Intrinsic Motivation in the Labor Market: Not Too Much, Thank You*

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Abstract

We study the screening problem of a firm that needs to hire a worker to produce output and that observes neither the productive ability nor the intrinsic motivation of the worker applying for the job. We completely characterize the set of optimal contracts and we show that it is always in the firm's interest to hire all types of worker, even the worst ones, and to offer different contracts to different types of employees. Interestingly, the highest social welfare attains when motivation is high but not so much as to become more significant than productive ability. Moreover, when motivation is very high, incentives force the firm to offer a strictly positive wage to workers who derive a positive utility from effort exertion and who become paid volunteers. These results prove that very high motivation is not a desirable workers' characteristic.

Jel classification: D82, D86, J31, M55.

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1 Introduction

In a labor market where potential workers can be intrinsically motivated for the job, as the market for health professionals, teachers and civil servants, firms are confronted with differences in abilities together

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with different levels of motivation, and their screening problem must account for both components. On the one side, intrinsic motivation can not be considered as a sufficient condition for a high-quality outcome, since vocation alone does not guarantee high skills. On the other side, selecting workers on the base of productive ability only can be suboptimal in a vocation-based market where workers enjoy, to a certain extent, their contribution to the firm's output. Indeed, the workers' overall performance derives from the combination of both intrinsic motivation and high productive ability. In a world where workers' characteristics are not observable, a firm should consider the interplay between intrinsic motivation and productive ability when designing optimal wage schemes in order to screen potential job applicant.

In this paper, we investigate the problem of the selection of workers whose overall performance results from the interplay of skills and motivation and we thus contribute to the existing literature by explicitly accounting for the bidimensional nature of workers' private information. The most closely related papers are Heyes (2005) and Delfgaauw and Dur (2007) that deal with the selection of workers who are privately informed about their motivation but that do not include skills' heterogeneity. Delfgaauw and Dur (2008) considers both attributes but do not explicitly solve the bidimensional screening problem since their principal only hires a limited set of types. A detailed description of the related literature is provided in a separate section which follows.

We consider a principal-agent relationship where agents differ in both skills (or productive ability) and intrinsic motivation for the task to be performed, with each characteristic being independently and discretely distributed, and taking two possible values. In practice, both ability and motivation decrease the cost of providing effort for the worker, although in a different way. Productive ability lowers the worker's cost of providing effort whereas motivation is interpreted as the worker's enjoyment of her personal contribution to the firm's outcome or as a non-monetary benefit accruing to the worker when performing a given task; in the latter case, the worker likes what she does, at least to a certain extent.

Since worker's characteristics can not be observed by the employer, they can not be contracted upon. Instead, we assume that the firm can observe and verify the effort levels provided by different types of workers. Thus, the employer offers a menu of contracts consisting in different combinations of wage rate and effort provision. Our goal is then to describe the set of contracts that are compatible with workers' self-selection in such an asymmetric information framework and, in particular, to analyze which types of workers are hired and which are the optimal incentive schemes that the firm offers them.

The complete characterization of optimal contracts allows us to deliver some novel and interesting results. Despite the fact that the firm only has one instrument available (the observable effort level), we show that it succeeds in solving the bidimensional screening problem. We also find that it is always optimal for the principal to offer contracts involving full separation and full participation of types. If possible, the principal does not resort to either bunching, offering the same contract to different types of employees, or exclusion of the worst type(s). Hence, in our setting, screening along two different dimensions of private information is not too costly, either in terms of information rents that the principal has to leave to the most motivated and/or able type, or in terms of distortions of effort levels that less motivated and/or able types are required to provide.[1]

Our main results are driven by the relative importance of the difference in motivation *vis à vis* the difference in ability, which influences the principal's "preference ordering" over the possible types. High-skilled motivated workers are unambiguously the best types, since they provide the highest possible level of effort, low-skilled non-motivated employees are the worst types while the ranking of intermediate types can differ. In any case, we show that the equilibrium ranking of different types of workers always coincides with the one that would arise under full information, being first- and second-best solutions aligned.

There are two possible states of the world to be studied. The first one is characterized by motivation prevailing over ability, in which case the low-ability motivated worker is asked to provide a higher effort than the high-ability non-motivated type. The second is characterized by ability being more significant than motivation so that high-skilled non-motivated workers are induced to exert higher effort than low-skilled motivated ones. When motivation prevails over ability, a unique fully separating and fully participating equilibrium emerges. When ability prevails over motivation, multiple solutions arise and anomalous incentive constraints might be binding.

There are two polar situations (one in which motivation prevails and the other in which ability prevails and motivation is very low) where our bidimensional screening problem is equivalent to the unidimensional one, the unique parameter of private information being the overall cost of effort exertion of employees. Such situations are fully intuitive and are characterized by information rents being increasing while effort distortions being decreasing in the effort provided by the different types. When motivation prevails, we obtain an intriguing result: low-skilled motivated workers may become "paid volunteers", as they enjoy a *net utility* from effort provision at the optimal contract even if their salary is always strictly positive because of the information rents that they necessarily receive for truthful revelation.

The most interesting cases occur when ability prevails over motivation but still vocation is high enough. Here a tension realizes: on the one hand, at any optimal contract, the high-skilled non-motivated worker is still required to provide a higher effort than the low-skilled motivated one; on the other hand, as motivation increases, the motivated worker faces a diminishing disutility of effort so that it becomes more and more convenient to increase her effort and more and more difficult to meet the previous monotonicity

[1] One may ask whether this screening problem could be analyzed in the simpler one-dimensional setup with different types of workers being characterized by a different "overall (un-)willingness to exert effort". The answer is no because ability and motivation influence effort provision in a different way so that it is not possible to represent them together using a single summary statistic. Only with bidimensional screening are we able to characterize all the possible classes of equilibria, some of which would disappear in a world with unidimensional adverse selection. Indeed, it is the combination of both ability uncertainty and motivation uncertainty that gives rise to the equilibria that are dominating in terms of total surplus.

condition. This tension causes a global rather than a local downward incentive constraint to bind. As a consequence, not only does the standard result of *no distortion at the top* hold, but we also find no distortion for the optimal effort provided by low-skilled motivated workers. We can thus conclude that, from a social point of view, it is better when productivity prevails over motivation, proving that very high motivation is not a desirable workers' characteristic. Moreover, precisely the same tension might induce an upward incentive constraint to bind: when the pressure exerted by such an upward incentive constraint is sufficiently high, we find a separating equilibrium with an upward distortion in the effort exerted by the high-skilled non-motivated type of worker and exclusion of the worst type (the low-skilled non-motivated worker).

As for the optimal wage schemes, we find that, under full information, high-ability non-motivated workers are always paid the highest wage, while motivated low-ability employees are always paid the lowest salary. In addition, the ordering of intermediate wages is not unique: either high-ability workers receive the largest salaries or motivated employees receive the lowest rewards. At the second-best, however, our model predicts a switch in the ranking of rewards: high-skilled motivated workers always receive the highest salary and low-skilled non-motivated ones receive the lowest wage rate. Thus, under asymmetric information, it is the type with the lowest overall cost of effort provision who receives the highest transfer, despite her positive motivation, while the type with the highest overall cost of providing effort obtains the lowest reward, even if she is not motivated. Motivated low-skilled workers and non-motivated high-skilled ones are in-between and the ranking between their salaries depends on the class of equilibria considered. We conclude that, when screening on both ability and motivation, a firm always offers to motivated workers a larger wage than to non-motivated ones, for given workers' ability. In different words, motivated workers are potential mimickers in our setting and thus they always receive an information rent which makes their salary increase. Moreover, except when motivation is very low, we also find that motivated workers enjoy a higher utility than non-motivated ones, irrespective of their ability.

The market for nurses, provides an environment that matches our setting and results. Here, the observable effort is the contractible number of hours nurses choose to work and the total salary is represented by an hourly wage that depends on number of hours worked per day. In the United States, for instance, the hospital typically offers contracts characterized by different number of hours (see Bae, 2012): part-time contracts require about 24 hours each week, full-time nurses work an average of about 43 hours a week; moreover nurses can choose paid voluntary overtime up to a total amount that cannot exceed 60 hours a week.[2] Generally, the total salary may encompass part-time penalties and/or overtime

[2] Bae (2012) presents quantitative survey data collected from registered nurses who worked in hospitals as staff nurses in North Carolina and West Virginia. In the sample, 33.2% of nurses working overtime are *choosing* to perform voluntary paid overtime. Among them, 42% are working overtime more than 12 hours a week. Interestingly, the survey also considers the reasons reported by nurses as to why they worked overtime. Nearly half (46.3%) of nurses choosing voluntary overtime declared that they "like to work overtime". This in line with our results when motivation prevails where low-ability

premia. Our model predicts that high-ability motivated workers choose the contract with the largest voluntary overtime and low-ability non-motivated nurses are targeted to part-time contracts.

Another possible interpretation of our model stems from considering workers' career concerns as a screening device. Typically workers self-select into different career paths: some of them accept tasks involving strong performance evaluation in exchange for more likely and faster promotions; some others prefer a slower progress up the job ladder together with lower pay and almost no performance evaluation. In the academia, for instance, junior professors can choose between tenure-track positions, which require them to demonstrate, within a short time span, a strong record of published research, grant funding, teaching and administrative service and positions off the tenure track (such as lecturer or adjunct professor), which require them to teach full- or part-time but with few or no research responsibilities. Again, tenure-track positions are targeted to attract the best researchers.

The rest of the paper is organized as follows. In the following subsection we describe the related literature. In Section 2, we set up the model, describe the first-best (Section 2.1.1) and two benchmark cases in which there is asymmetric information on one dimension only, be it ability (Section 2.1.2) or intrinsic motivation (Section 2.1.3). In Section 3, we consider the interaction between the two sources of incomplete information. We distinguish between the two polar cases in which: (*i*) motivation has a larger impact than ability on the worker's overall cost of effort provision (Section 3.1) or (*ii*) ability has a larger impact than motivation on the worker's cost of effort provision (Section 3.2). In the text, we provide a qualitative characterization of informational rents and optimal contracts with full separation and full participation of types and we compare the properties of the different classes of equilibria. All proofs are relegated to the Appendix as well as the formal analysis of bunching and/or exclusion and the profit comparisons among different classes of equilibria. Section 4 offers a summary of results and their economic interpretation. Finally, Section 5 concludes.

1.1 Related literature

Our work contributes to two different strands of literature: from an economic point of view, it adds to the recent and rapidly growing literature on the selection of workers with intrinsic motivation; from a technical point of view, it explicitly solves the principal-agent problem in a labor market where workers are characterized by two different dimensions of private information.

The problem of the design of optimal incentive schemes for intrinsically motivated workers has been tackled by Murdock (2002), Besley and Gathak (2005) and Ghatak and Mueller (2011), whose attention has been primarily devoted to moral hazard, while we consider the screening problem.

Heyes (2005) and Delfgaauw and Dur (2007) are the first papers that address the issue of the selection

motivated workers are *potential* volunteers.

of workers who are privately informed about their vocation. They show that, as a worker's motivation increases, the minimum wage that a worker is willing to accept decreases. Therefore, as the wage increases, the average motivation of the pool of active workers deteriorates, since workers with lower motivation are willing to accept the job. Delfgaauw and Dur (2007) examine how a firm can attract highly motivated workers using a directed search framework *à la* Diamond, Mortensen and Pissarides. The optimal wage scheme entails a trade-off between the probability of filling a vacancy, the rents left to the workers and the expected motivation of job applicants. Our analysis departs from this work because it includes a second source of imperfect information (namely, productive ability) and, most importantly, because it resorts to a direct revelation mechanism that allows the principal to infer the workers' true types.

Delfgaauw and Dur (2010) consider a richer framework where workers are heterogeneous with respect to both their intrinsic motivation to work at a firm and their ability. Their analysis focuses on the issue of managerial self selection into public vs private sectors in the case of full information on the workers' characteristics: the key result is that, if the demand for public sector output is not too high and if motivation is unrelated to either effort provision or to the firm's outcome, then the return to managerial ability is always lower in the public sector as compared to the private sector. Therefore, attracting a more able managerial workforce to the public sector by increasing remuneration up to the private sector levels is not cost-efficient. Finally, Barigozzi and Raggi (2013) and Barigozzi and Turati (2012) consider the lemons problem with asymmetric information about the two workers' characteristics. Given that a fixed wage, independent of the workers' unobservable types, must be offered to all potential applicants, they examine what happens to both average productivity and average motivation of active workers as the wage rate increases. It is shown that the lemons' problem might be exacerbated by the presence of multidimensional asymmetric information because an increase in the market wage can determine a simultaneous decrease in both expected vocation and expected productivity of applicants.

Our paper is also closely related to Handy and Katz (1998) and Delfgaauw and Dur (2008). The first authors set up a model to explain why in the not-for-profit sector there appears to be lower managerial and professional wages than in the for-profit sector. They argue that non-profits attract motivated managers by offering them compensation packages involving lower money wages and a larger component of institution specific fringe benefits. But, rather than considering an agency setup, they exogenously impose a ranking of effort levels and reservation wages for the different types of managers, which in turn drives the main results. Delfgaauw and Dur (2008) characterize the optimal incentive schemes offered by a cost minimizing public agency when workers differ in laziness (the opposite of our productive ability) and public service motivation. They show that, when effort is observable and contractible and when the production required by the public institution is sufficiently high so that at least two types of agents must be hired, the institution attracts dedicated and productive workers as well as the economy's laziest workers, by offering contracts that are both distorted. Indeed, dedicated workers are asked to exert higher effort than in the private, perfectly competitive sector whereas lazy workers' effort is distorted downwards in order to make their contract unattractive for dedicated workers. We depart from the previous model in the following ways. First we consider one sector in isolation, then our principal's objective function is profit maximization rather than cost minimization, and finally our screening problem is unrestricted: indeed, Delfgaauw and Dur (2008) simplify their analysis by considering that the principal is interested in hiring at most two types of agents rather the whole set of types, so that they study two alternative screening problems.

The literature on the analysis of optimal screening of agents with unknown characteristics has flourished in the last two decades of the twentieth century. Nonetheless, this problem has been examined under the assumption of unidimensional asymmetric information. The interesting and possibly more realistic cases where agents have several unobservable characteristics have been studied by few important works: Armstrong and Rochet (1999), Armstrong (1996), Rochet and Chonè (1998), Armstrong (1999), Basov (2001, 2005) and Deneckere and Severinov (2011). They show that it is almost impossible to extend to the multidimensional environment the qualitative results and the regularity conditions that make the unidimensional case easily tractable. This is eventually one of the reasons of the diminishing interest on multidimensional screening in the theoretical literature.

Armstrong and Rochet (1999) provide a complete characterization of the optimal contracts when the dimensionality of actions is the same as the dimensionality of private information and the type space is discrete. Our model too is characterized by a discrete type space (types distributions are assumed to be independent and no regularity requirements are imposed), but the number of dimensions of private information is larger than the number of instruments (namely the contractible effort level) available to the principal. When the dimensionality of actions is smaller than the dimensionality of private information and the type space is continuous, Laffont et al. (1987) explicitly solve a model of bidimensional optimal nonlinear pricing by a regulated monopoly when consumers' utility is linear quadratic (has our workers') and when consumers' types are uniformly distributed on a unit square. Moreover, for the continuous case, Armstrong (1996), Rochet and Chonè (1998), Basov (2001, 2005) and Deneckere and Severinov (2011) present several useful techniques to solve the problem of multidimensional screening. These papers provide existence proofs and characterization results showing that exclusion is generic and full separation of types is impossible. In other words, it is generally optimal for the principal not to serve the lower part of the agent's distribution and to offer the same contract to different (typically intermediate) types of agents.

Our analysis owes much to Armstrong (1999), who considers optimal price regulation of a monopoly that is privately informed about both its cost and demand functions. He solves a discrete model distinguishing between two main classes of problems. If cost uncertainty is relatively more important than demand uncertainty, then optimal prices are always weakly above marginal costs. Conversely, if demand uncertainty is more significant than cost uncertainty, then pricing below marginal cost could be optimal. And it is the combination of cost and demand uncertainty that brings about the interesting result of sub-marginal cost pricing.[3] To focus on this aspect, Armstrong (1999) explicitly ignores the problem of exclusion by restricting parameter values in such a way that it is never optimal for the regulator to shut down some types of firm. Notably, in our model, we do not need to impose analogous requirements: the problem is sufficiently well-behaved so that full participation always dominates exclusion and full separation of types always dominates pooling.

2 The model

We consider a principal-agent model with bidimensional adverse selection. Both the principal and the agent are risk neutral. The principal (he) is willing to hire only one agent (she) to perform a given task.

The production function is such that the only input is labor supplied by the agent. We call e the *observable and measurable* task level that the agent is asked to provide.[4] The production function displays constant returns to effort in such a way that

$$q(e) = e.$$

The principal's payoff function can be written as

$$\pi = e - w,$$

where the price of output is assumed to be exogenous and normalized to 1, and w is the salary paid to the hired worker. Obviously, the principal's profit depends on the type of the agent as will be clear in the sequel.

Suppose that agents differ in two characteristics, productive ability and intrinsic motivation. As for ability, we interpret a highly productive potential worker as an agent incurring in a low cost of providing a given effort level. Workers can have only two possible levels of ability $\theta_i \in \{\theta_L, \theta_H\}$. Employees can be highly productive, i.e. they can have a low cost of effort θ_L, with probability ν, or they can be less productive and have a high cost of effort θ_H, with probability $1-\nu$, where $\theta_H > \theta_L > 0$. As for intrinsic motivation, we mainly refer to Delfgaauw and Dur (2008) and assume that workers, to a certain extent, derive utility from exerting effort. Since there exists a one-to-one relationship between effort exerted and output produced by the firm, this interpretation is equivalent to considering intrinsic motivation as the

[3] Indeed, with either cost uncertainty or demand uncertainty alone, prices are always set weakly above marginal costs.

[4] In particular, the variable e can be interpreted as a job-specific requirement like the amount of hours of labor the agent is asked to devote to production or the speed at which a production line is run in a factory.

enjoyment of one's personal contribution to the firm's outcome.[5] A slightly different view of intrinsic motivation (which suits the model as well) is given by Delfgaauw and Dur (2007, page 607), who argue that intrinsic motivation might arise because "the firm has some unique trait that is valued differently by different workers, giving the firm monopsony power".[6] Paralleling ability, we assume that motivation can take only two possible values $\gamma_j \in \{\gamma_L, \gamma_H\}$. Workers can have either high motivation γ_H, with probability μ, or low motivation γ_L , with probability $1 - \mu$.

Without loss of generality, we normalize the lower bounds of the support of the distribution of both attributes, setting $\theta_L = 1$ and $\gamma_L = 0$. We will thus focus attention on situations in which agents can be either intrinsically motivated, with motivation parameter taking value $\gamma_H = \gamma$ or not motivated at all. Furthermore, we will impose that $0 < \gamma \leq 1$ and that $1 < \theta_H \leq 2$ (the reader is referred to Section 2.1.1 for the justification of such assumptions). Finally, we assume for simplicity that motivation and productivity have independent distributions. So, there are four types of agents denoted as $ij = \{LH, LL, HH, HL\}$ where the first index indicates the cost of effort provision and the second motivation. Importantly, allowing for more general distribution functions that admit correlation between ability and motivation does not alter our results, since all possible classes of equilibria that we find are still relevant with a more general distribution.

The agents' *reservation utility* is normalized to zero for all possible types.

Workers' utility is quasi-linear in income and takes the form

$$u_{ij} = w_{ij} - \frac{1}{2}\theta_i e^2 + \gamma_j e.$$

The marginal rate of substitution between effort and wage is given by

$$MRS_{e,w} = -\frac{\partial u_{ij}/\partial e}{\partial u_{ij}/\partial w} = \theta_i e - \gamma_j,$$

which is positive for $e > \frac{\gamma_j}{\theta_i}$ (it is always positive for non-motivated agents with $\gamma_j = 0$). Thus, when the effort required by the principal is sufficiently high, motivated workers' indifference curves have the standard positive slope in the space (e, w) and effort is a "bad". Alternatively we can say that, if $e > \frac{\gamma_j}{\theta_i}$, the agents' utility is decreasing in effort.

Also notice that productivity θ_i enters utility with a convex term, while motivation γ_j enters utility

[5] The same interpretation of intrinsic motivation can be found in Besley and Ghatak (2005) and Delfgaauw and Dur (2007, 2008, 2010-only as for Section 5) and traces back to the "warm-glow giving" or impure altruism theory in Andreoni (1990).

[6] Delfgaauw and Dur (2007, page 607) also add: "Monopsony power arises naturally when intrinsic motivation is firm-specific. When it is related to an occupation rather than to working at a particular firm, monopsony power arises only if there is no other firm (in the neighborhood) offering similar jobs". In turn, the link between workers' motivation and market power justifies our hypothesis concerning profit maximization and wage setting on the part of the principal.

with a linear term.[7] Providing effort represents a net cost when

$$-\frac{1}{2}\theta_i e^2 + \gamma_j e < 0.$$

The above condition is satisfied for any effort level $e > 0$ if workers are not motivated and $\gamma_j = 0$; if instead $\gamma_j > 0$, then it is satisfied for effort levels such that $e > \frac{2\gamma_j}{\theta_i}$. Thus, only if the effort required by the principal is sufficiently high (or motivation is sufficiently low) do motivated workers experience a disutility loss from effort provision and need a positive wage to be willing to exert such effort. Conversely, if the effort required is sufficiently low, motivated workers could perform their task also when receiving a non-positive reward (in other words they would be ready to volunteer to be hired by the firm).

Remark 1 *A motivated worker obtains a net positive utility from exerting the effort when* $e < \frac{2\gamma_j}{\theta_i}$. *We call "volunteer" a worker who is willing to receive a non-positive reward when* $e < \frac{2\gamma_j}{\theta_i}$.

