight (μ is low), the monopolist prefers to create outside options at the cost of having competition in the final market. As most of the analysis parallels that of price competition, we will expand only to point out the most relevant differences and we will introduce further notation only when needed.

To tackle the analysis, the first step is to invert the demand systems (2) and (4) to obtain the inverse demands, which are

$$p_m(D_m) = u_i(1 - D_m), \tag{32}$$

in the case of exclusive contract, and

$$p_h(D_h, D_l) = u_h(1 - D_h) - u_l D_l, \quad p_l(D_h, D_l) = u_l(1 - D_h - D_l) \tag{33}$$

in the case of non exclusive contracts. Consumer surplus in either case coincides with (3) and (5).

4.1.1 Exclusive contract

Under exclusive contracts the downstream firm is a monopolist and, as is well known, in this situation the distinction between price and quantity competition is immaterial. Accordingly, the analysis of Section 3.1 carries through unchanged, yielding the same results of Lemma 1.

4.1.2 Non-exclusive contracts

Quantity setting. As in the analysis of price competition, the contingency of contracts does not influence the last stage of the game. The downstream firms profits are the following.[10]

$$\pi_i^c(D_h, D_l, T_i) = D_i \left[p_i(\cdot) - w_i\right] - t_i, \quad i \in \{h, l\}. \tag{34}$$

Each profit function π_i is concave in D_i as long as $u_i > 0$, so by taking the first-order conditions and solving their system, the optimal quantities at the last stage are easily obtained

$$\hat{D}_h^c(w_h, w_l) \equiv \frac{2u_h - u_l - 2w_h + w_l}{4u_h - u_l}, \quad \hat{D}_l^c(w_h, w_l) \equiv \frac{u_h u_l - 2u_h w_l + u_l w_h}{u_l(4u_h - u_l)}. \tag{35}$$

The associated prices are obtained by substituting (35) back into (33) and are

$$\hat{p}_h^c(w_h, w_l) \equiv \frac{2u_h^2 - u_h(u_l - 2w_h - w_l) - u_l w_h}{4u_h - u_l}, \quad \hat{p}_l^c(w_h, w_l) \equiv \frac{u_h u_l + 2u_h w_l + u_l w_h - u_l w_l}{4u_h - u_l}, \tag{36}$$

and the downstream firms profits

$$\hat{\pi}_h^c(T_h, w_l) \equiv \frac{u_h(2u_h - u_l - 2w_h + w_l)^2}{(4u_h - u_l)^2} - t_h, \quad \pi_l^c(T_l, w_h) = \frac{[u_l(u_h + w_h) - 2u_h w_l]^2}{u_l(4u_h - u_l)^2} - t_l. \tag{37}$$

The profit of the upstream producer is defined similarly to price competition, namely $\Pi(D_h, D_l, T_h, T_l) \equiv D_h w_h + D_l w_l + t_h + t_l$. At the quantities $\hat{D}_h(\cdot)$ and $\hat{D}_l(\cdot)$ this profit boils down to

$$\hat{\Pi}^c(T_h, T_l) \equiv \frac{w_h u_l(2u_h - u_l - 2w_h + w_l) + w_l(u_h u_l - 2u_h w_l + u_l w_h)}{u_l(4u_h - u_l)} + t_h + t_l. \tag{38}$$

Bargaining. As in the analysis of price competition, we tackle non-contingent contracts first.

Non-contingent contracts. The approach under quantity competition is the same as that under price competition, therefore the outside option in the negotiation with firm i is $\hat{\Pi}^m(T_j^{cN})$, where T_j^{cN} is the equilibrium contract signed with firm $j; i, j \in \{h, l\}, i \neq j$. The

[10] The superscript c stands for "Cournot competition".