Finally, notice that agents' utility function is well-behaved in the sense that it satisfies the (double) single-crossing property.[8]

Remark 2 *The single-crossing property is satisfied both with respect to the productivity parameter and with respect to motivation. In fact* $MRS_{e,w}$ *is increasing in* θ *and decreasing in* γ.

By considering the impact of productivity and motivation together on the workers' effort and on the firm's output, we can say that the type exerting the highest effort is worker LH (with low effort cost and high motivation) whereas the type exerting the lowest effort is worker HL (with high effort cost and no motivation). Worker types LL and HH are in-between. Thus, we expect that the contracts offered by the principal will specify effort levels such that $e_{LH} \geq \max\{e_{LL}, e_{HH}\} \geq \min\{e_{LL}, e_{HH}\} \geq e_{HL}$. The ranking between workers LL and HH will be considered in the sequel.[9]

In what follows, we assume that the principal offers the agent a menu of contracts of the form $\{e, w(e)\}$. Applying the Revelation Principle, we will focus on four contracts such that a worker of type ij exerts effort e_{ij} and receives a wage $w(e_{ij}) = w_{ij}$.

[7] Note that this linear-quadratic specification of the utility function is widely used in the literature on workers' intrinsic motivation (see Besley and Ghatak 2005 and Delfgaauw and Dur 2010). The same objective function for the agent is also considered in the literature on multidimensional screening with a continuum of types (see Laffont et al. 1987, Basov 2003, and Deneckere and Severinov 2011). See also Footnote 8 for a general assessment of the properties of the utility function.

[8] All the properties of the utility function extend to the more general case in which the cost of effort is still convex while the benefit from exerting effort, due to intrinsic motivation, is concave. Moreover, it is possible to prove that all qualitative results concerning the second-best solutions carry on in this general case.

[9] Notice that, as mentioned in the Introduction, the existence of two possible orderings of effort levels is a consequence of the bidimensionality of our problem and could not be generated in a unidimensional set-up with, say, four different types of employees characterized by a different overall cost of providing effort.

2.1 Benchmark cases

2.1.1 Full information

At the first-best, both ability and motivation are observable. For $i = L, H$ and $j = L, H$, the principal solves

$$\max_{(e_{ij}, w_{ij})} \pi = e_{ij} - w_{ij} \tag{FB}$$
$$\text{s.t. } u_{ij} \geq 0$$

which is maximized for a level of effort equal to

$$e_{ij}^{FB} = \frac{1+\gamma_j}{\theta_i} \tag{1}$$

and where the wage levels are set such that each worker receives her zero reservation utility

$$w_{ij}^{FB} = \frac{\left(1+\gamma_j\right)\left(1-\gamma_j\right)}{2\theta_i}.$$

If $\gamma_j \leq 1$ is satisfied, then, at the first-best, all wages are non-negative and motivated workers are not volunteers since they face a net cost from exerting effort.[10]

Assumption 1 *Let $0 < \gamma \leq 1$. Then, motivated workers are not volunteers and always receive a non-negative salary at the first-best.*

The intuition for this requirement is straightforward. Given Program (FB) and first-order condition (1), we can interpret $1 + \gamma$ as the total marginal productivity of effort. When $\gamma \leq 1$, the contribution of worker's intrinsic motivation on the marginal productivity of effort does not dominate the standard one.

Importantly, at the second-best, effort levels might be distorted downwards for workers different from LH (because of the standard result of distortion for types different from the "top" one). This implies that Assumption 1 is no longer sufficient to ensure a net cost of the effort when type HH is considered. Thus, in the next Sections, it will be necessary to check whether $e_{HH}^{SB} \gtreqless \frac{2\gamma}{\theta_H}$ and we will show that the worker type HH can experience a net utility from the effort so that she may become a volunteer at the second-best.

It is immediate to check that $e_{LH}^{FB} > e_{HH}^{FB} > e_{HL}^{FB}$ and $e_{LH}^{FB} > e_{LL}^{FB} > e_{HL}^{FB}$ both hold. Also note that, for intermediate types, one has

$$e_{LL}^{FB} \leq e_{HH}^{FB} \text{ if and only if } \gamma \geq \Delta\theta, \tag{2}$$

[10] This assumption allows us to exclude situations where, at the first-best, motivated workers receive a negative wage while non motivated employees receive a positive salary. Our analysis can be easily extended to allow for volunteers and standard workers to coexist at the first-best. At the second-best, threshold values obtained when the difference in motivation is more important than the difference in productivity would change, whereas the classes of equilibria when productivity prevails over motivation would not be affected.

where $\Delta\theta = (\theta_H - \theta_L) = (\theta_H - 1)$, and

$$e_{LL}^{FB} \geq e_{HH}^{FB} \text{ if and only if } \gamma \leq \Delta\theta \equiv \gamma^*. \tag{3}$$

alternatively, (2) can be restated as

$$e_{LL}^{FB} \leq e_{HH}^{FB} \leq \frac{\gamma}{\Delta\theta} \tag{4}$$

while (3) is equivalent to[11]

$$e_{LL}^{FB} \geq e_{HH}^{FB} \geq \frac{\gamma}{\Delta\theta}. \tag{5}$$

Given Assumption 1, a necessary condition for (2) is that $\gamma^* \leq 1$ or else $\theta_H \leq 2$.

Remark 3 *The ordering of effort levels in first-best is as follows.*

1. *If $\theta_H \leq 2$ and $\gamma \geq \gamma^*$ both hold, then the ordering of optimal effort levels is $e_{LH}^{FB} > e_{HH}^{FB} \geq e_{LL}^{FB} > e_{HL}^{FB}$.*
2. *If $\gamma \leq \gamma^*$, then the ordering of optimal effort levels is $e_{LH}^{FB} > e_{LL}^{FB} \geq e_{HH}^{FB} > e_{HL}^{FB}$.*

Intuitively, the first (respectively, second) situation occurs when the difference in motivation $\Delta\gamma = \gamma - 0$ is higher (respectively, lower) than the difference in productivity $\Delta\theta = (\theta_H - 1)$, in which case the effort provided by type HH at the first-best (respectively, type LL) is higher than that of type LL (respectively, HH). Since both instances are economically relevant, we impose that $\gamma^* \leq 1$ which is equivalent to $\theta_H \leq 2$.

Assumption 2 *Let $1 < \theta_H \leq 2$. Then $0 < \gamma^* \leq 1$ holds and all orderings $e_{HH}^{FB} \gtreqless e_{LL}^{FB}$ are possible.*

Note that, when $\gamma = \gamma^*$, the type space corresponds to a square and types LL and HH are equivalent being $e_{LL}^{FB} = e_{HH}^{FB}$. We will show that the second-best equilibrium requires a pooling contract between types LL and HH in a whole region around $\gamma = \gamma^*$ (see Figure 4).

Let us consider the ranking of wages with perfect information.

Remark 4 *The ordering of wage levels at the first-best is as follows:*

$$w_{LL}^{FB} > \max\left\{w_{LH}^{FB}, w_{HL}^{FB}\right\} \geq \min\left\{w_{LH}^{FB}, w_{HL}^{FB}\right\} > w_{HH}^{FB} \geq 0$$

Hence, for fixed ability, motivated workers always obtain lower rewards than non-motivated ones. In addition, when $w_{HL}^{FB} > w_{LH}^{FB}$, motivated workers always earn less than non-motivated workers independently of their productivity.[12]

[11] Take $e_{HH}^{FB} \leq e_{LL}^{FB}$. This amounts to $\frac{1+\gamma}{\theta_H} \leq 1$ or else to $1 + \gamma \leq \theta_H$. It follows that $\gamma \leq \theta_H - 1 = \Delta\theta$ or else that $\frac{\gamma}{\Delta\theta} \leq 1 = e_{LL}^{FB}$. Similarly, starting from $\gamma \leq \Delta\theta$ and adding to both sides of the inequality $\gamma\Delta\theta$ yields $\frac{\gamma}{\Delta\theta} \leq \frac{1+\gamma}{\theta_H} = e_{HH}^{FB}$. The same reasoning can be applied to the opposite case in which $e_{HH}^{FB} \geq e_{LL}^{FB}$.

[12] The ranking of wages at the first-best is consistent with the theory of compensating wage differentials (Rosen 1986 and Hwang, Reed and Hubbard 1992) because motivated agents can be interpreted as those workers who have a high willingness to pay for a desired, non-monetary job attribute and are thus ready to accept lower wages.

2.1.2 Adverse selection on ability

Suppose that workers' motivation γ_j is observable to the principal but ability θ_i is not, we call this case Benchmark A, or BA. For fixed $j = L, H$ the principal solves

$$\max_{(e_{Hj},w_{Hj});(e_{Lj},w_{Lj})} E(\pi) = \nu(e_{Lj} - w_{Lj}) + (1-\nu)(e_{Hj} - w_{Hj}) \tag{BA}$$

subject to the two participation constraints and the two incentive compatibility constraints. According to standard solution techniques, for $j = L, H$, wage schedules satisfy

$$w_{Hj} = \frac{1}{2}\theta_H e_{Hj}^2 - \gamma_j e_{Hj}$$

and

$$w_{Lj} = \frac{1}{2}e_{Lj}^2 - \gamma_j e_{Lj} + \underbrace{\frac{1}{2}\Delta\theta e_{Hj}^2}_{\text{Info rent worker } Lj} \tag{6}$$

As expected, the information rent of agent Lj is increasing in $\Delta\theta$ and in e_{Hj}. This implies that the effort of agent e_{Hj} is distorted downward in the optimal second-best contract. Substituting the previous wage schedules into the principal's problem and solving for the effort levels, we find

$$e_{Lj}^{BA} = 1 + \gamma_j = e_{Lj}^{FB}$$

and

$$e_{Hj}^{BA} = \frac{(1+\gamma_j)(1-\nu)}{(\theta_H - \nu)},$$

where the results of *no distortion at the top* and downward distortion in the effort exerted by the low-productivity worker both hold. Finally, it is straightforward to show that full participation is always optimal or that it is never in the principal's interest to exclude low-productivity workers (type Hj).[13]

As for wages, we have $w_{HH}^{BA} > 0$ if and only if

$$\gamma < \frac{\theta_H(1-\nu)}{(\theta_H - \nu) + \nu\Delta\theta} \equiv \gamma^{BA} < 1,$$

meaning that, when productivity is workers' private information while motivation is observable, type HH can become a volunteer if motivation is high enough. Moreover, for any given level of employees' motivation, the wage rate is increasing in workers' productivity.

[13] In fact, the principal's benefit from keeping worker Hj is the expected profit from this worker $(1-\nu)(e_{Hj} - w_{Hj})$, whereas the cost from letting her participate is the information rent appearing in expression (6) multiplied by the proportion of workers receiving the rent, that is $\frac{1}{2}\nu\Delta\theta e_{Hj}^2$. By substituting expression (6) for the wage in $(1-\nu)(e_{Hj} - w_{Hj})$, it can be checked that the principal always offers a non-null contract to low-productivity workers, independently of their motivation.

2.1.3 Adverse selection on motivation

Suppose now that workers' productivity θ_i is observable to the principal but motivation γ_j is not, we call this case Benchmark M, or BM. For fixed $i = L, H$ the principal solves

$$\max_{(e_{iH}, w_{iH});(e_{iL}, w_{iL})} E(\pi) = \mu(e_{iH} - w_{iH}) + (1-\mu)(e_{iL} - w_{iL}) \tag{BM}$$

subject to the two participation constraints and the two incentive compatibility constraints. In fact, motivated agents have interest in mimicking non-motivated ones whenever the effort they are required to provide is sufficiently high so as to cause a disutility.

For $i = L, H$, wage schedules satisfy

$$w_{iL} = \frac{1}{2}\theta_i e_{iL}^2$$
$$w_{iH} = \frac{1}{2}\theta_i e_{iH}^2 - \gamma e_{iH} + \underbrace{\gamma e_{iL}}_{\text{Info rent worker } iH} \tag{7}$$

The information rent of agent iH is increasing in e_{iL}. This explains why the effort of agent iL is distorted downward in the optimal second-best contract. Substituting the previous wage schedules into the principal's problem and solving for effort levels we find

$$e_{iH}^{BM} = \frac{1+\gamma}{\theta_i} = e_{iH}^{FB}$$

and

$$e_{iL}^{BM} = \frac{(1-\mu) - \mu\gamma}{(1-\mu)\theta_i}$$

where the results of *no distortion at the top* and downward distortion in the effort exerted by the non-motivated worker both hold. Also, $e_{iL}^{BM} > 0$ for

$$\gamma < \frac{1-\mu}{\mu} \equiv \gamma^{BM}$$

where $\gamma < \gamma^{BM}$ always holds if $\mu < \frac{1}{2}$. In words, when γ is sufficiently high, the information rent that the principal must pay to the motivated types is so costly that he prefers to exclude non-motivated workers. However, the necessary condition for full participation, that is $e_{iL}^{BM} > 0$, is always satisfied if the proportion μ of motivated workers is sufficiently low. In fact, following the same procedure as in Footnote 13, it can be checked that $e_{iL}^{BM} > 0$ is both necessary and sufficient for full participation.

As for wages, we always have that they are increasing in motivation: $w_{iH} > w_{iL}$. Hence, when motivation is workers' private information and ability is observable, the ranking of salaries for workers who are equally productive but have different vocation is the opposite with respect to the first-best. Namely, under asymmetric information on motivation, an intrinsically motivated worker always receives a higher salary than a non-motivated one because the former has to be given information rents in order for her not to mimic the latter.

3 Screening on ability and motivation

Summing up, what the benchmark cases predict is the following. When the principal cannot observe workers' skills (but is perfectly informed about their motivation), he might take advantage of motivated workers and make them work for free. As we will see, this turns out to be impossible at the second-best. When the principal cannot observe workers' motivation (but is perfectly informed about their skills), he might find in his interest to exclude non-motivated employees, no matter whether they have high- or low-ability; again this will not be the case at the second-best. Furthermore, motivated employees are always offered a higher salary than non-motivated ones, this stands in contrast with the first-best but will be confirmed at the second-best.

Suppose now that both the workers' productivity θ_i and motivation γ_j are the agents' private information, we call this situation the second-best. The principal offers the worker a choice of four effort-wage combinations. For $i = L, H$ and $j = L, H$, the principal's program is

$$\begin{array}{rl} \max_{(e_{ij},w_{ij})} E\left(\pi\right) = & \nu\mu\left(e_{LH} - w_{LH}\right) + \nu\left(1-\mu\right)\left(e_{LL} - w_{LL}\right) + \\ & \left(1-\nu\right)\mu\left(e_{HH} - w_{HH}\right) + \left(1-\nu\right)\left(1-\mu\right)\left(e_{HL} - w_{HL}\right) \end{array} \tag{SB}$$

subject to four participation constraints PC_{ij} and twelve incentive compatibility constraints $IC_{ijvsi'j'}$ (which are listed in Appendix B). There, we show that incentive compatibility and participation constraints satisfy some regularity conditions. Moreover, the following monotonicity condition holds

$$e_{LH} \geq \max\left\{e_{LL}; e_{HH}\right\} \geq \min\left\{e_{LL}; e_{HH}\right\} \geq e_{HL}. \tag{8}$$

Concerning intermediate types HH and LL, one can add IC_{LLvsHH} and IC_{HHvsLL} and find that either

$$e_{HH}^{SB} > e_{LL}^{SB} \text{ and } e_{LL}^{SB} + e_{HH}^{SB} \leq \frac{2\gamma}{\Delta\theta}, \tag{9}$$

or

$$e_{LL}^{SB} > e_{HH}^{SB} \text{ and } e_{LL}^{SB} + e_{HH}^{SB} \geq \frac{2\gamma}{\Delta\theta} \tag{10}$$

holds. Although conditions (9) and (10) are less transparent than the corresponding first-best conditions (4) and (5), we can still observe that $e_{HH} > e_{LL}$ holds at the second-best when motivation has a larger impact than ability on effort and output provision. On the contrary, if $e_{LL} > e_{HH}$ holds at the second-best, then it is because ability has a larger impact than motivation on effort and output provision.[14]

We will then solve a relaxed program in which only PC_{HL} and some (mostly downward) incentive constraints will bind.

[14] Note that condition (4) is *per se* more restrictive than (9) and that condition (5) is again more restrictive than condition (10). Hence, one can in principle expect some misalignment between first- and second-best effort levels as for intermediate types. See Lemma 1 for further reference.

There are two different cases to be investigated according to whether condition (9) or condition (10) holds. In the Propositions that follow, we provide an interpretation of the two cases by considering which incentive constraints are binding and why.

Proposition 1 ***Motivation prevails (Case M)***. *When motivation has a higher impact on effort provision than ability, then condition (9) holds and a separating equilibrium with* $e_{HH} > e_{LL}$ *is attained. The binding downward incentive constraints specific to this case are those of high-productivity types mimicking low-productivity ones, that is* IC_{LHvsHH} *and* IC_{LLvsHL}*. The additional downward incentive constraint is* IC_{HHvsLL}*, connecting the previous ones.*

If motivation has a higher impact on effort and output provision than ability, then from the principal's viewpoint, types can be ordered as $LH \succ HH \succ LL \succ LH$. Now we have to solve a bidimensional screening problem which embeds and generalizes the two sub-problems with adverse selection on the workers' ability only (Benchmark BA in Subsection 2.1.2). The two sub-problems BA are now considered simultaneously and linked by incentive constraint IC_{HHvsLL}. Figure 1 describes this case. On the horizontal axis we represent effort cost or productivity while on the vertical axis we have motivation. Types are located at the corners of a rectangle whose width is the difference in effort cost, or $\Delta\theta$, and whose height is the difference in motivation, or simply γ. An arrow from one type to another represents that the incentive constraint that the former type does not choose the contract designed for the latter type is binding.

Insert Figure 1 and Figure 2a around here

Intuitively, when motivation uncertainty is more relevant than ability uncertainty, the rectangle on which types are located has height greater than width. Then, what we call the *rule of the short side* applies. Since types LH and HH as well as types LL and HL are close to each other, then it is natural that the incentive constraints that bind first are IC_{LHvsHH} and IC_{LLvsHL}. The remaining binding constraint is the one that concerns "intermediate" types, namely IC_{HHvsLL}. Indeed, note that, since Case M occurs when motivation γ is high enough, then type HH is asked to provide a relative high effort in exchange for a relatively low salary and she might find the contract (e_{LL}, w_{LL}) potentially convenient.

Proposition 2 ***Ability prevails (Case A)***. *When ability has a higher impact on effort provision than motivation, then condition (10) holds and a separating equilibrium with* $e_{LL} > e_{HH}$ *is attained. The binding downward incentive constraint specific to this case is that of highly productive and motivated types mimicking non-motivated ones, that is* IC_{LHvsLL}*. As for the other relevant binding constraints, three sub-cases must be considered: (1) Case A.1. The binding incentive constraints are the two adjacent ones* IC_{LLvsHH} *and* IC_{HHvsHL}*; (2) Case A.2. The binding incentive constraints are* IC_{LLvsHL} *and* IC_{HHvsHL}; (3) *Case A.3. The binding incentive constraints are* IC_{LLvsHL} *and the upward* IC_{HHvsLL}*.*

If ability has a higher impact on effort and output provision than motivation, then, from the principal's viewpoint, types can be ordered as $LH \succ LL \succ HH \succ LH$. Now we have a plurality of situations which arises because the principal faces a trade-off between the need to satisfy condition $e_{LL} > e_{HH}$ and the incentive to increase e_{HH} as motivation grows.

Case $A.1$ is the most natural one and is symmetric to Case M: it requires to solve a bidimensional screening problem that consists of the two sub-programs related to adverse selection on workers' motivation (as in Benchmark BM in Subsection 2.1.3) together with incentive constraint IC_{LLvsHH} (see Figure 2a). Now the rectangle on which types are located has height smaller than width, whereby the types that are closest to each other are LH and LL as well as HH and HL. Then the *rule of the short side* again applies implying that the incentive constraints that bind first are those of the closest pairs IC_{LHvsLL} and IC_{HHvsHL}. The remaining binding constraint is the one that concerns intermediate types, namely IC_{LLvsHH}. Here, the motivation level γ is sufficiently low and then type LL might be induced to mimic type HH because the former can benefit from a lower effort e_{HH} and still enjoy a salary w_{HH} which cannot be too low (given that motivation plays a minor role).

In Case $A.2$, motivation γ is growing with respect to Case $A.1$ and it becomes high enough so as to generate a small disutility from effort provision for worker of type HH. In turn, the wage offered to type HH becomes so small, relative to the level of effort exerted, that type LL rather prefers to mimic HL. Case $A.2$ represents a bidimensional screening problem consisting of the two sub-programs related to adverse selection on workers' motivation (as in Benchmark BM in Subsection 2.1.3) which are now connected by incentive constraint IC_{LLvsHL} (as in Figure 2b).

In Case $A.3$, motivation keeps increasing and the disutility from the effort exerted by type HH is even lower than in Case $A.2$. Thus not only does type LL mimic type HL rather than type HH, but it turns out that type HH mimics LL rather than HL, meaning that an upward incentive constraint is binding here. This occurs when the motivated type HH values a relatively higher wage associated with a higher effort (that she would obtain by mimicking LL) more than the combination of lower wage and lower effort (that she would get by mimicking HL). Case $A.3$ is represented in Figure 2c.

Insert Figure 2b and Figure 2c around here

Figure 3 illustrates the existing classes of equilibria just presented. Which class of equilibrium realizes depends on the relative position of the term $\frac{2\gamma}{\Delta\theta}$ with respect to the sum of different pairs of effort levels. The term $\frac{\gamma}{\Delta\theta}$ again reflects the relative importance of motivation uncertainty *vis à vis* ability uncertainty and it is doubled since it must be compared to the sum of two effort levels, exerted by different pairs of agents. In turn, such different pairs of effort levels can be singled out by examining the crucial incentive constraints in each case (see Propositions 1 and 2 and Appendices C and D for further details).

Interestingly, what emerges in the following Subsections is that Case M holds when $\gamma > \gamma^*$ while

Case A attains when $\gamma < \gamma^*$. Therefore, when effort levels are aligned in a given way at the first-best, then the same ordering of effort levels arises at the second-best.

Lemma 1 *The ranking of second-best effort levels is always the same as the first-best ranking.*

Proof. See Appendices C.1, D.1.1, D.2.1 and D.3.1. ■

But it might also happen that neither motivation nor ability prevail. Therefore, it might be unfeasible to separate intermediate types HH and LL and pooling equilibria with $e_{LL} = e_{HH} = e_p$ might occur. As in the separating equilibria, we must distinguish here two sub-cases, the first one where the binding incentive constraint is IC_{HHvsHL}, which is relevant when $e_p + e_{HL} \geq \frac{2\gamma}{\Delta\theta}$ and the second one where the binding incentive constraint is IC_{LLvsHL}, occurring when $e_p + e_{HL} \leq \frac{2\gamma}{\Delta\theta}$. We will show that, when motivation and productivity have a similar impact on effort provision, i.e. for values of γ close to γ^*, then separation of types LL and HH becomes impossible and Case $A.1$ converges to the pooling equilibrium with IC_{HHvsHL} binding, whereas Case M, Case $A.2$ and Case $A.3$ all converge to the pooling equilibrium with IC_{LLvsHL} binding (see also Figure 4 in Section 4).[15]

It is possible to show that the solution entailing full participation and full separation of types always yields the highest profits to the principal, who will then always implement it when possible.

Proposition 3 *Independently of whether motivation or ability prevail, the principal's profits are maximal at the solution with full participation and full separation of types.*

Proof. The procedure for the situation in which motivation prevails is illustrated in Appendix C.3. The proofs for the three possible cases that realize when ability prevails are equivalent and then omitted. ■

In what follows, we will focus on the characterization of the four possible classes of equilibria with full participation and full separation, relegating to the Appendix the analysis of the corresponding situations with pooling and/or exclusion.[16] For expositional reasons, we are going to start from Case M, then we will treat the symmetric Case $A.1$ and, finally, we will consider the intermediate Cases $A.2$ and $A.3$. For simplicity, in the text we just provide a qualitative description of the different solutions with economic intuitions; we will relegate quantitative results, technical statements and proofs to the Appendices.

A comprehensive overview of the main results is provided in Section 4, which abstracts from the heavy technicalities and procedural complexities and instead focuses on the economic intuitions and on the relevant insights.

[15] Pooling equilibria for the four classes of possible results will be analyzed in Appendices C.2, D.1.2, D.2.2 and D.3.2, respectively. Pooling equilibria will be treated in a general way in Appendix D.4.

[16] Note that, when considering contracts with some pooling or exclusion, we always find that the optimal effort for workers that are neither pooled nor excluded is the same as in the fully participating and fully separating contract of the same class.

3.1 The solution when motivation prevails (Case M)

In Case M, a separating equilibrium with $e_{HH} > e_{LL}$ occurs if and only if Condition (9) holds, that is if $e_{LL} + e_{HH} \leq \frac{2\gamma}{\Delta\theta}$. Then, the constraints that are expected to bind at the optimum are IC_{LHvsHH}, IC_{HHvsLL}, IC_{LLvsHL} and PC_{HL}, as in Figure 1 (see also Proposition 1). In this situation, motivation γ is high enough for type HH to be asked to provide a relative high effort in exchange for a salary that is quite low (in fact w_{HH} may also be lower than the salary offered to worker LL, as the inequality in Remark 5 below points out). Thus, type HH might find the contract (e_{LL}, w_{LL}) appealing.

Given the binding constraints, we can derive the wage schedules, which allow us to isolate the information rents received by each type of worker

$$w_{HL} = \frac{1}{2}\theta_H e_{HL}^2, \tag{11}$$

$$w_{LL} = \frac{1}{2}e_{LL}^2 \underbrace{+\frac{1}{2}\Delta\theta e_{HL}^2}_{\text{Info rent worker } LL}, \tag{12}$$

$$w_{HH} = \frac{1}{2}\theta_H e_{HH}^2 - \gamma e_{HH} \underbrace{-\frac{1}{2}\Delta\theta e_{LL}^2 + \gamma e_{LL} + \frac{1}{2}\Delta\theta e_{HL}^2}_{\text{Info rent worker } HH} \tag{13}$$

and finally

$$w_{LH} = \frac{1}{2}e_{LH}^2 - \gamma e_{LH} \underbrace{+\frac{1}{2}\Delta\theta e_{HH}^2 - \frac{1}{2}\Delta\theta e_{LL}^2 + \gamma e_{LL} + \frac{1}{2}\Delta\theta e_{HL}^2}_{\text{Info rent worker } LH} \tag{14}$$

All types except HL receive an information rent and information rents cumulate when moving from the worst type HL up to the best type LH. Since information rents are always increasing in the effort exerted by the types that can be mimicked, we observe a downward distortion with respect to the first-best for all effort levels except the one of worker LH. Moreover, all information rents include at least one expression of the form $\frac{1}{2}\Delta\theta e_{ij}^2$ as in Benchmark BA (see comments to Proposition 1). Only motivated types HH and LH receive information rents depending also on motivation γ; in particular, the rent received by type HH when mimicking LL is given by $-\frac{1}{2}\Delta\theta e_{LL}^2 + \gamma e_{LL}$ which is always positive and increasing in e_{LL} when motivation prevails.[17] This occurs since the binding constraint IC_{HHvsLL} is here linking the two programs analyzed in Benchmark BA.

Substituting the wage schedules into the principal's objective function and maximizing with respect to effort levels yields the optimal efforts, the optimal wage levels and the indirect utilities that we can rank as follows.

Remark 5 *When motivation prevails (Case M), at the solution with full separation and full participation, the ranking of effort levels is*

$$e_{LH}^{SBM} = e_{LH}^{FB} > e_{HH}^{SBM} = e_{HH}^{BA} > e_{LL}^{SBM} > e_{HL}^{SBM}; \tag{15}$$

[17] A similar expression holds for type LL in Case $A.1$ as described in Subsection 3.2.1.

the ranking of wages is

$$w_{LH}^{SBM} > \max\left\{w_{HH}^{SBM}, w_{LL}^{SBM}\right\} > \min\left\{w_{HH}^{SBM}, w_{LL}^{SBM}\right\} > w_{HL}^{SBM} > 0$$

and the ordering of indirect utilities (information rents) is

$$u_{LH}^{SBM} > u_{HH}^{SBM} > u_{LL}^{SBM} > u_{HL}^{SBM} = 0.$$

When motivation prevails, all effort levels, except the one of the most efficient type of agent LH, are strictly less than the corresponding first-best levels. Hence we have the familiar result of *no distortion at the top* and a downward distortion in effort levels for all other agent's types.[18] Interestingly, e_{HH}^{SBM} is equivalent to the effort level we obtained for type HH in the case of asymmetric information on workers' productivity only. This again confirms that we are studying a program which extends the two sub-programs analyzed in Benchmark BA. Nonetheless, effort levels required from workers LL and HL are characterized by a larger distortion than in program BA (see expressions for e_{Lj}^{BA} and e_{Hj}^{BA}). This occurs because of the bidimensional nature of adverse selection, which in turn determines the cumulative effect of information rents.

Information rents have the same ordering as effort levels, while there can be a twist in the ranking of the salary of intermediate types. In other words, the principal could offer the motivated but high-cost type HH a contract in which effort provision is higher and remuneration is lower than in the contract proposed to type LL. This result is not trivial and depends on the peculiarity of motivated workers' utility function, which admits voluntary work.[19] Moreover, when $w_{HH}^{SBM} < w_{LL}^{SBM}$ holds, then it is always the case that $e_{HH}^{SBM} < \frac{2\gamma}{\theta_H}$, implying that for motivated high-cost types HH effort provision has an overall positive impact on utility and does not represent a net cost (see expression 13).

Corollary 1 *When motivation prevails, worker HH might be a "paid volunteer": she is offered a positive wage, given the information rents she receives, but she enjoys a positive utility from effort exertion.*

Finally note that Case M corresponds to the situation where our bidimensional screening problem is equivalent to a unidimensional screening one with four types, the unidimensional parameter of private information being the workers' overall cost of effort exertion.

3.2 The solution when ability prevails (Case A)

In Case A, full separation with $e_{LL} > e_{HH}$ occurs if and only if Condition (10) holds, that is if $e_{LL}+e_{HH} \geq \frac{2\gamma}{\Delta\theta}$ (see Proposition 2). The participation constraint PC_{HL} is required to be binding and the relevant

[18] All quantitative results referring to this Section are contained in Appendix C.

[19] In particular, $w_{HH}^{SBM} < w_{LL}^{SBM}$ holds when the probability of motivation is low relative to the probability of low effort cost, when the difference in effort cost is high and when the level of motivation is high too.

incentive constraints that one assumes to be binding are IC_{LHvsLL}, IC_{LLvsHH} or eventually IC_{LLvsHL} (whichever one binds first), IC_{HHvsHL} or IC_{HHvsLL} (again whichever one binds first). Note that all incentive compatibility constraints considered are downward constraints except for IC_{HHvsLL} which points upwards. Since IC_{LLvsHH} and IC_{HHvsLL} cannot be simultaneously binding at a separating equilibrium, then the possible situations are the following: (1) all downward local ICs are binding and thus IC_{LHvsLL}, IC_{LLvsHH} and IC_{HHvsHL} hold with equality, as shown in Figure 2a; (2) the downward local constraints IC_{LHvsLL} and IC_{HHvsHL} and the global downward constraint IC_{LLvsHL} are all binding, as shown in Figure 2b; (3) constraints IC_{LHvsLL}, IC_{LLvsHL} and the upward IC_{HHvsLL} hold with equality, as shown in Figure 2c.

Such three possible cases will be analyzed in detail in what follows.[20]

3.2.1 Case $A.1$

Suppose that IC_{LLvsHH} (rather than IC_{LLvsHL}) and IC_{HHvsHL} are binding (Figure 2a), which occurs when $e_{HL}+ e_{HH} \geq \frac{2\gamma}{\Delta\theta}$ holds. This represents the most intuitive case where the downward incentive constraint between the intermediate types LL and HH is binding. This occurs when γ is sufficiently low so that worker HH receives a relatively high salary in exchange for a relatively low effort, and such a contract is attracting for type LL.

Solving the binding constraints for salaries, one obtains the following wage schedules

$$w_{HL} = \frac{1}{2}\theta_H e_{HL}^2, \tag{16}$$

$$w_{HH} = \frac{1}{2}\theta_H e_{HH}^2 - \gamma e_{HH} \underbrace{+\gamma e_{HL}}_{\text{Info rent worker } HH}, \tag{17}$$

$$w_{LL} = \frac{1}{2}e_{LL}^2 \underbrace{+\frac{1}{2}\Delta\theta e_{HH}^2 - \gamma e_{HH} + \gamma e_{HL}}_{\text{Info rent worker } LL} \tag{18}$$

and

$$w_{LH} = \frac{1}{2}e_{LH}^2 - \gamma e_{LH} \underbrace{+\gamma e_{LL} + \frac{1}{2}\Delta\theta e_{HH}^2 - \gamma e_{HH} + \gamma e_{HL}}_{\text{Info rent worker } LH}. \tag{19}$$

All information rents, except the one of type HL, are positive and have the usual cumulative structure. They all include at least one expression of the form γe_{ij} as in Benchmark BM where asymmetric information is on motivation only (see comments to Proposition 2). Only type LL receives an information rent which also depends on the difference in ability $\Delta\theta$: this comes from the fact that this program embeds the two subcases in Benchmark BM and links them through constraint IC_{LLvsHH}. Type LH cumulates this rent too when trying to mimic LL.

[20] All quantitative results referring to this Section are contained in Appendix D.

This case is peculiar because an additional constraint needs to be satisfied: the rent accruing to type LL when mimicking HH must be positive and this occurs if and only if $e_{HH} > \frac{2\gamma}{\Delta\theta}$ (which is obviously more stringent than condition 10). In different words, only when γ is sufficiently low, does type LL benefit from mimicking type HH. Otherwise, type LL will rather prefer to mimic type HL as in Case $A.2$ and Case $A.3$ that follow.[21]

Being the former requirement satisfied, it is immediate to observe that information rents are increasing in the effort exerted by the types that can be mimicked. Therefore, the result of *no distortion at the top* and downward distortion in effort levels for all other agent's types is still obtained.[22]

Remark 6 *When ability prevails and IC_{LLvsHH} and IC_{HHvsHL} are binding (Case $A.1$), at the solution with full separation and full participation, the ordering of effort levels is*

$$e_{LH}^{SBA1} = e_{LH}^{FB} > e_{LL}^{SBA1} = e_{LL}^{BM} > e_{HH}^{SBA1} > e_{HL}^{SBA1},$$

the ordering of wages is

$$w_{LH}^{SBA1} > w_{LL}^{SBA1} > w_{HH}^{SBA1} > w_{HL}^{SBA1}$$

and the ordering of indirect utilities (information rents) is

$$u_{LH}^{SBA1} > u_{LL}^{SBA1} > u_{HH}^{SBA1} > u_{HL}^{SBA1} = 0.$$

Note that e_{LL}^{SBA1} is equal to the effort level we obtained for type LL in the case of adverse selection on the workers' motivation only (Benchmark BM), while the effort levels required from the less efficient workers (here types HH and HL) are characterized by a larger downward distortion than in program BM.

Case $A.1$ represents the unique instance in which wages and information rents always have the same ordering as effort levels.

Together with Case M, this case corresponds to the situation where the bidimensional screening problem is equivalent to the unidimensional screening one with four types, the unidimensional parameter of private information being the workers' overall cost of effort exertion.

3.2.2 Case $A.2$

Suppose that, together with PC_{HL} and IC_{LHvsLL}, the binding incentive constraints are now IC_{HHvsHL} and IC_{LLvsHL} (Figure 2b), which happens when $e_{HL} + e_{HH} \leq \frac{2\gamma}{\Delta\theta}$ holds. Moreover IC_{HHvsLL} must be

[21] Note that condition $e_{HH} > \frac{2\gamma}{\Delta\theta}$ implies condition $e_{HH} > \frac{2\gamma}{\theta_H}$. Hence if LL receives a positive information rent when mimicking HH, then it must be that type HH is not a potential volunteer and that she is experiencing a net cost from providing effort.

[22] See Appendix D.1 for the complete analysis.

satisfied, which amounts to $e_{HL} + e_{LL} \geq \frac{2\gamma}{\Delta\theta}$. This represents one of the less intuitive subcases where type LL is able to obtain a higher information rent when mimicking type HL rather than type HH. This occurs since motivation γ is high enough so that type HH is asked to make a relatively high effort in exchange for a relatively low wage and her contract is not appealing to type LL.

The salaries of types HH and HL are the same as in Case $A.1$, and given by (17) and (16) respectively, whereas the other relevant wage levels are now

$$w_{LL} = \frac{1}{2}e_{LL}^2 \quad \underbrace{+\frac{1}{2}\Delta\theta e_{HL}^2}_{\text{Info rent worker } LL} \tag{20}$$

and

$$w_{LH} = \frac{1}{2}e_{LH}^2 - \gamma e_{LH} \underbrace{+\gamma e_{LL} + \frac{1}{2}\Delta\theta e_{HL}^2}_{\text{Info rent worker } LH}. \tag{21}$$

Here, no type is willing to mimic worker HH, so that it is useless for the principal to distort effort e_{HH} downwards in order to reduce the information rent of potential mimickers. Hence, we do not observe downward distortions with respect to the first-best effort levels neither for type LH nor for type HH.

Moreover, the information rent of worker LL has the same expression as the one obtained in Case M and is formed by one term only, which depends on the effort exerted by worker HL: this is because type LL mimics type HL directly, without "going through" type HH, and thus no rent depending on e_{HH} appears. For the same reason, information rents accruing to both types LH and LL are "shorter" than in Case $A.1$, as the paths of binding incentive constraints in Figure 2b show. More precisely, the information rents of both types HH (see expression 17) and LL depend on the effort of worker HL; however in w_{LL} the rent is $\frac{1}{2}\Delta\theta e_{HL}^2$ (as the one in expression 6 of Benchmark BA and in expression 12 of Case M), whereas in w_{HH} the rent is γe_{HL} (as the one in expression 7 of Benchmark BM). As a consequence, and as will be more clear when describing Case $A.3$, we can interpret this specific sub-case as a program that is in-between Case $A.1$ and Case $A.3$ which follows.[23]

Remark 7 *When ability prevails and constraints IC_{LLvsHL} and IC_{HHvsHL} are binding (Case A.2), at the solution with full separation and full participation, the ordering of effort levels is*

$$e_{LH}^{SBA2} = e_{LH}^{FB} > e_{LL}^{SBA2} = e_{LL}^{SBA1} = e_{LL}^{BM} > e_{HH}^{SBA2} = e_{HH}^{FB} > e_{HL}^{SBA2},$$

the ranking of wages is

$$w_{LH}^{SBA2} > w_{LL}^{SBA2} > w_{HH}^{SBA2} > w_{HL}^{SBA2} \tag{22}$$

and the ordering of information rents is

$$u_{LH}^{SBA2} > u_{HH}^{SBA2} > u_{LL}^{SBA2} > u_{HL}^{SBA2} = 0. \tag{23}$$

[23] See Appendix D.2 for the complete analysis.

Note that e_{LL}^{SBA2} has the same expression as e_{LL}^{SBA1} and as e_{LL}^{BM} in Benchmark BM with adverse selection on motivation. As already mentioned, both e_{LH}^{SBA2} and e_{HH}^{SBA2} are equal to their first-best levels, while both e_{LL}^{SBA2} and e_{HL}^{SBA2} are distorted downwards and e_{HL}^{SBA2} has a larger distortion than the corresponding term in program BM.

In Case $A.2$ (and also in Case $A.3$, as will be clarified later on), wages have the same ordering as effort levels, while the ranking of information rents is switched for intermediate types (and it is the same as in Case M). Such a switch of the indirect utilities of intermediate types depends on the value of γ which is higher than in Case $A.1$ and sufficiently high to substantially reduce the disutility from the effort for type HH.

Importantly, as stated in Result 5 of Appendix D.2.1, a fully separating and fully participating equilibrium in Case $A.2$ only exist if $\mu < \frac{1}{2}$, that is if the probability of motivated workers is sufficiently low. In fact, the information rents of workers HH and LH depend on γ, which is relatively large in Case $A.2$. Thus, this equilibrium exists if the total number of information rents that motivated workers receive is not too high.

Intuitively, this situation seems to have good welfare properties since effort levels are less distorted than in Cases M and $A.1$ which would lead to a higher total surplus; furthermore, the paths characterizing informational rents in this case are shorter than in Cases M and $A.1$, thus suggesting a distribution of total surplus in favor of the principal (see also Figure 2b). Despite our intuition, it is not possible to provide a clear-cut comparison between Case $A.2$ and Case M.[24] Nonetheless, Case $A.2$ and Case $A.1$ can be ordered in terms of total surplus.[25]

Remark 8 *The equilibrium allocation with full separation and full participation of types attained in Case $A.2$ Pareto-dominates the corresponding allocation in Case $A.1$.*

3.2.3 Case $A.3$

Suppose that, together with PC_{HL} and IC_{LHvsLL}, the binding incentive constraints are now IC_{LLvsHL} and the *upward* incentive constraint IC_{HHvsLL} (see Figure 2c). This results in inequality $e_{HL} + e_{LL} \leq \frac{2\gamma}{\Delta\theta} \leq e_{HH} + e_{LL}$.

[24] A sufficient condition for Case $A.2$ to yield higher total surplus than Case M would be $e_{ij}^{SBA2} \geq e_{ij}^{SBM}$ for each type ij. However, such inequality is not satisfied for type HL. The necessary and sufficient condition for Case $A.2$ to Pareto dominate Case M amounts to $\sum_{ij} e_{ij}^{SBA2} \geq \sum_{ij} e_{ij}^{SBM}$, but it is not possible to assess whether such requirement is satisfied or not.

[25] The sufficient condition for Case $A.2$ to dominate Case $A.1$ is $e_{HL}^{SBA2} \geq e_{HL}^{SBA1}$ which is always satisfied. See Appendix D.3.3.

The relevant wage levels are now

$$w_{HH} = \frac{1}{2}\theta_H e_{HH}^2 - \gamma e_{HH} \underbrace{- \frac{1}{2}\Delta\theta e_{LL}^2 + \gamma e_{LL} + \frac{1}{2}\Delta\theta e_{HL}^2}_{\text{Info rent worker } HH} \tag{24}$$

and

$$w_{LH} = \frac{1}{2}e_{LH}^2 - \gamma e_{LH} \underbrace{+ \gamma e_{LL} + \frac{1}{2}\Delta\theta e_{HL}^2}_{\text{Info rent worker } LH} \tag{25}$$

together with w_{HL} and w_{LL} as defined above by expressions (16) and (20), respectively.

The information rent of type HH in expression (24) is composed of two terms: the last one, $\frac{1}{2}\Delta\theta e_{HL}^2$, is the rent received through type LL mimicking HL (which accrues to all types except HL); the first one, $-\frac{1}{2}\Delta\theta e_{LL}^2 + \gamma e_{LL}$, is the part of the rent specific to type HH mimicking LL, and as we expected has the same expression as in Case M. Such expression is positive if and only if $e_{LL} < \frac{2\gamma}{\Delta\theta}$, which is always the case when $e_{LL} + e_{HL} \leq \frac{2\gamma}{\Delta\theta}$ is satisfied. Thus, all the terms appearing in the informational rent of type HH are strictly positive. Moreover, the information rent accruing to type LH has the same expression as in Case $A.2$. Also note that motivated types receive an information rent which depends both on the variability in productivity and on motivation, so that this case shares some features both with Benchmark BA and with Benchmark BM.