Nash products therefore are

$$NP_h^{cN}(T_h, T_l^{cN}) = \left[\hat{\Pi}^c(T_h, T_l^{cN}) - \hat{\Pi}_m(T_l^{cN})\right]^{\mu} \hat{\pi}_h^c(T_h, w_l^{cN})^{1-\mu}, \tag{39}$$

$$NP_l^{cN}(T_h^{cN}, T_l) = \left[\hat{\Pi}^c(T_h^{cN}, T_l) - \hat{\Pi}_m(T_h^{cN})\right]^{\mu} \hat{\pi}_l^c(T_l, w_h^{cN})^{1-\mu}. \tag{40}$$

As for price competition, standard maximization techniques lead to the following contracts (the solutions of the system of the FOCs of (39) and (40) are unique and the Nash products are locally concave at these solutions),

$$T_h^{cN} = (w_h^{cN}, t_h^{cN}) = \left(-\frac{u_h u_l}{4u_h - 2u_l}, \frac{[\mu(2u_h - u_l) + u_l]}{8}\right), \tag{41}$$

$$T_l^{cN} = (w_l^{cN}, t_l^{cN}) = \left(-\frac{u_l^2}{2(2u_h - u_l)}, \frac{u_h u_l[\mu(2u_h - u_l) + u_l]}{8(2u_h - u_l)^2}\right). \tag{42}$$

The first thing that is worth noticing is that the per unit fees are *below* the upstream marginal production cost (zero, in our case). As is well known, under quantity competition, given the contract signed with one downstream producer, the upstream firm and the other downstream firm have an incentive to reduce the per-unit input price in order to expand their production (see, e.g. Alipranti *et al.*, 2014). The fixed fees, by contrast are always positive and increasing in the bargaining weight of the upstream supplier. By substituting back into prices, demands and profits is it easy to ascertain that, although the prices, demands and downstream firms profits are non-negative for the relevant parameter constellations $u_h > u_l > 0 \cup 0 < \mu < 1$, the upstream firm profit is non negative only as long as $\mu^{cN} \equiv \frac{u_l^2(3u_h - u_l)}{(2u_h - u_l)\left(4u_h^2 - 3u_h u_l + u_l^2\right)} \leq \mu < 1$. This is intuitive, the upstream firm subventions the downstream ones through the per-unit fee, thus can only recoup this cost (and reap profits) through the fixed fees. When μ is low, however, the fixed fees are low and fail to compensate the outlays due to the per-unit price. Notice, furthermore, that because of the negativity of the unit input price, both the outside options for the upstream firm become negative when μ is low, thus contributing to reduce the equilibrium profit of the upstream firm.[11] It is immediate, therefore to conclude that non-exclusive, non-contingent contracts can be proposed by the upstream firm only when $\mu^{cN} \leq \mu < 1$. We state

Lemma 8. *Let $\mu^{cN} \leq \mu < 1$. Under non-contingent contracts and Cournot competition, the contractual terms are (41) and (42). The equilibrium quantities are $D_h^{cN} = \frac{1}{2}$ and $D_l^{cN} = \frac{u_h}{2(2u_h - u_l)}$, the induced equilibrium prices are $p_h^{cN} = \frac{u_h(u_h - u_l)}{2u_h - u_l}$ and $p_l^{cN} = \frac{u_l(u_h - u_l)}{2(2u_h - u_l)}$. The profit of the upstream firm are $\Pi^{cN} = \frac{8\mu u_h^3 - 10\mu u_h^2 u_l + (5\mu - 3)u_h u_l^2 + (1-\mu)u_l^3}{8(2u_h - u_l)^2}$ and those of the downstream ones are $\pi_h^{cN} = \frac{(1-\mu)(2u_h - u_l)}{8}$ and $\pi_l^{cN} = \frac{(1-\mu)u_h u_l}{8(2u_h - u_l)}$. Finally, the equilibrium consumer surplus*

[11] In this case the participation constraint of the upstream monopolist is violated, as in McAfee and Schwartz (1994).

is $CS^{cN} = \frac{u_h\left(4u_h^2+u_hu_l-u_l^2\right)}{8(2u_h-u_l)^2}$.