One can conclude that this program bridges Case A (in particular, Case $A.2$) and Case M. Indeed, the unique incentive constraint that is shared with Case $A.1$ is IC_{LHvsLL}; whereas the other two binding constraints are IC_{HHvsLL} and IC_{LLvsHL} as in Case M (see Figures 1, 2a and 2c).

Remark 9 *When ability prevails and constraints IC_{LLvsHL} and IC_{HHvsLL} are binding (Case $A.3$), at the solution with full separation and full participation, the ordering of effort levels is*

$$e_{LH}^{SBA3} = e_{LH}^{FB} > e_{LL}^{SBA3} > e_{HH}^{SBA3} = e_{HH}^{FB} > e_{HL}^{SBA3} = e_{HL}^{SBM}$$

while the ordering of wages and information rents is the same as in Case $A.2$.[26]

Both e_{LH}^{SBA3} and e_{HH}^{SBA3} are equal to their first-best levels and e_{HL}^{SBA3} has the same expression as e_{HL}^{SBM}. Moreover, the usual downward distortion holds for the effort provided by types HL and LL, the latter despite the upward incentive constraint IC_{HHvsLL} being binding.

Nonetheless, when the optimal contract calls for exclusion of type HL (occurring for motivation levels that are below the range in which full participation and full separation is guaranteed)[27], then it might well be that effort e_{LL}^{SBA3} is distorted upward with respect to its first-best level. The existence of an upward distortion in second-best effort levels parallels the result of sub-marginal cost pricing in Armstrong (1999)

[26] See Remark 7 and comments below.

[27] See also Figure 4 below.

who points out that it is the combination of two dimensions of asymmetric information which gives rise to this interesting finding. A difference with respect to Armstrong (1999) is that sub-marginal cost pricing can only be found when private and social incentives diverge (i.e. first- and second-best allocations are not aligned), while in our model full alignment always occurs (see Lemma 1).

Considering effort levels at equilibria with full participation and full separation, a clear-cut and interesting comparison across the different cases can be made.

Proposition 4 *The equilibrium allocation with full separation and full participation of types attained in Case $A.3$ Pareto-dominates the corresponding allocations in all other cases.*

Proof. See Appendix D.3.3. ■

Less distortions are usually coupled with higher profits to the principal. Unfortunately, it is not possible to assess whether the higher surplus generated in Case $A.3$ is distributed in favor of the principal or in favor of workers, since neither expected profits nor information rents are easily comparable across cases.[28]

4 Summary and interpretation of results

In this Section, we summarize our main findings and we offer some economic interpretations. We first consider the four dominating equilibria with full participation and full separation, we then provide intuitions on which equilibria with pooling or exclusion arise in-between the previous ones.

All our results have been presented as a function of the motivation parameter γ that, in our model, has economic meaning in the range $(0, 1]$. We solved two broad classes of problems: when motivation γ is high relative to the difference in ability $\Delta\theta$, then $e_{HH} > e_{LL}$ holds (Case M); conversely, when the difference in ability is high relative to motivation, then $e_{LL} > e_{HH}$ holds (Case A). We showed that the possible ranking of effort levels hold both at the first- and at the second-best, meaning that there is full alignment between first- and second-best allocations or else that the distortions imposed by bidimensional adverse selection are somehow limited.

We started by characterizing the two polar and most intuitive solutions of the model: Case M and Case $A.1$. In those situations, the binding incentive constraints of the principal's program are simple to be identified since the *rule of the short side* applies. In particular, only downward local incentive constraints are binding, which are the ones connecting types that are relatively closer to each other because they are located on the short sides of the rectangle representing the type space. In both environments, information

[28] Some partial results concerning the different distribution of surplus between principal and agent in Case $A.3$ and in Case M are presented at the end of Appendix D.3.3.

rents are monotonically increasing while effort distortions are monotonically decreasing with respect to the ranking of types.

Case M occurs when motivation takes very high values and is more important than the difference in ability, so that the short sides of the rectangle representing the type space are those connecting types along the dimension of productivity. Thus, such case embeds and generalizes to the bidimensional context the unidimensional screening programs with unobservable productivity and observable motivation (Benchmark BA). Notably, Case M is the unique situation in which low-skilled, motivated workers can become volunteers, that is they can be ready to work for free since they receive a *utility* instead of a disutility from effort provision. Nevertheless, we show that they are *paid volunteers* since they receive a strictly positive wage; this is due to the information rents required for truthful revelation (see Corollary 1). Put differently, optimal contracts are such that the ranking of wages is not fully aligned with the ranking of efforts and information rents (see Remark 5) since worker HH may receive a lower wage than worker LL even if she exerts a higher effort. Despite this fact, under Case M, type HH always enjoys a higher utility than type LL.

Case $A.1$ arises when motivation not only is less important than productivity variation but takes very low values, so that the *rule of the short side* becomes relevant and the binding constraints are those connecting types along the dimension of motivation. Thus, Case $A.1$ embeds and generalizes to the bidimensional context the unidimensional screening programs where productivity is observable and motivation is not (Benchmark BM).

Interestingly, Case M and Case $A.1$ correspond to the situation where our bidimensional screening problem is equivalent to a unidimensional screening one with four types, the unidimensional parameter of private information being the workers' overall cost of effort provision.

Between Case M and Case $A.1$, that is when motivation is still less important than ability variation but is not too low, the two less intuitive situations occur: Case $A.2$ and Case $A.3$. Here a tension realizes since, on the one hand, type LL is asked to provide a higher effort than type HH; on the other hand, as motivation increases, type HH workers face a diminishing disutility from the effort such that it becomes more and more convenient for the principal to ask them to provide a larger effort and to pay them a lower salary. This tension leads anomalous incentive constraints to bind. In particular, Case $A.2$ emerges when the downward incentive constraint IC_{LLvsHH} is not binding anymore and IC_{LLvsHL} is binding instead, so that the principal has no interest in distorting downward the effort required to worker HH. As a consequence, in Case $A.2$, together with the standard *no-distortion at the top*, we also find no-distortion for type HH. The tension described before has even more drastic consequences in Case $A.3$ where motivation is rising. Now we find that not only is the downward incentive constraint IC_{LLvsHH} not binding, so that no-distortion for type HH occurs, but the *upward* incentive constraint IC_{HHvsLL} is binding instead. In other words, the disutility from the effort for type HH is so low that she is asked to provide an effort level close to the one required from LL. Moreover, the latter worker is also receiving a higher wage, thus HH is willing to mimic LL rather than HL. Notably, Case $A.3$ is such that, when full participation is not viable and exclusion of type HL is necessary, then the solution might be characterized by an upward distortion in the effort provided by type LL. This results parallels the one concerning sub-marginal cost pricing in Armstrong (1999) and is peculiar to the bidimensional nature of asymmetric information.

Effort distortions are higher in Case M and Case $A.1$ as opposed to both Case $A.2$ and Case $A.3$, where one more effort, namely e_{HH}, is set at the first-best level. In particular, we show that Case $A.2$ Pareto-dominates Case $A.1$ and, most importantly, we are able to prove that in Case $A.3$ the highest possible surplus, among all second-best solutions, is reached. Nevertheless, the distribution of this higher surplus generated by Case $A.3$ might not necessarily be in favour of the principal, who might fail to appropriate the benefits from the higher efficiency.

We would expect Case $A.2$ to be characterized by less distortions and higher surplus than Case $A.3$, given that information rents are composed by a fewer number of parts that are added up (as can be seen following the paths highlighted in Figures 2b and 2c). Contrary to this intuition, Case $A.3$ ends up being the best from a social point of view. The reason for this counter-intuitive result is the following. Effort levels e_{LH} and e_{HH} are both set at their first-best levels in both Cases $A.2$ and $A.3$, while downward distortions for non-motivated workers are higher in Case $A.2$ than in Case $A.3$. In particular, in Case $A.2$, worker HL has two other types being attracted to him and therefore e_{HL} faces a stronger downward pressure; in Case $A.3$, effort e_{LL} is subject to two opposing of forces: on the one hand, a downward distortion is called for because of the potential mimicking by type LH, on the other hand an upward pressure, which partially off-sets the former downward distortion, is exerted by type HH.

Since Case $A.3$ allows to obtain the highest social surplus, we reach the unexpected conclusion that high motivation is detrimental to total effort provision and thus to total output production.

Concerning the different types of equilibria we study (with and without pooling and/or exclusion), an unexpected result is the following: whatever the value of motivation (and thus, whatever the class of results considered) the equilibrium involving full participation and full separation of types yields the highest profits to the principal, who will then always implement it when possible. As mentioned in the Related Literature, the strict dominance of fully separating and fully participating equilibria is unusual in models of multidimensional screening, both for discrete and for continuous types space.

Figure 4 describes emerging equilibria as a function of motivation, mainly focusing on the existence regions for fully separating and fully participating equilibria. In the Figure, we also consider the main equilibria involving pooling and exclusion that arise in-between the four classes of fully separating and fully participating equilibria. All equilibria are mutually exclusive, since for any given realization of the parameters $\gamma \in (0, 1]$ and $\theta_H \in (1, 2]$, a different solution is obtained. Moreover, according to either the probability distributions of motivation and productive ability or to the magnitude of the difference in ability, some situations could be discarded.[29]

Insert Figure 4 around here

When implementation of fully separating contracts is not viable, the principal resorts to different optimal contracts involving pooling of types. In particular, when motivation takes the lowest possible values (that is to the left of Case $A.1$) then a pooling equilibrium where the low-ability types HH and HL are given the same contract emerges. At the other extreme, for the highest possible values of motivation (that is to the right of Case M), a pooling equilibrium where the non-motivated types HL and LL are given the same contract emerges. Moreover, when neither motivation uncertainty nor productivity uncertainty strictly prevail, we obtain a solution with bunching for intermediate types HH and LL.

When full participation becomes impossible, then the principal resorts to exclusion of either the worst type or even the two worse types. As Figure 4 shows, the occurrence of equilibria with exclusion is really limited and essentially relegated to small regions lying in-between fully participating and fully separating Case $A.1$ and Case $A.2$ and in-between fully participating and fully separating Case $A.2$ and Case $A.3$. Comparing our results concerning exclusion with Delfgaauw and Dur (2008)'s, we can state the following. Our model suggests that, if exclusion is necessary, then it surely concerns the worst type of workers HL; conversely, Delfgaauw and Dur (2008) point out that, when the government needs to hire at most two types of workers and when motivation enters workers' utility in combination with a concave function of effort, types LH and HL are hired while types LL are left out of the public sector.

5 Conclusion

It is argued that the efficient selection of workers is more effective, from the principal's point of view, than optimally designing incentives once the worker has been hired. In different words, firms might partially solve their agency problems by hiring agents with specific preferences (see Brehm and Gates 1997, Prendergast 2007, 2008). This seems particularly relevant in a labor market where potential workers can be intrinsically motivated for the job.

We solve a bidimensional screening model with discrete types where a firm is willing to fill a vacancy and where workers have private information about both their intrinsic motivation for the task to be performed and their skills. Optimal contracts are derived and fully characterized when only one instrument is available to the principal, namely the observable level of effort required by the contract.

The existing literature on intrinsic motivation in the labor market has focused on two major issues:

[29] Appendix E considers the possible equilibria arising in the particular case in which the probability distribution of types is uniform.

(i) the lemons' problem, mainly investigating adverse (vs propitious) selection effects of workers' private information on the composition of the pool of active workers; (ii) the sorting of different workers' types into different sectors (vocational and non-vocational) of the labor market. We depart from the first strand of literature because we focus our attention at the individual level and examine a principal-agent relationship. We also depart from the second strand of literature because we consider a single sector in isolation. This allows us to examine bidimensional screening in all its essential features and to contribute to the existing literature, where the problem of workers' self-selection has either been avoided (because full information on the workers' attributes has been considered, as in Delfgaauw and Dur 2010), or has been modeled in a reduced form (with only a subset of workers being employed, as in Delfgaauw and Dur 2008).

We show that four different classes of equilibria with full separation and full participation exist, which always dominate the equilibria with pooling and/or exclusion. Thus, despite the number of instruments being lower than the number of dimensions of private information, effort distortions and information rents are not too costly, since the principal is willing to hire all workers' types and implements fully separating allocations whenever is possible. From this viewpoint, our results stand in contrast with the literature on multidimensional screening with a continuum of types (Laffont et al. 1987 and Basov 2003, 2005) which predicts that exclusion and bunching are inevitable.

We find that, in all equilibria, motivated workers always receive higher rewards than non-motivated ones, for a given ability level. This result does not match the key prediction of the previous literature on intrinsic motivation (see Handy and Katz 1998, Besley and Ghatak 2005, Delfgaauw and Dur 2007, 2008), namely that relatively low pay and weak monetary incentives endogenously emerge in jobs where intrinsic motivation matters.

In addition, optimal contracts may cause some non-monotonicities to arise, which are driven by the interplay of the two dimensions of private information. In Case M, motivated low-skilled workers might become paid volunteers because they experience a net utility from effort provision and, notwithstanding, they receive a sufficiently high wage due to the enjoyment of information rents. In Case $A.3$, effort distortions are minimal and the maximal levels of effort provision and output production are reached. So our model predicts that, from a social point of view, it is better when motivation is not too high and rather when workers' ability prevails over motivation. This result is reminiscent of Van den Steen (2006), who analyses the consequences of pay-for-performance incentives when principal and agent might disagree on the optimal course of action and concludes that motivation might be too high because it triggers agent's disobedience. Finally, Case $A.3$ is compatible with the existence of an upward distortion in the effort exerted by non-motivated, skilled types LL. This results is in line with Armstrong (1999)'s finding of sub-marginal cost pricing when demand uncertainty is more significant than cost uncertainty.

In our future research, we are willing to tackle the problem of sorting of different workers' types into different sectors of the labor market (being one of them vocation-based). In particular, we are going to consider two principals competing for workers who are characterized by different motivation and skill levels. One principal represents the vocational sector and is thus interested in screening potential workers with respect to both motivation and ability (as in the present analysis), while the other principal is only interested in workers' skills. The present work represents a first step towards the analysis of optimal sorting when both employers act strategically.

A Appendix

B Constraints

For type LH the constraints are

$$w_{LH} - \frac{1}{2}e_{LH}^2 + \gamma e_{LH} \geq 0 \tag{PC_{LH}}$$

and

$$w_{LH} - \frac{1}{2}e_{LH}^2 + \gamma e_{LH} \geq w_{LL} - \frac{1}{2}e_{LL}^2 + \gamma e_{LL} \tag{IC_{LHvsLL}}$$

$$w_{LH} - \frac{1}{2}e_{LH}^2 + \gamma e_{LH} \geq w_{HH} - \frac{1}{2}e_{HH}^2 + \gamma e_{HH} \tag{IC_{LHvsHH}}$$

$$w_{LH} - \frac{1}{2}e_{LH}^2 + \gamma e_{LH} \geq w_{HL} - \frac{1}{2}e_{HL}^2 + \gamma e_{HL}. \tag{IC_{LHvsHL}}$$

For type LL:

$$w_{LL} - \frac{1}{2}e_{LL}^2 \geq 0 \tag{PC_{LL}}$$

and

$$w_{LL} - \frac{1}{2}e_{LL}^2 \geq w_{LH} - \frac{1}{2}e_{LH}^2 \tag{IC_{LLvsLH}}$$

$$w_{LL} - \frac{1}{2}e_{LL}^2 \geq w_{HH} - \frac{1}{2}e_{HH}^2 \tag{IC_{LLvsHH}}$$

$$w_{LL} - \frac{1}{2}e_{LL}^2 \geq w_{HL} - \frac{1}{2}e_{HL}^2. \tag{IC_{LLvsHL}}$$

For type HH:

$$w_{HH} - \frac{1}{2}\theta_H e_{HH}^2 + \gamma e_{HH} \geq 0 \tag{PC_{HH}}$$

and

$$w_{HH} - \frac{1}{2}\theta_H e_{HH}^2 + \gamma e_{HH} \geq w_{LH} - \frac{1}{2}\theta_H e_{LH}^2 + \gamma e_{LH} \tag{IC_{HHvsLH}}$$

$$w_{HH} - \frac{1}{2}\theta_H e_{HH}^2 + \gamma e_{HH} \geq w_{LL} - \frac{1}{2}\theta_H e_{LL}^2 + \gamma e_{LL} \tag{IC_{HHvsLL}}$$

$$w_{HH} - \frac{1}{2}\theta_H e_{HH}^2 + \gamma e_{HH} \geq w_{HL} - \frac{1}{2}\theta_H e_{HL}^2 + \gamma e_{HL}. \tag{IC_{HHvsHL}}$$

Finally, for type HL one has

$$w_{HL} - \frac{1}{2}\theta_H e_{HL}^2 \geq 0 \tag{PC_{HL}}$$

and

$$w_{HL} - \frac{1}{2}\theta_H e_{HL}^2 \geq w_{LH} - \frac{1}{2}\theta_H e_{LH}^2 \tag{IC_{HLvsLH}}$$

$$w_{HL} - \frac{1}{2}\theta_H e_{HL}^2 \geq w_{LL} - \frac{1}{2}\theta_H e_{LL}^2 \tag{IC_{HLvsLL}}$$

$$w_{HL} - \frac{1}{2}\theta_H e_{HL}^2 \geq w_{HH} - \frac{1}{2}\theta_H e_{HH}^2. \tag{IC_{HLvsHH}}$$

One can show that participation constraint PC_{HH} is automatically satisfied when PC_{HL} and IC_{HHvsHL} are both satisfied. Also participation constraint PC_{LH} is automatically satisfied when PC_{LL} and IC_{LHvsLL} are. Finally, once incentive constraint IC_{LLvsHL} and participation constraint PC_{HL} are satisfied, then also participation constraint PC_{LL} is. So, when all worker types are expected to be hired by the principal, it is only necessary to consider the participation constraint of the worst type HL.

As for the incentive compatibility constraints, it is possible to sum them two by two, yielding a partial ranking of effort levels. In particular, adding IC_{LLvsHL} with IC_{HLvsLL} and IC_{HHvsLH} with IC_{LHvsHH} one has $e_{Lj} \geq e_{Hj}$ $\forall j = L, H$, meaning that, given motivation, effort required must be higher the lower the effort cost. In the same way, adding IC_{HHvsHL} with IC_{HLvsHH} and IC_{LHvsLL} with IC_{LLvsLH} yields $e_{iH} \geq e_{iL}$ $\forall i = L, H$. Namely, for a given effort cost, effort is higher the higher the motivation. Hence the monotonicity condition (8) in the main text holds. Condition (8) also allows us to eliminate some "global" downward incentive constraints and focus on "local" ones. Indeed, adding IC_{LHvsHH} and IC_{HHvsHL} one obtains

$$w_{LH} - \frac{1}{2}e_{LH}^2 + \gamma e_{LH} \geq w_{HL} - \frac{1}{2}\theta_H e_{HL}^2 + \gamma e_{HL} + \frac{1}{2}\Delta\theta e_{HH}^2.$$

But, when $e_{HH} \geq e_{HL}$, the right-hand side of the above inequality is greater than $w_{HL} - \frac{1}{2}e_{HL}^2 + \gamma e_{HL}$, which in turn implies that the global downward incentive constraint IC_{LHvsHL} is satisfied when the two local incentives constraints IC_{LHvsHH} and IC_{HHvsHL} are.[30]

What about intermediate types HH and LL? Adding IC_{LLvsHH} and IC_{HHvsLL} one has

$$\frac{1}{2}\Delta\theta\left(e_{LL} - e_{HH}\right)\left(e_{LL} + e_{HH}\right) - \gamma\left(e_{LL} - e_{HH}\right) \geq 0,$$

which is satisfied either under condition (9) or under condition (10) in the main text.

Using the same arguments as before, one can get rid of other global constraints. Suppose that condition (9) is verified: then, it is easy to show that the sum of the local constraints IC_{LHvsHH} and IC_{HHvsLL} implies that the global constraint IC_{LHvsLL} is satisfied as well. In addition, IC_{HHvsLL} and IC_{LLvsHL} imply IC_{HHvsHL}. By the same token, suppose that condition (10) holds: then, one can prove

[30] The same conclusion holds taking the two local incentives IC_{LHvsLL} and IC_{LLvsHL}.

that constraints IC_{LHvsLL} and IC_{LLvsHH} imply constraint IC_{LHvsHH} and also that IC_{LLvsHH} and IC_{HHvsHL} can be used to eliminate IC_{LLvsHL}.

C Motivation prevails (Case M)

C.1 Full separation and full participation

Let us impose that IC_{LHvsHH}, IC_{HHvsLL}, IC_{LLvsHL} and PC_{HL} hold with equality and let us solve for the wage levels: this yields expressions from (11) to (14). Substituting such wage schedules into the principal's objective function and maximizing with respect to effort levels gives

$$e_{LH}^{SBM} = 1 + \gamma, \tag{26}$$

$$e_{HH}^{SBM} = \frac{(1-\nu)(1+\gamma)}{(\theta_H - \nu)}, \tag{27}$$

$$e_{LL}^{SBM} = \frac{\nu(1-\mu) - \mu\gamma}{(1-(1-\nu)(1-\mu)) - \mu\theta_H} \tag{28}$$

and

$$e_{HL}^{SBM} = \frac{(1-\nu)(1-\mu)}{\theta_H - (1-(1-\nu)(1-\mu))}. \tag{29}$$

Observe that all effort levels are always strictly positive, except for e_{LL}^{SBM}. In order for e_{LL}^{SBM} to be a maximum of the principal's expected profits, it is necessary to impose that both the numerator and the denominator in expression (28) be positive,[31] that is it must be that both

$$\gamma < \frac{\nu(1-\mu)}{\mu} = \gamma_0, \tag{30}$$

where $\gamma_0 > 1$ for $\mu > \frac{\nu}{1+\nu} = \mu_0$ (thus $\mu > \mu_0$ implies that $\gamma < \gamma_0$ is always verified) and

$$\theta_H < \frac{(1-(1-\nu)(1-\mu))}{\mu} = \rho_1,$$

with $\rho_1 > 1$, hold.

As far as the monotonicity conditions are concerned, $e_{HH}^{SBM} > e_{LL}^{SBM}$ is satisfied if and only if

$$\gamma > \frac{(\mu(1-\nu) + \nu(1-\mu))\Delta\theta}{\nu\mu\Delta\theta + (1-\nu)(1-(1-\nu)(1-\mu))} = \underline{\gamma}^{SBM},$$

where $\underline{\gamma}^{SBM} < 1$ is always the case for $(3\mu\nu - \nu - \mu) \geq 0$, that is for $\nu > \frac{1}{3}$ and $\mu \geq \frac{\nu}{(3\nu-1)}$, whereas, for $(3\mu\nu - \nu - \mu) < 0$, inequality $\underline{\gamma}^{SBM} < 1$ is true when

$$\theta_H < \frac{\mu + \nu - 3\mu\nu + (1-\nu)(1-(1-\nu)(1-\mu))}{\mu + \nu - 3\mu\nu} = \rho_2$$

[31] This can be easily seen by collecting e_{LL} in the principal's objective function, once the wage schedules have been substituted, and observing the sign of the coefficient of e_{LL}^2.

with $\rho_2 > \rho_1$ if and only if $\mu > \mu_0$ (with $\mu_0 < \frac{1}{2}$). Hence, it must be that $\theta_H < \min\{\rho_1, \rho_2\}$. Moreover, $e_{HL}^{SBM} < e_{LL}^{SBM}$ holds for

$$\gamma < \frac{(1-\mu)(1-(1-\nu)(1-\mu))\Delta\theta}{\mu(\Delta\theta + (1-\mu)(1-\nu))} = \overline{\gamma}^{SBM},$$

with $\overline{\gamma}^{SBM} < 1$ being always the case for $\mu \geq \mu_0$.

Recall that condition (9) must be satisfied and this amounts to $e_{LL}^{SBM} + e_{HH}^{SBM} \leq \frac{2\gamma}{\Delta\theta}$ which is equivalent to

$$\gamma \geq \frac{(\theta_H - 1)(2\nu(1-\mu)(1-\nu) + (\nu-\mu)(\theta_H - 1))}{2\nu(1-\nu)(1-\mu) + (\theta_H - 1)\nu(2-\mu(\theta_H + 1)) - (\theta_H - 1)(1-\nu)(1-(1-\nu)(1-\mu))} = \gamma_1^{SBM},$$

where $\gamma_1^{SBM} < \underline{\gamma}^{SBM}$ if and only if $\theta_H < \rho_1$, which must be the case. Finally, note that the chain of inequalities $\gamma_1^{SBM} < \gamma^* < \underline{\gamma}^{SBM} < \overline{\gamma}^{SBM} < \gamma_0$ holds provided that the denominator of e_{LL}^{SBM} is positive (which is our starting requirement), that is provided that $\theta_H < \rho_1$.

Result 1 ***Full participation and full separation when motivation prevails.*** *A solution to the principal's program, which entails full participation and full separation of types, which satisfies the monotonicity condition* $e_{LH}^{SBM} > e_{HH}^{SBM} > e_{LL}^{SBM} > e_{HL}^{SBM} > 0$, *and which is such that effort levels are given by expressions from (26) to (29), exists if and only if* $\theta_H < \min\{\rho_1, \rho_2\}$ *and* $\underline{\gamma}^{SBM} < \gamma < \overline{\gamma}^{SBM}$ *with*

$$\begin{aligned}
\underline{\gamma}^{SBM} &\equiv \frac{(\mu(1-\nu)+\nu(1-\mu))\Delta\theta}{(\nu\mu\Delta\theta+(1-\nu)(1-(1-\nu)(1-\mu)))} \\
\overline{\gamma}^{SBM} &\equiv \frac{(1-\mu)(1-(1-\nu)(1-\mu))\Delta\theta}{\mu(\theta_H-(1-(1-\nu)(1-\mu)))} \\
\rho_1 &\equiv \frac{(1-(1-\nu)(1-\mu))}{\mu} \\
\rho_2 &\equiv \frac{((\mu+\nu-3\mu\nu)+(1-\nu)(1-(1-\nu)(1-\mu)))}{(\mu+\nu-3\mu\nu)}
\end{aligned}.$$

Interestingly, both $\gamma^* < \underline{\gamma}^{SBM}$ and $\min\{\rho_1, \rho_2\} < 2$ hold, so that the alignment of second-best effort levels with the ranking obtained in first-best under Condition (4) necessarily holds.

C.2 Pooling and exclusion

When the equilibrium with full participation and full separation of types is not viable, meaning that the conditions in Result 1 are not respected, the principal will have to resort to different optimal contracts involving pooling of types and eventually exclusion of some workers' types. In particular, notice that the range of existence of a fully separating and fully participating solution is characterized by a lower bound $\underline{\gamma}^{SBM}$, which comes from the condition $e_{HH}^{SBM} > e_{LL}^{SBM}$, and by an upper bound $\overline{\gamma}^{SBM}$, which corresponds to $e_{LL}^{SBM} > e_{HL}^{SBM}$. Therefore, if $\gamma \leq \underline{\gamma}^{SBM}$, the principal is forced to offer the same contract to both types HH and LL, whereas if $\gamma \geq \overline{\gamma}^{SBM}$, we expect a pooling equilibrium where types HL and LL receive the same contract. We refer the reader to Appendix D.4.2 for the detailed analysis of the first situation, while we consider the second one in what follows.

Suppose that there's pooling between non motivated types and that $e_{LL} = e_{HL} = e_{\overline{p}}$. Then the ordering of effort levels is $e_{LH} > e_{HH} > e_{LL} = e_{HL} = e_{\overline{p}}$ and the relevant downward incentive constraints that one expects to be binding are IC_{LHvsHH} and IC_{HHvsLL} (or IC_{HHvsHL}, which is equivalent) together with participation constraint PC_{HL}. Since here worker types LL and HL receive the same wage and provide the same effort, $u_{LL} > u_{HL}$ necessarily holds. The wages are

$$w_{LL} = w_{HL} = w_{\overline{p}} = \frac{1}{2}\theta_H e_{\overline{p}}^2, \tag{31}$$

$$w_{HH} = \frac{1}{2}\theta_H e_{HH}^2 - \gamma e_{HH} + \underbrace{\gamma e_{\overline{p}}}_{\text{Info rent worker } HH}$$

and

$$w_{LH} = \frac{1}{2}e_{LH}^2 - \gamma e_{LH} + \underbrace{\frac{1}{2}\Delta\theta e_{HH}^2 + \gamma e_{\overline{p}}}_{\text{Info-rent worker } LH}.$$

Substituting the wage functions into the objective function of the principal and maximizing yields

$$e_{LH}^{SBM} = e_{LH}^{FB} = 1 + \gamma,$$

$$e_{HH}^{SBM} = \frac{(1-\nu)(1+\gamma)}{(\theta_H - \nu)}$$

and

$$e_{LL} = e_{HL} = e_{\overline{p}}^{SBM} = \frac{(1-\mu) - \mu\gamma}{(1-\mu)\theta_H} = e_{HL}^{BM}$$

Note that the expressions for e_{LH} and e_{HH} are the same as in Case M, meaning that *no distortion at the top* is verified and that the effort of individual HH is lower than the corresponding first-best level. Moreover, $e_{LH} > e_{HH}$ still holds. Concerning $e_{\overline{p}}^{SBM}$, it is the same as in Benchmark BM, it is strictly positive for $\gamma < \gamma^{BM}$ and it is such that $e_{HH} > e_{\overline{p}}^{SBM}$ holds if and only if

$$\gamma > \frac{\nu(1-\mu)\Delta\theta}{\theta_H(1-\nu) + \mu\nu\Delta\theta} = \gamma_{\overline{p}}$$

where $\gamma_{\overline{p}} < \underline{\gamma}^{SBM}$ always holds. Therefore, a solution characterized by full participation and pooling between types LL and HL always exists when $\gamma_{\overline{p}} < \gamma < \gamma^{BM}$. Observe that the conditions of existence of an equilibrium with full participation and pooling of workers HL and LL are less stringent than the ones we obtained in Result 1 because the requirement $e_{HL}^{SBM} < e_{LL}^{SBM}$ is no longer relevant. Also note that the pooled effort $e_{\overline{p}}^{SBM}$ is always in-between expressions (28) and (29), in particular $e_{HL}^{SBM} > e_{\overline{p}}^{SBM} > e_{LL}^{SBM}$ holds if and only if $\gamma > \overline{\gamma}^{SBM}$.

Result 2 (i) ***Full participation and Pooling between types HH and LL when motivation prevails.*** *A solution to the principal's program, which entails full participation and pooling between types HH and LL and IC_{LLvsHL} binding, which satisfies the monotonicity condition $e_{LH}^{SBM} > e_{\underline{p}}^{SBM} > e_{HL}^{SBM} > 0$, and which is such that effort levels are given by expressions (26), (29) and*

$$e_{LL} = e_{HH} \equiv e_{\underline{p}}^{SBM} = \frac{\nu(1-\mu) + \mu(1-\nu) - \mu\nu\gamma}{\nu(1-\mu) + \mu(1-\nu)},$$

is chosen if and only if $\theta_H < \rho_1$ *and* $\gamma^* \leq \gamma \leq \underline{\gamma}^{SBM}$.

(ii) ***Full participation and Pooling between types LL and HL when motivation prevails.*** *A solution to the principal's program, which entails full participation and pooling between types LL and HL, which satisfies the monotonicity condition* $e_{LH}^{SBM} > e_{HH}^{SBM} > e_{\overline{p}}^{SBM} > 0$, *and which is such that effort levels are given by expressions (26), (27) and*

$$e_{LL} = e_{HL} \equiv e_{\overline{p}}^{SBM} = \frac{(1-\mu) - \mu\gamma}{(1-\mu)\,\theta_H},$$

is chosen if and only if $\theta_H < \rho_1$ *and* $\overline{\gamma}^{SBM} \leq \gamma \leq \min\left\{\gamma^{BM}, 1\right\}$.

Notice that $\gamma^{BM} \geq 1$ if and only if $\mu \leq \frac{1}{2}$, therefore the principal always proposes a pooling contract to types LL and HL when motivation is sufficiently high (i.e. for $\gamma \geq \overline{\gamma}^{SBM}$) and the probability of being motivated is sufficiently low (i.e. for $\mu \leq \frac{1}{2}$). Conversely, when $\mu > \frac{1}{2}$ and $\gamma^{BM} < 1$, then for $\gamma \geq \gamma^{BM}$ the principal will exclude type HL and fully separate the remaining types, since the probability of motivated types is high and the productivity loss from type HL is low.

As for exclusion, the necessary and sufficient condition for full participation requires in general that, for any type ij, the expected profit from employing type ij be higher than the expected information rents that have to be paid her; this condition is satisfied as long as type ij's effort is strictly positive. However, the condition $e_{ij} > 0$ might call for some restrictions on the parameter space, as in the Benchmark case BM (see footnote 13).

Corollary 2 ***Exclusion of type HL when motivation prevails.*** *A solution to the principal's program, which entails separation and exclusion of type HL, which satisfies the monotonicity condition* $e_{LH}^{SBM} > e_{HH}^{SBM} > e_{LL}^{SBM} > e_{HL} = 0$ *and which is such that effort levels are given by expressions from (26) to (28), is chosen if and only if* $\mu > \frac{1}{2}$, $\theta_H < \rho_1$ *and* $\gamma^{BM} < \gamma \leq 1$.

Note that, to derive the conditions for existence and to characterize the equilibrium with exclusion of type HL, we proceed as in the case with full participation, but we obviously drop worker HL from the principal's maximization program and we omit the constraint $e_{HL}^{SBM} < e_{LL}^{SBM}$. Since the upper bound $\overline{\gamma}^{SBM}$ of the existence range for an equilibrium with full participation comes precisely from the condition $e_{HL}^{SBM} < e_{LL}^{SBM}$, the range for the existence of a separating equilibrium with exclusion of HL is broader on the right side with respect to the interval $(\underline{\gamma}^{SBM}, \overline{\gamma}^{SBM})$. In particular, a solution with separation and exclusion of type HL exists for $\underline{\gamma}^{SBM} < \gamma < \gamma_0$ and $\theta_H < \rho_1$. Moreover, the optimal effort levels of the remaining types are given by the same expressions from (26) to (28), even with exclusion. Instead, the optimal wages of the remaining types will be lower than expressions from (12) to (14), since the portions of the three information rents that depend on e_{HL}^{SBM} disappear.

C.3 Proof of Proposition 3

We want to show that the solution entailing full participation and full separation of types dominates both full separation but exclusion of at least worker HL and full participation but pooling of two workers' type. Moreover, we prove that full participation and pooling of two different types dominates full separation and exclusion of (at least) worker HL, whenever the two solutions coexist. We consider the situation in which motivation prevails over ability (Case M). The same line of reasoning applies to Case A as well, which is therefore omitted.

Start with the comparison between full participation and full separation of types and exclusion of at least worker HL. The first solution dominates the second if and only if it guarantees higher profits to the principal. As in Benchmark BA in Section 2.1.2, we must compare the costs and benefits from participation of the worst worker type HL. The principal's benefit from employing worker HL is the expected profit

$$(1-\mu)(1-\nu)(e_{HL}-w_{HL}), \tag{32}$$

whereas the cost from participation of HL is represented by the information rents paid to the three remaining workers' types, which add up to

$$\frac{1}{2}\left(1-(1-\mu)(1-\nu)\right)\Delta\theta e_{HL}^{2} \tag{33}$$

Thus, the principal prefers full participation to exclusion of type HL if and only if (32) is strictly greater than (33). Taking into account expression (11) for the wage w_{HL} and expression (29) for e_{HL} in Case M, such inequality reduces to $2e_{HL}^{SBM} > e_{HL}^{SBM}$, which is obviously satisfied as long as $e_{HL}^{SBM} > 0$. Similar conclusions can be drawn considering exclusion of both workers HL and LL.

Consider now the comparison between full separation and full participation of types and full participation but pooling of workers HH and LL. Now the comparison between costs and benefits from full separation becomes more complicated, so let us resort directly to the comparison between the principal's profits under the two solutions. The principal's payoffs under full separation and full participation of types are

$$\pi_{FS,FP}^{SBM} = \frac{1}{2}\left(\nu\mu\left(1+\gamma\right)^{2}+\mu\frac{(1-\nu)^{2}(1+\gamma)^{2}}{\theta_{H}-\nu}+\frac{(\nu(1-\mu)-\mu\gamma)^{2}}{(1-(1-\nu)(1-\mu))-\mu\theta_{H}}+\frac{(1-\nu)^{2}(1-\mu)^{2}}{\theta_{H}-(1-(1-\nu)(1-\mu))}\right)$$

whereas, under full participation but pooling of workers HH and LL, they amount to

$$\pi_{FP,HH=LL}^{SBM} = \frac{1}{2}\left(\nu\mu\left(1+\gamma\right)^{2}+\frac{(\nu(1-\mu)+\mu(1-\nu)-\gamma\mu\nu)^{2}}{\nu(1-\mu)+\mu(1-\nu)}+\frac{(1-\nu)^{2}(1-\mu)^{2}}{\theta_{H}-(1-(1-\nu)(1-\mu))}\right)$$

It is immediate to check that $\pi_{FS,FP}^{SBM} > \pi_{FP,HH=LL}^{SBM}$ always holds.

Consider now the comparison between full separation and full participation of types and full participation but pooling of workers HL and LL. The principal's payoffs under full participation but pooling of workers HL and LL amount to

$$\pi^{SBM}_{FP,HL=LL} = \tfrac{1}{2}\left(\nu\mu\left(1+\gamma\right)^2 + \mu\tfrac{(1-\nu)^2(1+\gamma)^2}{\theta_H-\nu} + \tfrac{((1-\mu)-\mu\gamma)^2}{\theta_H(1-\mu)}\right)$$

and, again, it is straightforward to check that $\pi^{SBM}_{FS,FP} > \pi^{SBM}_{FP,HL=LL}$ always holds.

Finally, consider the comparison between full participation but pooling of workers HL and LL and full separation but exclusion of worker HL. Note that these two equilibria only coexist for $\overline{\gamma}^{SBM} < \gamma < \gamma_0$. The principal's profits at the latter solution are

$$\pi^{SBM}_{FS,HL=0} = \tfrac{1}{2}\left(\nu\mu\left(1+\gamma\right)^2 + \tfrac{\mu(1-\nu)^2(1+\gamma)^2}{(\theta_H-\nu)} + \tfrac{(\nu(1-\mu)-\mu\gamma)^2}{\nu(1-\mu)-\mu(\theta_H-1)}\right)$$

and $\pi^{SBM}_{FP,HL=LL} > \pi^{SBM}_{FS,HL=0}$ if and only if

$$\left((1-\mu)-\mu\gamma\right)e^{SBM}_{\overline{p}} > \left(\nu\left(1-\mu\right)-\mu\gamma\right)e^{SBM}_{LL}.$$

The above inequality is always verified since, above $\overline{\gamma}^{SBM}$, one always has $e^{SBM}_{\overline{p}} > e^{SBM}_{LL}$.

Note that the comparison between full participation but pooling of workers HH and LL and full separation but exclusion of worker HL is meaningless because, below $\underline{\gamma}^{SBM}$, it is never feasible to separate types HH and LL. So we are done.

D Ability prevails (Case A)

D.1 Case $A.1$

D.1.1 Full separation and full participation

When ability prevails, condition (10) holds and $e_{LL} > e_{HH}$ together with $e_{LL} + e_{HH} \geq \frac{2\gamma}{\Delta\theta}$ must be satisfied. Suppose that IC_{LLvsHH} and IC_{HHvsHL} are binding, together with constraints PC_{HL} and IC_{LHvsLL}. Note that IC_{LLvsHH} rather than IC_{LLvsHL} binds if and only if $e_{HL} + e_{HH} \geq \frac{2\gamma}{\Delta\theta}$ holds, which in turn implies $e_{LL} + e_{HH} \geq \frac{2\gamma}{\Delta\theta}$. Substituting the wage schedules given by expressions from (16) to (19) into the objective function and deriving with respect to effort levels we obtain

$$e^{SBA1}_{LH} = 1+\gamma \tag{34}$$

$$e^{SBA1}_{LL} = \frac{(1-\mu)-\mu\gamma}{(1-\mu)} = e^{BM}_{LL}, \tag{35}$$

$$e^{SBA1}_{HH} = \frac{(1-\nu)\,\mu + (1-(1-\nu)\,(1-\mu))\,\gamma}{(1-(1-\nu)\,(1-\mu))\,\theta_H - \nu} \tag{36}$$

and

$$e^{SBA1}_{HL} = \frac{(1-\nu)\,(1-\mu) - (1-(1-\nu)\,(1-\mu))\,\gamma}{(1-\nu)\,(1-\mu)\,\theta_H}. \tag{37}$$

Observe that e_{LH}^{SBA1} and e_{HH}^{SBA1} are strictly positive, while $e_{LL}^{SBA1} > 0$ if and only if $\gamma < \gamma^{BM}$, and $e_{HL}^{SBA1} > 0$ if and only if

$$\gamma < \frac{(1-\nu)(1-\mu)}{(1-(1-\nu)(1-\mu))} = \gamma_1^{SBA1}.$$

Actually, $e_{LL}^{SBA1} > 0$ always holds when $\mu \leq \frac{1}{2}$ or when e_{HL}^{SBA1} is strictly positive, since $e_{HL}^{SBA1} > 0$ implies $e_{LL}^{SBA1} > 0$ (being $\gamma^{BM} > \gamma_1^{SBA1}$).

As for the monotonicity conditions, it can easily be checked that $e_{LH}^{SBA1} > e_{LL}^{SBA1}$ always holds, that $e_{LL}^{SBA1} > e_{HH}^{SBA1}$ is true for

$$\gamma < \frac{(1-\mu)(1-(1-\nu)(1-\mu))\Delta\theta}{\mu(1-(1-\nu)(1-\mu))\Delta\theta+(\nu(1-\mu)+\mu(1-\nu))} = \gamma_2^{SBA1}$$

and that inequalities $e_{LH}^{SBA1} > e_{HH}^{SBA1}$, $e_{LL}^{SBA1} > e_{HL}^{SBA1}$ and $e_{HH}^{SBA1} + e_{LL}^{SBA1} > \frac{2\gamma}{\Delta\theta}$ all hold when $\gamma < \gamma_2^{SBA1}$. Note that $\gamma_2^{SBA1} < \gamma_1^{SBA1}$ if and only if

$$\theta_H < \frac{\mu(1-\nu(1-\nu))+\nu(1-\mu)}{\nu(1-(1-\nu)(1-\mu))} \equiv \rho_6.$$

Finally, $e_{HH}^{SBA1} > e_{HL}^{SBA1}$ for

$$\gamma > \frac{\nu(1-\nu)(1-\mu)\Delta\theta}{(1-(1-\nu)(1-\mu))(\theta_H-\nu)} = \underline{\gamma}^{SBA1}$$

where it is always the case that $\underline{\gamma}^{SBA1} < \min\left\{\gamma_1^{SBA1}, \gamma_2^{SBA1}\right\}$.