By comparing the outcomes reported in Lemmata 1 and 8 we state

Proposition 4. *Under quantity competition the upstream monopolist never selects non-exclusive, non-contingent contracts.*

Proof. Follows from the observation that, for all $0 < \mu < 1$, $\Pi_m^* > \Pi_m^{cN}$. □

As observed by Alipranti *et al.* (2014), contrary to the case of single-layer industries, in multi-layer industries the upstream firm may enjoy lower profits under Cournot competition than under Bertrand competition, due to the fact that this firm subventions the downstream ones. It is easy to ascertain that, in our case as well, $\Pi^N > \Pi^{cN} \forall \mu \in]0,1[$. The decrease in the profit of the upstream firm is so large that the profit under non-exclusive, non-contingent, contract may become negative ($0 < \mu < \mu^{cN}$) and is always lesser than the profit earned under exclusive contracts. In this case, in fact, aside the absence of competition downstream, both the fixed and the variable part of the tariff are always positive.

Contingent contracts. Under contingent contracts the Nash products are

$$NP_h^{cC}(T_h, T_l^{cC}) = \left[\hat{\Pi}^c(T_h, T_l^{cC}) - \frac{u_l}{4}\mu\right]^\mu \hat{\pi}_h^c(T_h, w_l^{cC})^{1-\mu}, \tag{43}$$

$$NP_l^{cC}(T_h^{cC}, T_l) = \left[\hat{\Pi}^c(T_h^{cC}, T_l) - \frac{u_h}{4}\mu\right]^\mu \hat{\pi}_l^c(T_l, w_h^{cC})^{1-\mu}. \tag{44}$$

Like under price competition, two separate cases have to be dealt with in this case. In fact the Nash products (43) and (44) are concave at the unique solutions of the system of the FOCs for $0 < \mu < \frac{3u_h^2-4u_hu_l+u_l^2}{(2u_h-u_l)^2} \equiv \mu^{cC}$. In this case the optimal contrats are

$$T_h^{cC} = (w_h^{cC}, t_h^{cC}) = \left(-\frac{u_hu_l}{4u_h-2u_l}, \frac{4(2-\mu)\mu u_h^3 - \Phi u_h^2u_l + \Gamma u_hu_l^2 + (1-\mu)u_l^3}{4(2-\mu)(2u_h-u_l)^2}\right), \tag{45}$$

$$T_l^{cC} = (w_l^{cC}, t_l^{cC}) = \left(-\frac{u_l^2}{2(2u_h-u_l)}, \frac{u_l\left[-\Psi u_h^2 + 4(1-\mu)^2u_hu_l - (1-\mu)^2u_l^2\right]}{4(2-\mu)(2u_h-u_l)^2}\right), \tag{46}$$

where $\Phi \equiv (13-4\mu)\mu - 5, \Gamma \equiv (6-\mu)\mu - 4$ and $\Psi \equiv 1 - 6\mu + 4\mu^2$. It is straightforward to observe that the (negative) unit prices do not change relative to the non-contingent contracts scenario. This is again due to the fact that the input prices are set to so as to maximize the profit of the pair upstream-downstream firm, which do not depend on the contingency of contracts.

When $\mu^{cC} < \mu < 1$, the profit of the low-quality firm at the contracts (45) and (46) becomes negative notwithstanding the subvention by the upstream through the input price. In this parameter region, as under price competition (see (26)), it is possible to construct a non-exclusive, contingent contract that satisfies with equality the participation constraint of the low-quality firm and maximizes its joint profit with the upstream producer by solving the problem

$$\max_{w_h, t_h} NP_h^{cC}(T_h, T_l^{cC}), \quad \hat{\pi}_l^c(T_l, w_h^{cC}) \overset{t_l}{=} 0, \quad \max_{w_l}[\hat{\Pi}^c(T_h^{cC}, T_l) + \hat{\pi}_l^c(T_l, w_h^{cC})], \tag{47}$$

which yields

$$T_h^{cC} = (w_h^{cC}, t_h^{cC}) = \left(-\frac{u_h u_l}{4u_h - 2u_l}, \frac{4\mu u_h^3 + \Delta u_h^2 u_l - (3-4\mu)\mu u_h u_l^2 + (1-\mu)\mu u_l^3}{4(2u_h - u_l)^2}\right), \tag{48}$$

$$T_l^{cC} = (w_l^{cC}, t_l^{cC}) = \left(-\frac{u_l^2}{2(2u_h - u_l)}, \frac{u_h^2 u_l}{4(2u_h - u_l)^2}\right), \tag{49}$$

with $\Delta \equiv 1 - \mu - 4\mu^2$. By plugging back into the relevant functions, we state

Lemma 9. *Under contingent contracts and quantity competition*

(i) In the region $\mu \in]0, \mu^{cC}]$ the contractual terms are as in (45) and (46). The equilibrium output levels are $D_h^{cC} = \frac{1}{2}$ and $D_l^{cC} = \frac{u_h}{4u_h - u_l}$, which induce the prices $p_h^{cC} = \frac{u_h(u_h - u_l)}{2u_h - u_l}$ and $p_l^{cC} = \frac{u_l(u_h - u_l)}{2(2u_h - u_l)}$. The downstream firms profits are $\pi_h^{cC} = \frac{(1-\mu)\left[4(2-\mu)u_h^3 - (13-4\mu)u_h^2 u_l + (6-\mu)u_h u_l^2 - u_l^3\right]}{4(2-\mu)(2u_h - u_l)^2}$ and $\pi_l^{cC} = \frac{(1-\mu)u_l\left[3u_h^2 - \mu(2u_h - u_l)^2 - 4u_h u_l + u_l^2\right]}{4(2-\mu)(2u_h - u_l)^2}$. The upstream firm profits are $\Pi^{cC} = \frac{\mu\left[4(2-\mu)u_h^3 - 5u_h^2 u_l - (2-3\mu)u_h u_l^2 + (1-\mu)u_l^3\right]}{4(2-\mu)(2u_h - u_l)^2}$.

(ii) In the region $\mu \in [\mu^{cC}, 1[$ the contractual terms are as in (48) and (49). The equilibrium output levels are $D_h^{cC} = \frac{1}{2}$ and $D_l^{cC} = \frac{u_h}{4u_h - u_l}$, which induce the prices $p_h^{cC} = \frac{u_h(u_h - u_l)}{2u_h - u_l}$ and $p_l^{cC} = \frac{u_l(u_h - u_l)}{2(2u_h - u_l)}$. The downstream firms profits are $\pi_h^{cC} = \frac{(1-\mu)\left[u_h\left(4u_h^2 - 5u_h u_l + u_l^2\right) - \mu u_l(2u_h - u_l)^2\right]}{4(2u_h - u_l)^2}$ and $\pi_l^{cC} = 0$. The upstream firm profits are $\Pi^{cC} = \frac{\mu\left[4u_h^3 - u_h^2 u_l - 3u_h u_l^2 - \mu u_l(2u_h - u_l)^2 + u_l^3\right]}{4(2u_h - u_l)^2}$.

(iii) The consumer surplus is $CS^{cC} = \frac{u_h\left(4u_h^2 + u_h u_l - u_l^2\right)}{8(2u_h - u_l)^2}$.

As in Lemma 5, the optimal quantities (and therefore the prices and consumer surplus), to not change in the two regions described in the foregoing Lemma. Firms profits, conversely, are different, again for the same reasons reported in Lemma 5. It is interesting to observe that in the region $\mu \in]0, \mu^{cC}]$ the optimal fixed fee in the high-quality contract is always positive, but the fixed fee in the low-quality contract may become negative, implying that the upstream firm subventions the low quality producer.[12] For $\mu \in [\mu^{cC}, 1[$, by contrast, the optimal fixed fees are always positive.

[12] In particular, this happens when $0 < \mu < \frac{3u_h^2 + u_l^2 - u_h\left(\sqrt{5u_h^2 - 4u_h u_l + u_l^2} + 4u_l\right)}{(2u_h - u_l)^2} < \mu^{cC}$.