In addition, it must be true that $e_{HH}^{SBA1} > \frac{2\gamma}{\Delta\theta}$ (such condition ensures not only that IC_{LLvsHH} binds while IC_{LLvsHL} does not bind but also that the information rent obtained by type LL when mimicking type HH is positive), which is equivalent to

$$\gamma < \frac{\mu(1-\nu)\Delta\theta}{\nu\Delta\theta+\mu(1-\nu)(\theta_H+1)} = \gamma_3^{SBA1},$$

where $\gamma_3^{SBA1} > \underline{\gamma}^{SBA1}$ holds if and only if $\mu > \frac{\nu}{1+\nu} = \mu_0$. Importantly, full participation and full separation in Case $A.1$ is possible only if $\mu > \mu_0$, or if the probability of motivated workers is sufficiently high, implying that information rents are not too costly. Thus, $\mu > \mu_0$ is a necessary condition ensuring that the two requirements $e_{HH}^{SBA1} > e_{HL}^{SBA1}$ and $e_{HH}^{SBA1} > \frac{2\gamma}{\Delta\theta}$ can both be met. Finally, $\gamma_3^{SBA1} < \gamma_1^{SBA1}$ if and only if

$$\theta_H < \frac{\mu(1+\nu)-\nu}{(2\mu-1)(1-(1-\mu)(1-\nu))} \equiv \rho_7,$$

which is always the case for $\mu \leq \frac{1}{2}$. Observe that $\gamma_3^{SBA1} < \gamma_2^{SBA1}$ if and only if $\mu < \frac{(1-2\nu)+\sqrt{1+4\nu(1-\nu)}}{4(1-\nu)} \equiv \mu_2$, with $\mu_2 > \frac{1}{2}$ and that $\rho_6 < \rho_7$ if and only if $\mu < \mu_2$.

We are then able to state the following Result.

Result 3 ***Full participation and full separation when ability prevails and IC_{LLvsHH} and IC_{HHvsHL} are binding.*** *A solution to the principal's program, which entails full participation, full separation* *of types and constraints IC_{LLvsHH} and IC_{HHvsHL} binding, which satisfies the monotonicity condition $e_{LH}^{SBA1} > e_{LL}^{SBA1} > e_{HH}^{SBA1} > e_{HL}^{SBA1} > 0$ and which is such that effort levels are given by expressions from (34) to (37), exists if and only if $\mu > \frac{\nu}{1+\nu} \equiv \mu_0$ and $\underline{\gamma}^{SBA1} < \gamma < \overline{\gamma}^{SBA1}$ with*

$$\begin{aligned} \underline{\gamma}^{SBA1} &\equiv \frac{\nu(1-\nu)(1-\mu)\Delta\theta}{(1-(1-\nu)(1-\mu))(\theta_H-\nu)} \\ \overline{\gamma}^{SBA1} &= \min\left\{\gamma_1^{SBA1}, \gamma_2^{SBA1}, \gamma_3^{SBA1}\right\} \end{aligned}$$

and

$$\begin{aligned} \gamma_1^{SBA1} &\equiv \frac{(1-\nu)(1-\mu)}{(1-(1-\nu)(1-\mu))} \\ \gamma_2^{SBA1} &\equiv \frac{(1-\mu)(1-(1-\nu)(1-\mu))\Delta\theta}{\mu(1-(1-\nu)(1-\mu))\Delta\theta+(\nu(1-\mu)+\mu(1-\nu))} \\ \gamma_3^{SBA1} &\equiv \frac{\mu(1-\nu)\Delta\theta}{(\nu\Delta\theta+\mu(1-\nu)(\theta_H+1))} \end{aligned} .$$

Finally note that $\gamma^* > \max\left\{\gamma_1^{SBA1}, \gamma_2^{SBA1}, \gamma_3^{SBA1}\right\}$ is always true, therefore Case $A.1$ with full participation and full separation is always a subset of the first-best state of the world in which condition (3) holds.

D.1.2 Pooling and exclusion

Consider now the instances in which the equilibrium with full participation and full separation of types is not viable.

First of all, observe that the lower bound $\underline{\gamma}^{SBA1}$ corresponds to condition $e_{HH}^{SBA1} > e_{HL}^{SBA1}$. Thus, if $\gamma \leq \underline{\gamma}^{SBA1}$, then we expect a pooling equilibrium where types HH and HL receive the same contract. Suppose that there's pooling between the less productive types and that $e_{HH} = e_{HL} = e_{\underline{p}}$ holds. Then the ordering of effort levels is $e_{LH} > e_{LL} > e_{\underline{p}} > 0$ and the relevant downward incentive constraints that one assumes to be binding are IC_{LHvsLL} and IC_{LLvsHL} (or IC_{LLvsHH}, which is equivalent) with participation constraint PC_{HL}. Since here the incentive constraints IC_{LLvsHH} and IC_{LLvsHL} are both binding by construction (meaning that $w_{LL} - \frac{1}{2}e_{LL}^2 = w_{HH} - \frac{1}{2}e_{HH}^2 = w_{HL} - \frac{1}{2}e_{HL}^2$), we do not need any condition on the sum of e_{HH} and e_{HL}. Moreover, since the two types of workers receive the same wage and provide the same effort, $u_{HH} > u_{HL}$ necessarily holds. The wages are

$$w_{HH} = w_{HL} = w_{\underline{p}} = \frac{1}{2}\theta_H e_{\underline{p}}^2,$$

$$w_{LL} = \frac{1}{2}e_{LL}^2 + \underbrace{\frac{1}{2}\Delta\theta e_{\underline{p}}^2}_{\text{Info rent worker } LL}$$

and

$$w_{LH} = \frac{1}{2}e_{LH}^2 - \gamma e_{LH} + \underbrace{\gamma e_{LL} + \frac{1}{2}\Delta\theta e_{\underline{p}}^2}_{\text{Info rent worker } LH} .$$

Substituting the wage functions into the objective function of the principal and maximizing with respect to effort levels yields

$$e_{LH}^{SBA1} = 1 + \gamma,$$

$$e_{LL}^{SBA1} = \frac{(1-\mu) - \mu\gamma}{(1-\mu)} = e_{LL}^{BM}$$

and

$$e_{HH} = e_{HL} = e_{\underline{p}}^{SBA1} = \frac{(1-\nu)}{(\theta_H - \nu)} = e_{HL}^{BA}$$

Note that the expressions for e_{LH} and e_{LL} are the same as in Case $A.1$ (and $A.2$) with full separation, meaning that *no distortion at the top* is verified and that the effort of individual LL is lower than the corresponding first-best level. Moreover, $e_{LH} > e_{LL}$ still holds. Concerning $e_{\underline{p}}^{SBA1}$, which is strictly positive, we expect that this effort lies in-between the effort exerted by types HH and HL in Case $A.1$ with full separation. One can easily check that $e_{HH}^{SBA1} < e_{\underline{p}}^{SBA1} < e_{HL}^{SBA1}$ if and only if $\gamma < \underline{\gamma}^{SBA1}$. Finally, $e_{LL}^{SBA1} > e_{\underline{p}}^{SBA1}$ if and only if

$$\gamma < \frac{(1-\mu)\,\Delta\theta}{\mu\,(\theta_H - \nu)} = \gamma_{\underline{p}}$$

where $\gamma_{\underline{p}} > \underline{\gamma}^{SBA1}$ always holds, so that a pooling equilibrium with $e_{HH} = e_{HL} = e_{\underline{p}}^{SBA1}$ always exists in Case $A.1$ for $\gamma \leq \underline{\gamma}^{SBA1}$.

Now consider the upper bounds (recall that condition $\gamma < \gamma_1^{SBA1}$ is equivalent to $e_{HL}^{SBA1} > 0$, that inequality $\gamma < \gamma_2^{SBA1}$ is equivalent to $e_{LL}^{SBA1} > e_{HH}^{SBA1}$ and finally that $\gamma < \gamma_3^{SBA1}$ ensures that requirement $e_{HH}^{SBA1} > \frac{2\gamma}{\Delta\theta}$ holds): if $\gamma \geq \overline{\gamma}^{SBA1}$, we expect an equilibrium in which either types HH and LL are pooled together or exclusion occurs or both.[32]

Result 4 (i) ***Full participation and pooling between types HH and HL when ability prevails.*** *A solution to the principal's program which entails full participation, pooling between types HH and HL, which satisfies the monotonicity condition* $e_{LH}^{SBA1} > e_{LL}^{SBA1} > e_{\underline{p}}^{SBA1} > 0$ *and which is such that effort levels are given by expressions (34), (35) and*

$$e_{HH} = e_{HL} \equiv e_{\underline{p}}^{SBA1} = \frac{(1-\nu)}{(\theta_H - \nu)},$$

is chosen if and only if $0 < \gamma \leq \underline{\gamma}^{SBA1}$.

(ii) ***Full participation and pooling between types HH and LL when ability prevails.*** *A solution to the principal's program which entails full participation, pooling between types HH and LL and* IC_{HHvsHL} *binding, which satisfies the monotonicity condition* $e_{LH}^{SBA1} > e_{\overline{p}}^{SBA1} > e_{HL}^{SBA1} > 0$ *and which is such that effort levels are given by expressions (34), (37) and*

$$e_{HH} = e_{LL} \equiv e_{\overline{p}}^{SBA1} = \frac{(\nu\,(1-\mu) + \mu\,(1-\nu))\,(1+\gamma)}{\nu\mu\Delta\theta + (\nu\,(1-\mu) + \mu\,(1-\nu))\,\theta_H},$$

is chosen only if $\overline{\gamma}^{SBA1} \neq \gamma_1^{SBA1}$ *and* $\overline{\gamma}^{SBA1} < \gamma < \min\left\{\overline{\gamma}^{SBPa}, \gamma_1^{SBA1}\right\}$ *with*

$$\overline{\gamma}^{SBPa} \equiv \frac{(\nu(1-\mu)+\mu(1-\nu))\Delta\theta}{2\nu\mu\Delta\theta+(\nu(1-\mu)+\mu(1-\nu))(\theta_H+1)}.$$

[32] We refer the reader to Appendix D.4.1 for the detailed analysis of this situation.

Note that when $\overline{\gamma}^{SBA1} = \gamma_1^{SBA1}$ and $\gamma_1^{SBA1} < \gamma < \min\left\{\gamma_2^{SBA1}, \gamma_3^{SBA1}\right\}$, the principal will necessarily exclude worker HL. This would lead us to consider alternative solutions where either full separation but exclusion of type HL (and where IC_{LLvsHH} and PC_{HH} are binding), or pooling of types HH and LL and exclusion of type HL, or else exclusion of both types HL and HH are implemented.[33]

D.2 Case $A.2$

D.2.1 Full separation and full participation

Suppose now that the incentive constraints IC_{HHvsHL} and IC_{LLvsHL} are binding, together with PC_{HL} and IC_{LHvsLL}, and that $e_{HL} + e_{HH} \leq \frac{2\gamma}{\Delta\theta}$ holds. In addition, IC_{HHvsLL} is satisfied if and only if $e_{HL} + e_{LL} \geq \frac{2\gamma}{\Delta\theta}$ so that Case $A.2$ is relevant when $e_{HL} + e_{HH} \leq \frac{2\gamma}{\Delta\theta} \leq e_{HL} + e_{LL}$. Substituting the wage functions listed in the main text into the principal's expected profits and deriving with respect to effort levels, we obtain

$$e_{LH}^{SBA2} = 1 + \gamma, \tag{38}$$

$$e_{LL}^{SBA2} = \frac{(1-\mu) - \gamma\mu}{(1-\mu)} = e_{LL}^{SBA1} = e_{LL}^{BM}, \tag{39}$$

$$e_{HH}^{SBA2} = \frac{1+\gamma}{\theta_H} = e_{HH}^{FB} \tag{40}$$

and

$$e_{HL}^{SBA2} = \frac{(1-\nu)\left((1-\mu) - \gamma\mu\right)}{\nu\Delta\theta + \theta_H (1-\mu)(1-\nu)}. \tag{41}$$

Observe that both $e_{HL}^{SBA2} > 0$ and $e_{LL}^{SBA2} > 0$ hold provided that $\gamma < \gamma^{BM}$, that $e_{LH}^{SBA2} > e_{LL}^{SBA2} > e_{HL}^{SBA2}$ and $e_{HH}^{SBA2} > e_{HL}^{SBA2}$ always hold while $e_{LL}^{SBA2} > e_{HH}^{SBA2}$ if and only if

$$\gamma < \frac{(1-\mu)\Delta\theta}{1+\mu\Delta\theta} = \gamma_1^{SBA2}.$$

It is immediate to check that the condition $\gamma < \gamma_1^{SBA2}$ implies both $e_{HL}^{SBA2} > 0$ and $e_{LL}^{SBA2} > 0$, being $\gamma_1^{SBA2} < \gamma^{BM}$, and also that $\gamma_1^{SBA2} < \gamma_2^{SBA1}$ always holds, being the requirement $e_{LL}^{SBA2} > e_{HH}^{SBA2} = e_{HH}^{FB}$ more restrictive than $e_{LL}^{SBA1} > e_{HH}^{SBA1}$, the corresponding requisite in Case $A.1$. Then, all monotonicity conditions are satisfied provided that $\gamma < \gamma_1^{SBA2}$. Moreover, it is easy to check that the condition $\gamma < \gamma_1^{SBA2}$ suffices for $e_{HH}^{SBA2} + e_{LL}^{SBA2} \geq \frac{2\gamma}{\Delta\theta}$.

There remains to check that incentive constraint IC_{LLvsHL} is binding rather than IC_{LLvsHH} and that IC_{HHvsHL} is binding rather than IC_{HHvsLL}, which amounts to $e_{HL} + e_{HH} \leq \frac{2\gamma}{\Delta\theta} \leq e_{HL} + e_{LL}$. As for $e_{HL}^{SBA2} + e_{LL}^{SBA2} \geq \frac{2\gamma}{\Delta\theta}$, it holds if and only if

$$\gamma \leq \frac{\Delta\theta(1-\mu)(\Delta\theta(1-\mu(1-\nu)) + 2(1-\mu)(1-\nu))}{2(1-\mu)^2(1-\nu) + \Delta\theta^2\mu(1-\mu(1-\nu)) + 2\Delta\theta(1-\mu)} = \gamma_2^{SBA2},$$

[33] In the region $\gamma \geq \overline{\gamma}^{SBA1}$, we do not provide the full characterization of the optimum (available upon request though) because several different cases might arise and the analysis becomes cumbersome without being very insightful.

conversely $e_{HH}^{SBA2} + e_{HL}^{SBA2} \leq \frac{2\gamma}{\Delta\theta}$ holds if and only if

$$\gamma \geq \frac{\Delta\theta(2\theta_H(1-\mu)(1-\nu)+\nu\Delta\theta)}{(\nu\Delta\theta+\theta_H(1-\mu)(1-\nu))(\theta_H+1)+\theta_H\Delta\theta(1-\nu)\mu} = \underline{\gamma}^{SBA2},$$

whereby a solution exists for $\underline{\gamma}^{SBA2} \leq \gamma < \min\left\{\gamma_1^{SBA2}, \gamma_2^{SBA2}\right\} \equiv \overline{\gamma}^{SBA2}$. Now, $\underline{\gamma}^{SBA2} < \gamma_2^{SBA2} < \gamma_1^{SBA2}$ is true if and only if $\mu < \frac{1}{2}$ and

$$\theta_H > \frac{(1-\mu(1+\nu))}{(1-2\mu)(1-\mu(1-\nu))} = \rho_3,$$

hence a separating equilibrium exists for $\mu < \frac{1}{2}$, $\theta_H > \rho_3$ and $\underline{\gamma}^{SBA2} \leq \gamma < \overline{\gamma}^{SBA2} = \gamma_2^{SBA2}$, while a solution with full separation and full participation under Case $A.2$ does not exist for $\mu \geq \frac{1}{2}$.

We are then able to state the following Result.

Result 5 ***Full participation and full separation when ability prevails and IC_{LLvsHL} and IC_{HHvsHL} are binding.*** *A solution to the principal's program, which entails full participation, full separation of types and IC_{LLvsHL} and IC_{HHvsHL} binding, which satisfies the monotonicity condition $e_{LH}^{SBA2} > e_{LL}^{SBA2} > e_{HH}^{SBA2} > e_{HL}^{SBA2} > 0$ and which is such that effort levels are given by expressions from (38) to (41), exists if and only if $\mu < \frac{1}{2}$, $\theta_H > \rho_3$ and $\underline{\gamma}^{SBA2} \leq \gamma < \overline{\gamma}^{SBA2}$, with*

$$\begin{aligned}
\underline{\gamma}^{SBA2} &\equiv \frac{\Delta\theta(2\theta_H(1-\mu)(1-\nu)+\nu\Delta\theta)}{(\nu\Delta\theta+\theta_H(1-\mu)(1-\nu))(\theta_H+1)+\theta_H\Delta\theta(1-\nu)\mu} \\
\overline{\gamma}^{SBA2} &\equiv \frac{\Delta\theta(1-\mu)(\Delta\theta(1-\mu(1-\nu))+2(1-\mu)(1-\nu))}{2(1-\mu)^2(1-\nu)+\Delta\theta^2\mu(1-\mu(1-\nu))+2\Delta\theta(1-\mu)} \\
\rho_3 &\equiv \frac{(1-\mu(1+\nu))}{(1-2\mu)(1-\mu(1-\nu))}
\end{aligned}.$$

Finally, observe that $\gamma_2^{SBA2} = \overline{\gamma}^{SBA2} < \gamma^*$ always holds, thus implying that this solution is attained when, at the first-best, condition (3) holds.

D.2.2 Pooling and exclusion

What happens when full participation and full separation is not viable? Below $\underline{\gamma}^{SBA2}$ one expects the principal to exclude the less efficient types, namely HL and possibly HH too, while above $\overline{\gamma}^{SBA2}$, one expects to have a pooling equilibrium where types LL and HH are given the same contract and, possibly, the worst type HL is excluded.[34]

Suppose then that the principal excludes type HL and offers him the null contract. The principal's program must be slightly modified with respect to full participation, the main differences being that monotonicity constraint $e_{HH}^{SBA2} > e_{HL}^{SBA2}$ is omitted and PC_{HH} (rather than PC_{HL}) is assumed to be binding. Moreover, the requirement that incentive constraint IC_{LLvsHL} rather than IC_{LLvsHH} be binding reduces to the need that PC_{LL} be binding and that $e_{HH}^{SBA2} \leq \frac{2\gamma}{\Delta\theta}$ holds, which is true if and only if

$$\gamma \geq \frac{\Delta\theta}{\theta_H + 1} = \underline{\underline{\gamma}}^{SBA2},$$

[34] We refer the reader to Appendix D.4.2 for the detailed analysis of the latter situation.

where $\underline{\underline{\gamma}}^{SBA2} < \underline{\gamma}^{SBA2}$ always holds when $\mu < \frac{1}{2}$. Furthermore, the requirement that incentive constraint IC_{HHvsHL} rather than IC_{HHvsLL} is binding reduces to the need that PC_{HH} binds and that $e_{LL}^{SBA2} \geq \frac{2\gamma}{\Delta\theta}$ holds, which is true for

$$\gamma \leq \frac{(1-\mu)\,\Delta\theta}{2\,(1-\mu)+\mu\Delta\theta} = \overline{\overline{\gamma}}^{SBA2},$$

with $\underline{\underline{\gamma}}^{SBA2} < \min\left\{\overline{\overline{\gamma}}^{SBA2}, \underline{\gamma}^{SBA2}\right\}$. Hence a solution characterized by exclusion of type HL, separation of the remaining types and both PC_{LL} and PC_{HH} binding exists for $\underline{\underline{\gamma}}^{SBA2} \leq \gamma < \min\left\{\overline{\overline{\gamma}}^{SBA2}, \underline{\gamma}^{SBA2}\right\}$.

Result 6 *(i)* ***Separation and exclusion of (at least) type HL when ability prevails.*** *A solution to the principal's program, which entails full separation but exclusion of type HL, both PC_{HH} and PC_{LL} binding, which satisfies the monotonicity condition $e_{LH}^{SBA2} > e_{LL}^{SBA2} > e_{HH}^{SBA2} > e_{HL} = 0$ and which is such that effort levels are given by expressions from (38) to (40) is chosen when $\mu < \frac{1}{2}$ and $\underline{\underline{\gamma}}^{SBA2} \leq \gamma \leq \min\left\{\underline{\gamma}^{SBA2}, \overline{\overline{\gamma}}^{SBA2}\right\}$, where*

$$\begin{array}{lll} \underline{\underline{\gamma}}^{SBA2} & \equiv & \frac{\Delta\theta}{(\theta_H+1)} \\ \overline{\overline{\gamma}}^{SBA2} & \equiv & \frac{(1-\mu)\Delta\theta}{2(1-\mu)+\mu\Delta\theta} \end{array}.$$

The equilibrium characterized by exclusion of both types HL and HH is chosen either when $\gamma < \underline{\underline{\gamma}}^{SBA2}$ or when $\overline{\overline{\gamma}}^{SBA2} < \gamma < \underline{\gamma}^{SBA2}$.

(ii) ***Full participation and Pooling between HH and LL when ability prevails and IC_{LLvsHL} is binding.*** *An equilibrium with full participation and pooling between types LL and HH and IC_{LLvsHL} binding, with effort levels described by expressions (38), (41) and*

$$e_{LL} = e_{HH} \equiv e_{\overline{p}}^{SBA2} = \frac{(\nu\,(1-\mu)+\mu\,(1-\nu)) - \gamma\mu\nu}{(\nu\,(1-\mu)+\mu\,(1-\nu))} = e_{\underline{p}}^{SBM} \tag{42}$$

is chosen when $\gamma \geq \gamma^{SBPb}$, where

$$\gamma^{SBPb} \equiv \frac{(\nu(1-\mu)+\mu(1-\nu))\Delta\theta(\Delta\theta+2(1-\nu)(1-\mu))}{(\theta_H-(1-(1-\nu)(1-\mu)))(2(\nu(1-\mu)+\mu(1-\nu))+\mu\nu\Delta\theta)} > \overline{\gamma}^{SBA2}.$$

(iii) ***Pooling between HH and LL and exclusion of HL when ability prevails.*** *An equilibrium with pooling between types LL and HH, exclusion of type HL and PC_{LL} binding, with effort levels described by expressions (38) and (42) is chosen when $\overline{\gamma}^{SBA2} \leq \gamma < \gamma^{SBPb}$.*

Observe that Result 6 (*ii*) describes precisely the same pooling equilibrium obtained in Case M for motivation levels below the threshold $\underline{\gamma}^{SBM}$.