4.1.3 Contract Choice.

By Proposition 4 we know that, for the upstream firm, the non-exclusive, non-contingent contract is always a choice dominated by the exclusive contract. As a consequence, the choice of the type of contract reduces to the comparison of the profits reaped from the exclusive and the non-exclusive, contingent contracts. We state

Proposition 5. *Under quantity competition the upstream producer offers*

(i) A non-exclusive, contingent contract for $\mu \in]0, \mu^{cC}]$.

(ii) An exclusive contract to the high-quality producer for $\mu \in [\mu^{cC}, 1[$.

Proof. Follows from Proposition 4 and observing that $\Pi_m^* \gtreqless \Pi^{cC} \Leftrightarrow \mu \gtreqless \frac{3u_h^2 - 4u_h u_l + u_l^2}{(2u_h - u_l)^2} = \mu^{cC}$. □

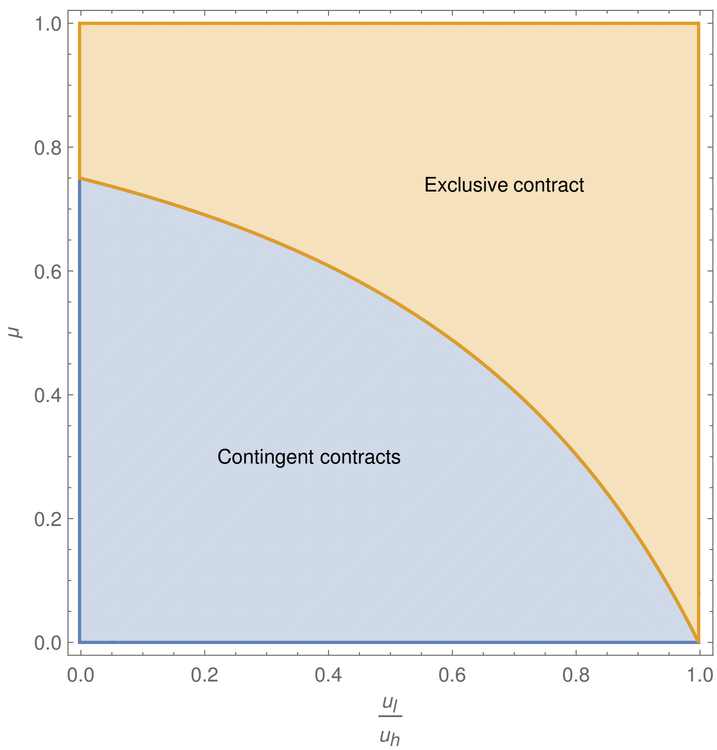


Figure 2: Equilibrium contract partition under quantity competition.

Under quantity competition the upstream firm profit with non-exclusive contracts are reduced relative to Bertrand competition, due to the commitment effect that pushes the input prices below their marginal production cost. Non-contingent contracts are such that, when the bargaining weight of the upstream tends to zero, this firm cannot recoup the outlay due to the negative input price through the –small– fixed fee, and it prefers an exclusive contract with the high-quality producer relative to the non-contingent contract. Yet, for μ low enough (case (i)), the upstream firm prefers the non-exclusive contingent contract (which always guarantees positive outside options) to the exclusive one because, in this case, it uses the outside options to improve its bargaining position towards the downstream firms. This, as already explained, comes at the cost of generating competition downstream. When μ is large (case (ii)), conversely, this firm prefers to avoid creating competition, at the cost of having a lower outside option. Figure 2 depicts the equilibrium contract partition in the product differentiation-upstream bargaining weight space. The foregoing Figure shows that, relative to the case of quantity competition, the parameter region where the upstream firm prefers the exclusive contract to the non-exclusive, contingent one is larger. This is again due to the fact that, under price quantity competition the profits the upstream firm reaps with non-exclusive relationships are generally lower than under price competition, which increases the incentive to enter exclusive relationships instead.

4.2 Alternative utility specification

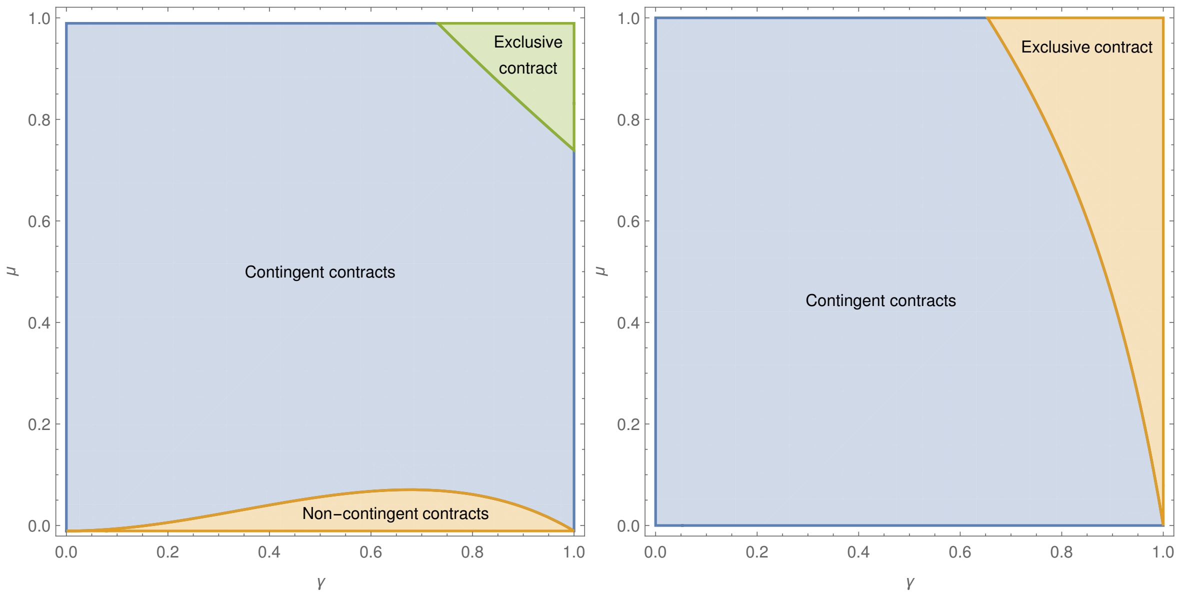
One natural question that may arise at this point is to which extent our results rely on the assumption of vertically differentiated products with Mussa and Rosen (1978) utility function (MU, henceforth). In this section we claim that the same qualitative results hold for the alternative approach to product differentiation followed by Bowley (1924), Spence (1976) and Dixit (1979) (BSD, henceforth), for example. Under this approach there is a representative consumer that may select the quantities D_i of two goods $i \in \{1, 2\}$ to consume. This consumer is endowed with the following quadratic utility function

$$U(D_1, D_2) = \alpha D_1 + \alpha D_2 - \frac{1}{2}\left(D_1^2 + D_2^2 + 2\gamma D_1 D_2\right), \tag{50}$$

resulting in the following linear direct demand system

$$D_1(p_1, p_2) = \frac{\alpha(1-\gamma) - p_1 + \gamma p_2}{1-\gamma^2}, \quad D_2(p_1, p_2) = \frac{\alpha(1-\gamma) - p_2 + \gamma p_1}{1-\gamma^2}, \tag{51}$$

where p_1 and p_2 are the prices of goods 1 and 2 respectively, $\alpha > 0$ and $\gamma \in]0, 1[$ represents the degree of substitutability between the goods. Clearly, when γ is close to zero the goods are independent from one another, whereas when γ tends to one the goods become perfectly substitutable, corresponding to the cases of completely different products and homogeneous products respectively. The most remarkable differences between this approach and the one of the previous Sections are that under the utility function in (50) (i) the goods are symmetric from the point of view of the consumer and (ii) the consumer is not restricted to purchase



(a) Price competition (b) Quantity competition

Figure 3: Equilibrium contract partition with BSD utility.

discrete quantities of the commodities.