D.3 Case $A.3$

D.3.1 Full separation and full participation

Suppose that constraints IC_{LHvsLL}, IC_{HHvsLL}, IC_{LLvsHL} and PC_{HL} are all binding and that inequality $e_{HL} + e_{LL} \leq \frac{2\gamma}{\Delta\theta} \leq e_{HH} + e_{LL}$ holds. Substituting the wage functions listed in the main text into the principal's expected profits and deriving with respect to effort levels we obtain

$$e_{LH}^{SBA3} = 1 + \gamma, \tag{43}$$

$$e_{LL}^{SBA3} = \frac{\nu (1 - \mu) - \mu\gamma}{\nu (1 - \mu) - \mu (1 - \nu) \Delta\theta}, \tag{44}$$

$$e_{HH}^{SBA3} = \frac{1 + \gamma}{\theta_H} = e_{HH}^{SBA2} = e_{HH}^{FB} \tag{45}$$

and

$$e_{HL}^{SBA3} = \frac{(1 - \mu)(1 - \nu)}{\theta_H - (1 - (1 - \mu)(1 - \nu))} = e_{HL}^{SBM}. \tag{46}$$

Observe that all effort levels are always strictly positive, except for e_{LL}^{SBA3}. In order for e_{LL}^{SBA3} to be a maximum of the principal's expected profits, it is necessary to impose that both the numerator and the denominator of its expression be positive: the numerator of e_{LL}^{SBA3} is positive for $\gamma < \gamma_0$ (see expression 30) and the denominator of e_{LL}^{SBA3} is positive when

$$\theta_H < \frac{(\mu (1 - \nu) + \nu (1 - \mu))}{\mu (1 - \nu)} = \rho_4.$$

Note that $\rho_4 > 2$ if and only if $\mu < \nu$, thus under Assumption 2 the requirement $\theta_H < \rho_4$ is always satisfied when $\mu < \nu$.

As for the monotonicity conditions, it must be that $e_{LL}^{SBA3} > e_{HH}^{SBA3}$, which holds if and only if

$$\gamma < \frac{(\mu (1 - \nu) + \nu (1 - \mu)) \Delta\theta}{\mu\nu\theta_H + (\mu (1 - \nu) + \nu (1 - \mu))} = \overline{\gamma}^{SBA3}$$

where $\overline{\gamma}^{SBA3} < \gamma^*$ and $\overline{\gamma}^{SBA3} < \gamma_0$ are always true. Moreover, $e_{HH}^{SBA3} > e_{HL}^{SBA3}$ always holds and $e_{LL}^{SBA3} > e_{HL}^{SBA3}$ is always satisfied when $e_{LL}^{SBA3} > e_{HH}^{SBA3}$ is (namely when $\gamma < \overline{\gamma}^{SBA3}$). Notice that e_{LL}^{SBA3} is distorted downwards if and only if

$$\gamma > (1 - \nu) \Delta\theta = \gamma_1^{SBA3}$$

where $\gamma_1^{SBA3} < \overline{\gamma}^{SBA3}$. Hence if motivation is not too high, Case $A.3$ could be compatible with an upward distortion in the effort of the productive but non-motivated worker LL.

Consider now the additional constraints $e_{LL} + e_{HL} \leq \frac{2\gamma}{\Delta\theta} \leq e_{LL} + e_{HH}$. As for $\frac{2\gamma}{\Delta\theta} \leq e_{LL} + e_{HH}$, it is easy to check that it is always satisfied provided that $\gamma < \overline{\gamma}^{SBA3}$, while $e_{LL} + e_{HL} \leq \frac{2\gamma}{\Delta\theta}$ holds if and only if

$$\gamma \geq \frac{\Delta\theta(1-\mu)\left(2\nu(1-\nu)(1-\mu)+\left(\nu-\mu(1-\nu)^2\right)\Delta\theta\right)}{(\theta_H-(1-(1-\mu)(1-\nu)))(2\nu(1-\mu)-\mu(1-2\nu)\Delta\theta)} = \underline{\gamma}^{SBA3}$$

where $\underline{\gamma}^{SBA3} > \gamma_1^{SBA3}$ (implying that e_{LL}^{SBA3} is always distorted downwards when full participation and full separation is possible) and $\underline{\gamma}^{SBA3} < \overline{\gamma}^{SBA3}$ when

$$\theta_H > \frac{(\mu (1 - \nu) + \nu (1 - \mu) - \nu\mu ((1 - (1 - \mu)(1 - \nu))))}{(\mu (1 - \nu) + \nu (1 - \mu) - \nu\mu ((1 + (1 - \mu)(1 - \nu))))} = \rho_5,$$

with $\rho_5 < \rho_4$ if and only if

$$\mu < \frac{\left(4\nu-\nu^2-1\right)-\sqrt{((4\nu-\nu^2-1))^2-4\nu(3\nu-2)(1-\nu)}}{2(3\nu-2)(1-\nu)} = \mu_1 > \frac{1}{2}$$

(for $\nu \neq \frac{2}{3}$ or for $\mu < \frac{\mu}{4\nu-\nu^2-1}$ if $\nu = \frac{2}{3}$).

Result 7 ***Full participation and full separation when ability prevails and*** IC_{HHvsLL} ***and*** IC_{LLvsHL} ***are binding.*** *A solution to the principal's program, which entails full participation, full separation of types and* IC_{HHvsLL} *and* IC_{LLvsHL} *binding, which satisfies the monotonicity condition* $e_{LH}^{SBA3} > e_{LL}^{SBA3} > e_{HH}^{SBA3} > e_{HL}^{SBA3} > 0$ *and which is such that effort levels are given by expressions from (43) to (46), exists if and only if* $\mu < \mu_1$, $\rho_5 < \theta_H < \rho_4$ *and* $\underline{\gamma}^{SBA3} \leq \gamma < \overline{\gamma}^{SBA3}$, *with*

$$\begin{aligned}
\underline{\gamma}^{SBA3} &\equiv \frac{\Delta\theta(1-\mu)\left(2\nu(1-\nu)(1-\mu)+\left(\nu-\mu(1-\nu)^2\right)\Delta\theta\right)}{(\theta_H-(1-(1-\mu)(1-\nu)))(2\nu(1-\mu)-\mu(1-2\nu)\Delta\theta)} \\
\overline{\gamma}^{SBA3} &\equiv \frac{\Delta\theta(\mu(1-\nu)+\nu(1-\mu))}{\mu\nu\theta_H+(\mu(1-\nu)+\nu(1-\mu))} \\
\mu_1 &\equiv \frac{\left(4\nu-\nu^2-1\right)-\sqrt{((4\nu-\nu^2-1))^2-4\nu(3\nu-2)(1-\nu)}}{2(3\nu-2)(1-\nu)} > \frac{1}{2} \\
\rho_4 &\equiv \frac{(\mu(1-\nu)+\nu(1-\mu))}{\mu(1-\nu)} \\
\rho_5 &\equiv \frac{(\mu(1-\nu)+\nu(1-\mu)-\nu\mu((1-(1-\mu)(1-\nu))))}{(\mu(1-\nu)+\nu(1-\mu)-\nu\mu((1+(1-\mu)(1-\nu))))}
\end{aligned}.$$

Since $\overline{\gamma}^{SBA3} < \gamma^*$ always holds, this solution is attained when condition (3) holds at the first-best.

D.3.2 Pooling and exclusion

What happens when full participation and full separation is not viable? Above $\overline{\gamma}^{SBA3}$, one expects to have a pooling equilibrium where types LL and HH are given the same contract. And also below $\underline{\gamma}^{SBA3}$ one still finds that this solution is relevant.

Result 8 ***Full participation and Pooling between HH and LL when productivity prevails and*** IC_{LLvsHL} ***is binding.*** *An equilibrium with full participation and pooling between types* LL *and* HH *and* IC_{LLvsHL} *binding, with effort levels described by expressions (43), (46)*[35] *and (42) is chosen when* $\overline{\gamma}^{SBA3} \leq \gamma \leq \gamma^*$ *and when* $\gamma^{SBPb} \leq \gamma \leq \underline{\gamma}^{SBA3}$.

Below $\underline{\gamma}^{SBA3}$ one also finds pooling between types HH and LL and exclusion of type HL and (possibly) a solution with separation but exclusion of type HL. Interestingly, in the latter case, it is possible to have an *upward distortion* of the effort required to type LL, but not so important as to allow for a pooling equilibrium where types LH and LL are given the same contract.

Suppose that type HL is left out. In this circumstance, the optimal levels of effort are the same as under full participation, except for $e_{HL} = 0$, and all relevant constraints are satisfied whenever the chain of inequalities $e_{LL} \leq \frac{2\gamma}{\Delta\theta} \leq e_{LL} + e_{HH}$ holds.

[35] Note that (46) is the same as (29).

Now, $\frac{2\gamma}{\Delta\theta} \leq e_{LL}^{SBA3} + e_{HH}^{SBA3}$ is always satisfied when $\gamma < \overline{\gamma}^{SBA3}$, whereas $e_{LL}^{SBA3} \leq \frac{2\gamma}{\Delta\theta}$ is true if and only if

$$\gamma \geq \frac{\nu(1-\mu)\Delta\theta}{(2\nu(1-\mu) - \mu\Delta\theta(1-2\nu))} = \underline{\underline{\gamma}}^{SBA3}$$

where $\underline{\underline{\gamma}}^{SBA3} < \underline{\gamma}^{SBA3}$ always holds and where $\underline{\underline{\gamma}}^{SBA3} > \gamma_1^{SBA3}$ if and only if $\nu > \frac{1}{2}$. Hence a solution with exclusion of type HL under Case $A.3$ exists for $\underline{\underline{\gamma}}^{SBA3} \leq \gamma < \overline{\gamma}^{SBA3}$ and $\theta_H < \rho_4$. Observe that, when $\nu < \frac{1}{2}$ and $\underline{\underline{\gamma}}^{SBA3} \leq \gamma < \gamma_1^{SBA3}$, the solution entails an upward distortion in the level of effort provided by type LL.

Result 9 *(i)* ***Pooling between HH and LL and exclusion of type HL when ability prevails and PC_{LL} is binding.*** *An equilibrium with pooling between types LL and HH and exclusion of type HL, with PC_{LL} binding, with effort levels described by expressions (43) and (42), is chosen when $\underline{\gamma}^{SBPb} < \gamma < \min\left\{\underline{\underline{\gamma}}^{SBA3}, \gamma^{SBPb}\right\}$, where*

$$\begin{array}{rcl} \underline{\underline{\gamma}}^{SBA3} & \equiv & \frac{\nu(1-\mu)\Delta\theta}{(2\nu(1-\mu)-\mu\Delta\theta(1-2\nu))} \\ \underline{\gamma}^{SBPb} & \equiv & \frac{\Delta\theta(\nu(1-\mu)+(1-\nu)\mu)}{(\nu\mu\Delta\theta+2(\nu(1-\mu)+(1-\nu)\mu))} \end{array}.$$

(ii) ***Separation and exclusion of type HL when ability prevails and IC_{HHvsLL} and PC_{LL} are binding.*** *An equilibrium with exclusion of type HL and IC_{HHvsLL} and PC_{LL} binding, with effort levels described by expressions from (43) to (45) is chosen only if $\underline{\underline{\gamma}}^{SBA3} < \gamma^{SBPb}$ and $\underline{\underline{\gamma}}^{SBA3} \leq \gamma < \gamma^{SBPb}$.*

Observe that Result 9 describes precisely the same pooling equilibrium obtained in Case M and Case $A.2$.

D.3.3 Proof of Proposition 4

Considering the comparison between total effort exerted in Case $A.3$ and Case M, it is immediate to check that the following chain of inequalities holds

$$e_{LH}^{SBA3} = e_{LH}^{SBM} = e_{LH}^{FB} > e_{LL}^{SBA3} > e_{HH}^{SBA3} = e_{HH}^{FB} > e_{HH}^{SBM} > e_{LL}^{SBM} > e_{HL}^{SBA3} = e_{HL}^{SBM}.$$

As for the comparison between Case $A.3$ and Case $A.2$, we have that $e_{LH}^{SBA3} = e_{LH}^{SBA2} = e_{LH}^{FB}$ and $e_{HH}^{SBA3} = e_{HH}^{SBA2} = e_{HH}^{FB}$ hence a sufficient condition for Case $A.3$ to Pareto dominate Case $A.2$ in terms of effort provision is that both $e_{LL}^{SBA3} > e_{LL}^{SBA2}$ and $e_{HL}^{SBA3} > e_{HL}^{SBA2}$ hold. Now, $e_{LL}^{SBA3} > e_{LL}^{SBA2}$ is true if and only if

$$\gamma < \frac{\Delta\theta(1-\mu)}{(\mu\Delta\theta + (1-\mu))} = \gamma_{LL}$$

while $e_{HL}^{SBA3} > e_{HL}^{SBA2}$ is true if and only if

$$\gamma > \frac{\Delta\theta(1-\mu)(1-\nu)}{\Delta\theta} = \gamma_{HL}$$

where $\gamma_{HL} < \underline{\gamma}^{SBA2} < \overline{\gamma}^{SBA3} < \gamma_{LL} < \gamma^*$ always holds. Hence, when both Case $A.3$ and Case $A.2$ are relevant, the sufficient conditions are met.

Finally, considering Case $A.3$ and Case $A.1$, we have that $e_{LH}^{SBA3} = e_{LH}^{SBA1} = e_{LH}^{FB}$ and $e_{HH}^{SBA3} = e_{HH}^{FB} > e_{HH}^{SBA1}$, hence a sufficient condition for Case $A.3$ to Pareto dominate Case $A.1$ is that both $e_{LL}^{SBA3} > e_{LL}^{SBA1}$ and $e_{HL}^{SBA3} > e_{HL}^{SBA1}$ hold. Now, $e_{LL}^{SBA3} > e_{LL}^{SBA1}$ is true if and only if $\gamma < \gamma_{LL}$ while $e_{HL}^{SBA3} > e_{HL}^{SBA1}$ is true if and only if $\gamma > \gamma_{HL}$ where $\gamma_{HL} < \gamma_1^{SBA1} < \overline{\gamma}^{SBA3} < \gamma_{LL} < \gamma^*$ always holds. Hence, when both Case $A.3$ and Case $A.1$ are relevant, the sufficient conditions are still met.

Concerning distributional issues, notice that information rents of non-motivated agents are the same in both Case M and Case $A.3$, being $u_{LL}^{SBA3} = u_{LL}^{SBM}$ and $u_{HL}^{SBA3} = u_{HL}^{SBM}$, whereas information rents of productive and motivated workers are higher in Case $A.3$, being $u_{LH}^{SBA3} > u_{LH}^{SBM}$. Hence, independently of which of the mutually exclusive cases realizes, the above-mentioned workers are always weakly better-off in Case $A.3$ than in Case M. As for motivated, low-productive types HH, the ranking between u_{HH}^{SBA3} and u_{HH}^{SBM} depends on whether Case M or Case $A.3$ attains: in particular, $u_{HH}^{SBM} > u_{HH}^{SBA3}$ holds when Case $A.3$ is relevant. Since the surplus is larger in Case $A.3$ but $u_{LH}^{SBA3} > u_{LH}^{SBM}$ holds, it is not possible to conclude whether the principal is better-off in Case $A.3$ or Case M.

D.4 Pooling between intermediate types HH and LL

Suppose that the principal offers a single contract to both agents LL and HH. Then one has $e_{LL} = e_{HH} = e_p$ and $w_{LL} = w_{HH} = w_p$. The relevant constraints are

$$w_{LH} - \frac{1}{2}e_{LH}^2 + \gamma e_{LH} \geq w_p - \frac{1}{2}e_p^2 + \gamma e_p$$

for type LH,

$$w_p - \frac{1}{2}e_p^2 \geq w_{HL} - \frac{1}{2}e_{HL}^2 \tag{47}$$

for type LL or

$$w_p - \frac{1}{2}\theta_H e_p^2 + \gamma e_p \geq w_{HL} - \frac{1}{2}\theta_H e_{HL}^2 + \gamma e_{HL} \tag{48}$$

for type HH. Finally, for type HL

$$w_{HL} - \frac{1}{2}\theta_H e_{HL}^2 \geq 0.$$

The binding participation constraint is the one of type HL above, while all other participation constraints are satisfied provided that PC_{HL} is. The monotonicity condition

$$e_{LH} \geq e_p \geq e_{HL}$$

holds; but which incentive compatibility constraint between 47 (that is IC_{LLvsHL}) and 48 (or else IC_{HHvsHL}) binds first? Taking into account the binding participation constraint of type HL, it must be that

$$w_p \geq \max\left\{\frac{1}{2}\theta_H e_p^2 - \gamma e_p + \gamma e_{HL}; \frac{1}{2}e_p^2 + \frac{1}{2}\Delta\theta e_{HL}^2\right\}.$$

Thus, 48 or IC_{HHvsHL} is binding first when

$$\frac{1}{2}\theta_H e_p^2 - \gamma e_p + \gamma e_{HL} \geq \frac{1}{2}e_p^2 + \frac{1}{2}\Delta\theta e_{HL}^2 \Longleftrightarrow e_p + e_{HL} \geq \frac{2\gamma}{\Delta\theta},$$

whereas 47 or IC_{LLvsHL} is binding when

$$\frac{1}{2}\theta_H e_p^2 - \gamma e_p + \gamma e_{HL} \leq \frac{1}{2}e_p^2 + \frac{1}{2}\Delta\theta e_{HL}^2 \Longleftrightarrow e_p + e_{HL} \leq \frac{2\gamma}{\Delta\theta}$$

In the following we study the two subcases separately.

D.4.1 Pooling between intermediate types with IC_{HHvsHL} binding

Suppose that when pooling occurs, IC_{HHvsHL} is binding while IC_{LLvsHL} is not. We call this situation Case $P(a)$. Then one has $e_p + e_{HL} \geq \frac{2\gamma}{\Delta\theta}$. Wages must satisfy

$$w_{HL} = \frac{1}{2}\theta_H e_{HL}^2, \tag{49}$$

$$w_p = \frac{1}{2}\theta_H e_p^2 - \gamma e_p + \underbrace{\gamma e_{HL}}_{\text{Info rent worker } HH}. \tag{50}$$

and

$$w_{LH} = \frac{1}{2}e_{LH}^2 - \gamma e_{LH} + \underbrace{\frac{1}{2}\Delta\theta e_p^2 + \gamma e_{HL}}_{\text{Info rent worker } LH}. \tag{51}$$

Note that wage w_p has the same expression as w_{HH} in Cases $A.1$ and $A.2$ (see equation 17). This is not surprising since IC_{HHvsHL} is binding in all Cases $A.1$, $A.2$ and $P(a)$. Thus, as in Benchmark BM, the information rent of type HH depends on γ. Since IC_{HHvsHL} is binding whereas IC_{LLvsHL} is not, we expect that the information rent of worker LL is higher than the one of worker HH. The information rent of worker LL is equivalent to $\frac{1}{2}\Delta\theta e_p^2 - \gamma e_p + \gamma e_{HL}$, where $\frac{1}{2}\Delta\theta e_p^2 - \gamma e_p > 0$ for $e_p > \frac{2\gamma}{\Delta\theta}$. This requirement is more stringent than $e_p + e_{HL} \geq \frac{2\gamma}{\Delta\theta}$ and it must be imposed ex-post, as was done in Case $A.1$.

Substituting again the wage schedules into the principal's program we find

$$e_{LH}^{SBPa} = 1 + \gamma, \tag{52}$$

$$e_p^{SBPa} \equiv e_p^{SBA1} = \frac{(\nu(1-\mu) + \mu(1-\nu))(1+\gamma)}{\nu\mu\Delta\theta + (\nu(1-\mu) + (1-\nu)\mu)\theta_H} \tag{53}$$

and

$$e_{HL}^{SBPa} = \frac{(1-\nu)(1-\mu) - \gamma(1-(1-\nu)(1-\mu))}{(1-\nu)(1-\mu)\theta_H}. \tag{54}$$

Note that $e_{LH}^{SBPa} > e_{p}^{SBPa}$ and $e_{LH}^{SBPa} > e_{HL}^{SBPa}$ always hold. Moreover e_{HL}^{SBPa} is the same as e_{HL}^{SBA1} since in both cases participation constraint of worker HL is binding. Also observe that e_{HL}^{SBPa} is strictly positive if and only if $\gamma < \gamma_1^{SBA1}$, and $e_p^{SBPa} > e_{HL}^{SBPa}$ if and only if

$$\gamma > \frac{\nu\mu(1-\nu)(1-\mu)\Delta\theta}{\nu\mu(1-(1-\nu)(1-\mu))\Delta\theta+\theta_H(\mu(1-\nu)+\nu(1-\mu))} = \underline{\gamma}^{SBPa} ,$$

where $\underline{\gamma}^{SBPa} < \underline{\gamma}^{SBA1}$ always holds. Moreover, $e_{LL}^{SBA1} < e_{p}^{SBPa} < e_{HH}^{SBA1}$ if and only if $\gamma > \gamma_2^{SBA1}$ and the condition $e_p > \frac{2\gamma}{\Delta\theta}$ holds if and only if

$$\gamma < \frac{(\nu(1-\mu)+\mu(1-\nu))\Delta\theta}{2\nu\mu\Delta\theta+(\nu(1-\mu)+\mu(1-\nu))(\theta_H+1)} = \overline{\gamma}^{SBPa}$$

where $\overline{\gamma}^{SBPa} > \underline{\gamma}^{SBPa}$ is always true, $\overline{\gamma}^{SBPa} < \gamma_1^{SBA1}$ if and only if

$$\theta_H < \frac{(\nu(1-\mu)(1-\mu(1-\nu))+\mu(1-\nu)(1-\nu(1-\mu)))}{\left((2\nu-1)(\nu(1-\mu)+\mu(1-\nu))+2(1-\nu)^2\mu^2\right)} = \rho_8$$

(always for $\nu < \frac{1}{2}$ and $\mu < \frac{(1-2\nu)^2+\sqrt{(1-2\nu)(1+2\nu-4\nu^2)}}{4(1-\nu)^2} \equiv \mu_3 < \frac{1}{2}$), where $\rho_6 < \rho_8 < \rho_7$ if and only if $\mu < \mu_2$, and $\gamma_3^{SBA1} < \overline{\gamma}^{SBPa} < \gamma_2^{SBA1}$ if and only if $\mu < \mu_2$.