A model identical to the one presented in the previous sections of this paper can be set-up and solved along the same lines under this alternative utility specification. We will not bother the reader here with an analysis that replicates the foregoing one and limit ourselves to show the results diagrammatically.[13] Figure 3 depicts the equilibrium contract partition under the quadratic utility function, panel (3a) represents price competition and panel (3b) quantity competition. In both cases the x-axis reports the degree of product substitutability $\gamma \in]0,1[$ and the y-axis the upstream firm bargaining weight $\mu \in]0,1[$.

As can easily be seen, the results obtained under the MR utility specification hold qualitatively unchanged under the BSD one. The most remarkable difference is the shape of the region where an exclusive contract is selected by the upstream firm. In fact, with the MR utility function, this region exists, under both price and quantity competition, *for any* degree of product differentiation *provided that* μ is large enough (namely $\mu > \frac{3}{4}$). Conversely, with the BSD utility function, the region exists *only* for "high enough" substitutability, *even if* μ is "large" (compare Figures (1) and (2), (3)). The reason for this difference rests on the type of product differentiation that underlies the MR and BSD utility functions. Under MR the goods are vertically differentiated, so, by definition all consumers *a priori* prefer the high-quality product to the low-quality one. The co-existence of high- and low-quality variants of the good, in this case, has a profit erosion effect that is particularly detrimental on the revenues

[13]Needless to say, the whole calculations are available upon request.

obtained from the sales of the high-quality good, which is the good that allows for a higher extraction of consumer surplus. As a consequence, when the upstream firm can extract most of the surplus from downstream (μ is "high"), it prefers to avoid profit-dissipating competition and to opt for an exclusive contract with the high-quality firm, as pointed out above. Under the BSD utility specification goods are differentiated but symmetrical, instead, so that the representative consumer has no *a priori* preference for one or the other. As a consequence, when the degree of substitutability between them is "low" (γ is close to zero, meaning that the degree of differentiation is "high") the loss in revenues due to competition is small (zero, when $\gamma \to 0$). This entails that the upstream firm prefers improving its bargaining power by creating outside options even if it can extract most of the consumer surplus (μ is "high"). An exclusive contract becomes interesting only when the substitutability is "high". In this case competition downstream has dramatic effects on producer surplus and the upstream firm prefers to avoid profit-eroding downstream competition through an exclusive contract.

5 Conclusion

In this paper we have analyzed the choice of an upstream monopolist concerning the types of contracts (exclusive, non-exclusive and non-contingent and non-exclusive, non contingent), to offer to two price-competing downstream firms. The downstream firms use the input sold by the upstream firm to produce vertically differentiated products, that they sell to final consumers. When the upstream firm opts for non-exclusive contracts, we have analyzed the pairwise proof equilibria of the game and we let the upstream monopolist decide whether to offer non-contingent or contingent contracts. In the first case, the contract signed between the upstream firm and a downstream one cannot contain terms that are contingent on the breakdown of the negotiation in the pair composed by the upstream firm itself and the other downstream firm. In the second case they can. We show that the relative bargaining power of the upstream and downstream firms plays a crucial role in determining the equilibrium outcome. In particular, when the bargaining power of the upstream firm is "high" relative to that of the downstream ones, that firm prefers to sign an exclusive contract with the high-quality producer. In this way, it avoids to put the downstream firms in competition, which erodes aggregate producer surplus, and, thanks to its high bargaining power, can extract most of the surplus generated by the downstream firm. For "lower" levels of bargaining power, the upstream firm switches to non-exclusive contracts. Acting this way, it creates an outside option in the negotiation with the downstream partners, at the cost of increasing the downstream competition. This improves its bargaining position and thus its payoff. Further, we show that, when the upstream firm opts for non-exclusive contracts, it prefers contingent ones for "intermediate" levels of bargaining power, and non-contingent ones for "low" levels of bargaining power. We have also shown that under quantity competition the upstream firm never finds it optimal to offer a non-exclusive, non-contingent contract, because the commitment effect is such that the per unit fees are negative, which makes this type of contract dominated by the exclusive one. By contrast, the upstream firm may offer a non-exclusive, contingent contract when it has most of the bargaining weight, and the intuition is the same as for the case of price competition. Finally, we have shown that the message conveyed by our paper is not restricted to industries featuring vertically differentiated products, but is also valid for symmetrically differentiated ones.