Thus, an equilibrium with full participation and pooling between types LL and HH and IC_{HHvsHL} binding exists if and only if $\underline{\gamma}^{SBPa} < \gamma < \min\left\{\gamma_1^{SBA1}, \overline{\gamma}^{SBPa}\right\}$. Instead, notice that an equilibrium with pooling between types LL and HH and exclusion of type HL and such that PC_{HH} is binding exists for $\gamma < \overline{\gamma}^{SBPa}$.

D.4.2 Pooling between intermediate types with IC_{LLvsHL} binding

Suppose now that when pooling occurs, IC_{LLvsHL} is binding while IC_{HHvsHL} is not. We call this situation Case $P(b)$, in which $e_p + e_{HL} \leq \frac{2\gamma}{\Delta\theta}$. Wages must satisfy

$$w_{HL} = \frac{1}{2}\theta_H e_{HL}^2, \tag{55}$$

$$w_p = \frac{1}{2}e_p^2 \quad + \underbrace{\frac{1}{2}\Delta\theta e_{HL}^2}_{\text{Info rent worker } LL} \tag{56}$$

and

$$w_{LH} = \frac{1}{2}e_{LH}^2 - \gamma e_{LH} + \underbrace{\gamma e_p + \frac{1}{2}\Delta\theta e_{HL}^2}_{\text{Info rent worker } LH}. \tag{57}$$

Note that wage w_p has the same expression as w_{LL} in Case $A.2$ (see equation 20) and in Case M. This is not surprising since IC_{LLvsHL} is binding in all the mentioned cases. Thus, as in Benchmark BA, the information rent of worker LL depends on $\Delta\theta$. Moreover, in the expression for w_{LH}, the information rent of worker LH has the same expression as in Case $A.2$ (with the term γe_p being equivalent to γe_{LL}). Since IC_{LLvsHL} is binding whereas IC_{HHvsHL} is not, the information rent of worker HH is higher than the one of worker LL and is equivalent to $\frac{1}{2}\Delta\theta e_{HL}^{2}-\frac{1}{2}\Delta\theta e_{p}^{2}+\gamma e_{p}$, where $-\frac{1}{2}\Delta\theta e_{p}^{2}+\gamma e_{p}>0$ for $e_{p}<\frac{2\gamma}{\Delta\theta}$. The latter inequality always holds given that, in this case, it must be $e_{p}+e_{HL}\leq\frac{2\gamma}{\Delta\theta}$.

Substituting the wage schedules into the program and deriving yields

$$e_{LH}^{SBPb}=1+\gamma, \tag{58}$$

$$e_{p}^{SBPb}\equiv e_{\underline{p}}^{SBM}=e_{\overline{p}}^{SBA2}=\frac{(\nu(1-\mu)+\mu(1-\nu))-\gamma\mu\nu}{(\nu(1-\mu)+\mu(1-\nu))} \tag{59}$$

and

$$e_{HL}^{SBPb}=\frac{(1-\nu)(1-\mu)}{\theta_{H}-(1-(1-\nu)(1-\mu))}, \tag{60}$$

where e_{HL}^{SBPb} is equal to e_{HL}^{SBM} and e_{HL}^{SBA3} since in all cases the incentive constraint IC_{LLvsHL} is binding. Note that $e_{p}^{SBPb}>0$ if and only if

$$\gamma<\frac{\nu(1-\mu)+\mu(1-\nu)}{\mu\nu}=\overline{\gamma}^{SBPb},$$

which is always the case for $\mu<\frac{\nu}{3\nu-1}$, and which is such that $\overline{\gamma}^{SBPb}>\gamma^{*}$ if and only if $\theta_{H}<\rho_{1}$ and such that $\overline{\gamma}^{SBPb}>\underline{\gamma}^{SBM}$ and $\overline{\gamma}^{SBPb}>\overline{\gamma}^{SBA2}$ always hold. Furthermore, observe that $e_{LH}^{SBPb}>e_{p}^{SBPb}$ and $e_{LH}^{SBPb}>e_{HL}^{SBPb}$ always hold, while $e_{p}^{SBPb}>e_{HL}^{SBPb}$ holds whenever $e_{p}^{SBPb}>0$ is true. Finally, the condition $e_{p}^{SBPb}+e_{HL}^{SBPb}\leq\frac{2\gamma}{\Delta\theta}$ holds if and only if

$$\gamma\geq\frac{(\nu(1-\mu)+\mu(1-\nu))\Delta\theta(\Delta\theta+2(1-\nu)(1-\mu))}{(\theta_{H}-(1-(1-\nu)(1-\mu)))(2(\nu(1-\mu)+\mu(1-\nu))+\mu\nu\Delta\theta)}=\gamma^{SBPb}$$

where $\gamma^{SBPb}<\min\left\{\gamma^{*},\overline{\gamma}^{SBPb}\right\}$ is always true and where $\overline{\gamma}^{SBA2}<\gamma^{SBPb}$ and $\underline{\gamma}^{SBA3}<\gamma^{SBPb}<\overline{\gamma}^{SBA3}$ are also true. Thus, an equilibrium with full participation and pooling between types LL and HH and IC_{LLvsHL} binding exists if and only if $\gamma^{SBPb}\leq\gamma<\overline{\gamma}^{SBPb}$.

Concerning exclusion of the worst type, we need to consider a similar program where, instead of having IC_{LLvsHL} binding while IC_{HHvsHL} not binding, we need PC_{LL} to be binding and PC_{HH} not binding. In this case, the requirement $e_{p}^{SBPb}+e_{HL}\leq\frac{2\gamma}{\Delta\theta}$ reduces to the more general condition $e_{p}^{SBPb}\leq\frac{2\gamma}{\Delta\theta}$, which is satisfied if and only if

$$\gamma\geq\frac{\Delta\theta(\nu(1-\mu)+(1-\nu)\mu)}{(\nu\mu\Delta\theta+2(\nu(1-\mu)+(1-\nu)\mu))}=\underline{\gamma}^{SBPb}$$

where $\underline{\gamma}^{SBPb}<\gamma^{SBPb}$, and $\underline{\gamma}^{SBPb}$ is smaller than $\overline{\gamma}^{SBA2}$ provided that $\theta_{H}\leq 2$, namely provided that Assumption 2 holds.

E Example

Let $\gamma_{L}=0$ and $\gamma_{H}=\gamma\in(0,1]$ and let $\theta_{L}=1$ and $\theta_{H}\in(1,2]$. Assume that motivation and skills are uniformly distributed across workers, so that $\mu=\nu=\frac{1}{2}$. Case M is attained for $1<\theta_{H}<\frac{3}{2}$, Case $A.2$

does not exist, while Case $A.3$ holds for $\frac{5}{3} < \theta_H < 2$. Hence one can have three classes of problems: (i) productivity variation is as low as $1 < \theta_H < \frac{3}{2}$, and either motivation prevails and Case M is attained or productivity prevails and Case $A.1$ holds; (ii) productivity variation is as high as $\frac{5}{3} < \theta_H \leq 2$, productivity always prevails and either Case $A.1$ or Case $A.3$ hold depending on the value taken by γ; (iii) productivity variation is intermediate $\frac{3}{2} < \theta_H < \frac{5}{3}$, productivity prevails and only Case $A.1$ holds.

In situation (i), productivity uncertainty is low and such that $1 < \theta_H < \frac{3}{2}$ and one observes the following solutions: when $0 < \gamma \leq \frac{\Delta\theta}{3(2\theta_H - 1)} = \underline{\gamma}^{SBA1}$ the principal offers a pooling contract to low-skilled types HH and HL, when $\underline{\gamma}^{SBA1} < \gamma < \overline{\gamma}^{SBA1} = \gamma_3^{SBA1} = \frac{\Delta\theta}{3\theta_H - 1}$ full participation and full separation under Case $A.1$ is implemented, when $\overline{\gamma}^{SBA1} \leq \gamma < \overline{\gamma}^{SBPa} = \frac{\Delta\theta}{2\theta_H}$ the principal offers a pooling contract to intermediate types HH and LL, which is such that IC_{HHvsHL} is binding, when $\overline{\gamma}^{SBPa} \leq \gamma < \underline{\gamma}^{SBPb} = \frac{2\Delta\theta}{\theta_H + 3}$ there is exclusion of both types HH and HL, when $\underline{\gamma}^{SBPb} < \gamma < \gamma^{SBPb} = \frac{4(2\theta_H - 1)\Delta\theta}{(4\theta_H - 3)(\theta_H + 3)}$ there is pooling between intermediate types HH and LL with the constraint IC_{LLvsHL} binding and exclusion of type HL. Note that $\gamma^{SBPb} < \gamma^*$ so that we still are in the domain in which ability prevails and $e_{LL} > e_{HH}$. When $\gamma^{SBPb} \leq \gamma \leq \underline{\gamma}^{SBM} = \frac{4\Delta\theta}{2\theta_H + 1}$ we have pooling between intermediate types HH and LL with the constraint IC_{LLvsHL} binding but full participation is attained, and we cross γ^* so that motivation prevails and $e_{HH} > e_{LL}$. When $\underline{\gamma}^{SBM} < \gamma < \frac{3\Delta\theta}{4\theta_H - 3} = \overline{\gamma}^{SBM} < \frac{1}{2}$, full separation and full participation is attained under Case M. When $\overline{\gamma}^{SBM} \leq \gamma < 1$ the principal offers a pooling contract to non-motivated types LL and HL.

In situation (ii), productivity uncertainty is high and such that $\frac{5}{3} < \theta_H < 2$, then one observes the following: when $0 < \gamma < \gamma^{SBPb}$ there are the same equilibria as in (i), when $\gamma^{SBPb} \leq \gamma < \underline{\gamma}^{SBA3} = \frac{(3\theta_H - 1)\Delta\theta}{2(4\theta_H - 3)}$ we have pooling between intermediate types HH and LL with the constraint IC_{LLvsHL} binding and full participation, when $\underline{\gamma}^{SBA3} < \gamma < \overline{\gamma}^{SBA3} = \frac{2\Delta\theta}{\theta_H + 2}$ there is full participation and full separation under Case $A.3$, when $\overline{\gamma}^{SBA3} \leq \gamma \leq 1$, we have pooling between intermediate types HH and LL with the constraint IC_{LLvsHL} binding and full participation.

In situation (iii), productivity uncertainty is intermediate since $\frac{3}{2} \leq \theta_H \leq \frac{5}{3}$, and one observes the following solutions: when $0 < \gamma < \gamma^{SBPb}$ there are the same equilibria as in (i) and (ii), when $\gamma^{SBPb} \leq \gamma < 1$ we have pooling between intermediate types HH and LL with the constraint IC_{LLvsHL} binding and full participation.

References

[1] Andreoni J. (1990), "Impure Altruism and Donations to Public Goods: a Theory of Warm-Glow Giving", *Econometrica*, 64, 51-75.

[2] Armstrong M. (1996), "Multiproduct Nonlinear Pricing", *Econometrica*, 64, 51-75.

[3] Armstrong M. and J-J. Rochet (1999), "Multi-dimensional Screening: A User's Guide", *European*

Economic Review, 43, 959-979.

[4] Armstrong M. (1999), “Optimal Regulation with Unknown Demand and Cost Functions”, *Journal of Economic Theory* 84, 196-215.

[5] Bae S-H (2012), "Nursing Overtime: Why, How Much, and Under What Working Conditions?, *Nursing Economics* 30(2), 60-71.

[6] Barigozzi F. and G. Turati (2012), “Human Health Care and Selection Effects. Understanding Labour Supply in the Market for Nursing”, *Health Economics* 21(4), 477-483.

[7] Barigozzi F. and D. Raggi (2013), “The Lemons Problem in a Vocation-Based Labor Market: When Higher Salaries Pay Worse Workers”, *mimeo*, University of Bologna.

[8] Basov S. (2003), *Multidimensional Screening*, Springer-Verlag.

[9] Basov S. (2001), “Hamiltonian Approach to Multidimensional Screening” *Journal of Mathematical Economics*, 36, 77-94

[10] Besley T. and M. Ghatak (2005), “Competition and Incentives with Motivated Agents”, *American Economic Review* 95(3), 616-636.

[11] Brehm J. and S. Gates (1997), *Working, Shirking, and Sabotage: Bureaucratic Response to a Democratic Public*, Michigan Studies in Political Analysis, Ann. Arbor: University of Michigan Press.

[12] Delfgaauw J. and R. Dur (2007), “Signaling and Screening of Workers’ Motivation”, *Journal of Economic Behavior and Organization* 62, 605-624.

[13] Delfgaauw J. and R. Dur (2008), “Incentives and Workers’ Motivation in the Public Sector”, *The Economic Journal* 118, 171-191.

[14] Delfgaauw J. and R. Dur (2009), “From Public Monopsony to Competitive Market: More Efficiency but Higher Prices”, *Oxford Economic Papers* 61, 586-602.

[15] Delfgaauw J. and R. Dur (2010), “Managerial Talent, Motivation, and Self-selection into Public Management”, *Journal of Public Economics* 94(9-10), 654-660.

[16] Deneckere R. and S. Severinov (2011), “Multi-Dimensional Screening: A Solution to a Class of Problems", *mimeo,* econ.ucsb.edu.

[17] Francois P. (2000), “Public Service Motivation as an Argument for Government Provision”, *Journal of Public Economics* 78(3), 275-299.

[18] Francois P. (2003), “Not-for-profit Provision of Public Services”, *Economic Journal* 113(486), C53-C61.

[19] Handy F. and E. Katz (1998), "The Wage Differential Between Nonprofit Institutions and Corporations: Getting More by Paying Less?", *Journal of Comparative Economics* 26, 246-261.

[20] Heyes A. (2005) "The Economics of Vocation *or* -Why is a Badly Paid Nurse a Good Nurse-?", *Journal of Health Economics* 24, 561-569.

[21] Hwang H., W.R. Reed and C. Hubbard (1992), "Compensating Wage Differentials and Unobserved Productivity", *Journal of Political Economy* 100(4), 835-858.

[22] Laffont J-J., E. Maskin and J.-C. Rochet (1987), "Optimal Non-Linear Pricing with Two-dimensional Characteristics" in T. Groves, R. Radner and S. Reiter, eds., *Information, Incentives and Economic Mechanisms*, University of Minnesota Press, Minneapolis, 256-266.

[23] Murdock K. (2002), "Intrinsic Motivation and Optimal Incentive Contracts", *RAND Journal of Economics,* 33(1), 650-671.

[24] Prendergast C. (2007), "The Motivation and Bias of Bureaucrats", *American Economic Review* 97(1), 180-196.

[25] Prendergast C. (2008), "Intrinsic Motivation and Incentives", *American Economic Review: Papers & Proceedings* 98(2), 201-205.

[26] Rochet J.-C. and P. Chonè (1998), "Ironing, Sweeping, and Multidimensional Screening" *Econometrica*, 66, 783-826.

[27] Rosen, S. (1986), "The Theory of Equalizing Differences" in O. Ashenfelter, R. Layard eds., *Handbook of Labor Economics I*, North Holland, 246-263.

[28] Van den Steen (2006), "Too Motivated?", Sloan School of Management Working Paper Series, No. 4547–05.

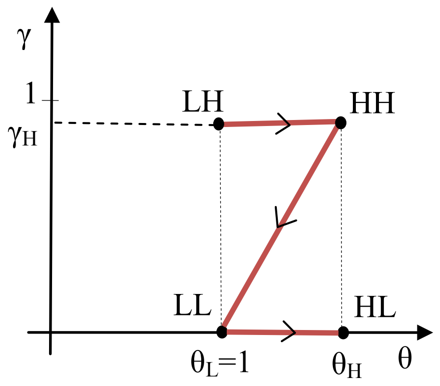

Figure 1. Case M ($e_{HH} > e_{LL}$): $2\gamma/\Delta\theta > e_{HH} + e_{LL}$.

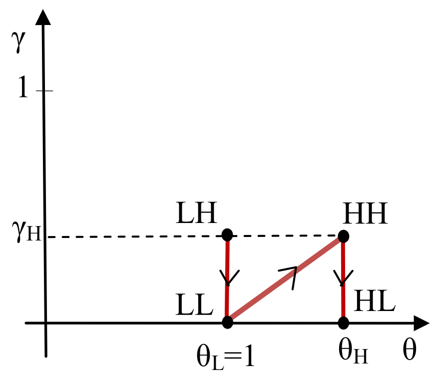

Figure 2a. Case A.1 ($e_{HH} < e_{LL}$): $2\gamma/\Delta\theta < e_{HH} + e_{HL}$.

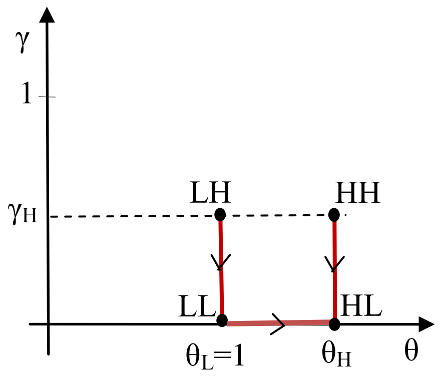

Figure 2b. Case A.2 ($e_{HH} < e_{LL}$): $e_{HH} + e_{HL} < 2\gamma/\Delta\theta < e_{LL} + e_{HL}$.

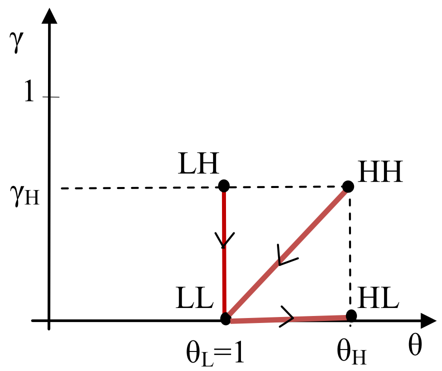

Figure 2c. Case A.3 ($e_{HH} < e_{LL}$): $e_{LL} + e_{HL} < 2\gamma/\Delta\theta < e_{LL} + e_{HH}$

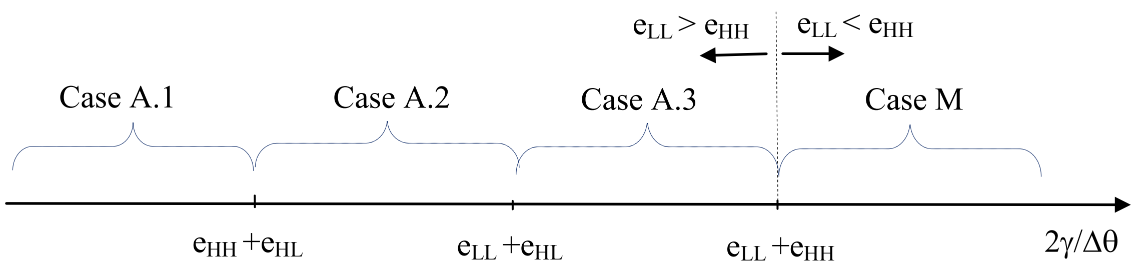

Figure 3. Existing classes of equilibria as a function of $2\gamma/\Delta\theta$.

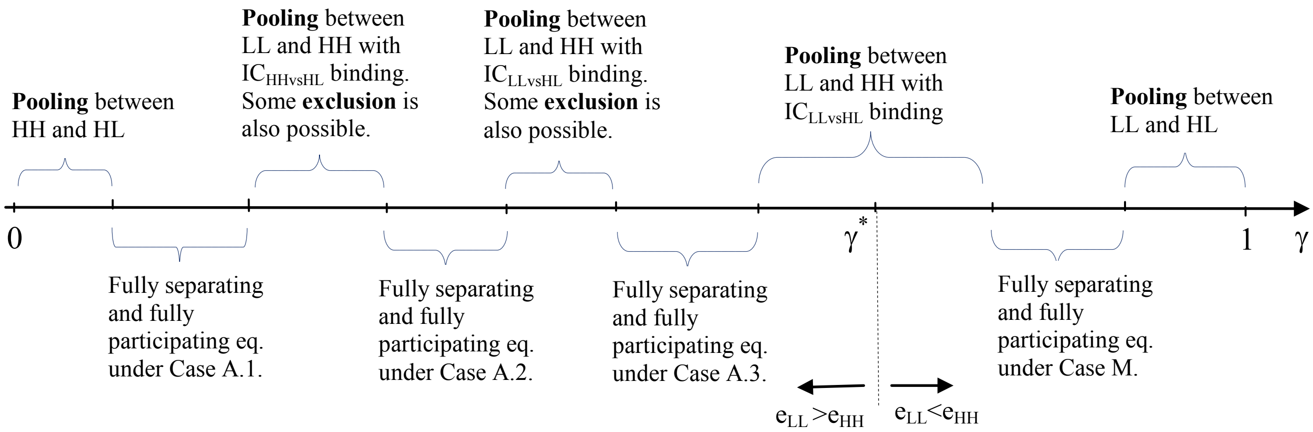

Figure 4. Equilibria with pooling and/or exclusion of some workers' types as a function of γ.