References

Acharyya, R. (1998). "Monopoly and product quality: Separating or pooling menu?" *Economics Letters*, **61**(2):187–194.

Alipranti, M., Milliou, C., and Petrakis, E. (2014). "Price vs. quantity competition in a vertically related market". *Economics Letters*, **124**(1):122–126.

Bacchiega, E., Bonroy, O., and Mabrouk, R. (2013). "Paying not to sell". *Economics Letters*, **121**(1):137–140.

Bazerman, M. H. and Gillespie, J. J. (1998). "Betting on the future: the virtues of contingent contracts." *Harvard Business Review*, **77**(5):155–60.

Bowley, A. L. (1924). *The mathematical groundwork of economics.* Oxford University Press, reprinted by A.M. Kelley New York, 1965.

Chambolle, C. and Villas-Boas, S. B. (2015). "Buyer power through the differentiation of suppliers". *International Journal of Industrial Organization*, **43**:56–65.

de Fontenay, C. C. and Gans, J. S. (2005). "Vertical Integration in the Presence of Upstream Competition". *RAND Journal of Economics*, pages 544–572.

Dixit, A. K. (1979). "A model of duopoly suggesting a theory of entry barriers". *The Bell Journal of Economics*, **10**(1):20–32.

Faulí-Oller, R., González, X., and Sandonís, J. (2013). "Optimal two-part tariff licensing contracts with differentiated goods and endogenous R&D". *The Manchester School*, **81**(5):803–827.

Gabszewicz, J. J. and Thisse, J.-F. (1979). "Price competition, quality and income disparities". *Journal of Economic Theory*, **20**(3):340–359.

Horn, H. and Wolinsky, A. (1988). "Bilateral Monopolies and Incentives for Merger". *The RAND Journal of Economics*, **19**(3):pp. 408–419.

Inderst, R. and Wey, C. (2003). "Bargaining, Mergers, and Technology Choice in Bilaterally Oligopolistic Industries". *RAND Journal of Economics*, pages 1–19.

Inderst, R. and Wey, C. (2007). "Buyer power and supplier incentives". *European Economic Review*, **51**(3):647–667.

Iozzi, A. and Valletti, T. (2014). "Vertical bargaining and countervailing power". *American Economic Journal: Microeconomics*, **6**(3):106–135.

McAfee, R. P. and Schwartz, M. (1994). "Opportunism in Multilateral Vertical Contracting: Nondiscrimination, Exclusivity, and Uniformity". *The American Economic Review*, **84**(1):pp. 210–230.

McAfee, R. P. and Schwartz, M. (1995). "The non-existence of pairwise-proof equilibrium". *Economics Letters*, **49**(3):251–259.

Miklós-Thal, J., Rey, P., and Vergé, T. (2011). "Buyer power and intrabrand coordination". *Journal of the European Economic Association*, **9**(4):721–741.

Milliou, C. and Petrakis, E. (2007). "Upstream horizontal mergers, vertical contracts, and bargaining". *International Journal of Industrial Organization*, **25**(5):963–987.

Mussa, M. and Rosen, S. (1978). "Monopoly and Product Quality". *Journal of Economic Theory*, **18**(2):301–317.

O'Brien, D. P. and Shaffer, G. (1992). "Vertical Control with Bilateral Contracts". *The RAND Journal of Economics*, **23**(3):pp. 299–308.

Rey, P. and Vergé, T. (2004). "Bilateral Control with Vertical Contracts". *The RAND Journal of Economics*, **35**(4):pp. 728–746.

Shaked, A. and Sutton, J. (1983). "Natural oligopolies". *Econometrica*, **51**(5):1469–1483.

Singh, N. and Vives, X. (1984). "Price and Quantity Competition in a Differentiated Duopoly". *The RAND Journal of Economics*, **15**(4):546–554.

Spence, M. (1976). "Product Differentiation and Welfare". *American Economic Review*, **66**(2):407–14.