R&D competition, product differentiation and enforcement against IPR violation

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Abstract

This paper analyses public enforcement policies against IPR violation in an environment where there is innovation competition between two firms with asymmetric R&D efficiencies that is captured by a contest function framework. The IPR violation is in the form of users who may choose to make illegal copies for personal consumption. The equilibrium enforcement policy induces the patent-winner to choose a price that allows buying and copying. Increases in R&D efficiency increases the equilibrium enforcement but has ambiguous effects on the incentive to innovate. Lowering of product differentiation has non-monotonic effect on the equilibrium enforcement but unambiguously reduces the incentive to innovate, which cannot be reversed by the positive equilibrium enforcement.

Keywords: Contest Function, Enforcement, Product Differentiation, R&D Efficiency

JEL Classification: D21, D43, L13, L21, L26.

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1. Introduction

There is a vast body of literature that studies the impact of competitive pressure on firms' incentive to innovate when there are multiple firms engaged in R&D competition.[1] The same research question also applies when completion arises due to violation of Intellectual Property Rights (IPR). However, in this case, the added question that arises is regarding the role of enforcement authorities in protecting IPR and its efficacy in preserving the innovators' incentive to innovate.

The literature addressing this issue considers a *single* innovator and uses product quality, in general, as a measure for the incentive to innovate. Using this framework various authors have shown an array of policy implications to counter copyright infringement in the context of digital products like software.

For example, Lahiri and De (2013), and Jaisingh (2009) show that stricter copyright enforcement policy may initially lower the incentive to innovate before increasing it. Yao (2005) shows that high levels of enforcement policy are warranted only for high levels of infringement. Bae and Choi (2006) consider limit pricing and copying regimes and show that violation of IPR lowers the product quality and in the copying regime tighter copyright protection policies reduce welfare. In Waldman and Novos (1984), an increase in copyright infringement causes the innovating firm to expend more on the protection of the product resulting in the product quality choices to be below the socially optimal level. Similarly, Qiu (2006) shows that weak copyright protection policy results in the development of customized rather than general software products.

However, this literature does not consider R&D competition among innovators. The relevance of considering R&D competition is the fact that product market competition can have ambiguous effects on the incentive to innovate when there is R&D competition (Vives, 2008).[2] Thus the question is whether R&D competition among innovating firms is sufficient to preserve their incentive to innovate in the face of competition arising from violation of IPR or

[1] See Vives (2008) for a detailed literature review.
[2] Even Schumpeter and Arrow (1962) obtain conflicting results with regard to the effect of competitive pressure on innovation.

enforcement policies against such violation required? If so can such policies generate the appropriate incentive?

The focus of this paper is to address these issues, which is yet to be addressed in the literature.[3] For the purpose, we consider R&D competition with asymmetric R&D efficiencies between two firms using a contest function approach. The patent winner faces IPR violation by end-users, that is, the users can either buy the product or make illegal copies for personal consumption. Further the copied product is an inferior substitute of the legitimate one whose quality is normalised to one. The difference between the two qualities measures the degree of product differentiation. Thus there is vertical product differentiation. This is different from the literature studying the impact of product market competition on the incentive to innovate when there is R&D competition, which in general uses a horizontal product differentiation approach (Morita et. al., 2015). In our paper competition is measured by the degree of product differentiation. The greater the product differentiation the lower is the competition. The enforcement policy is captured by the expected penalty faced by consumers violating IPR. Using this framework we analyse, how the R&D efficiency and the degree of product differentiation determines the optimal enforcement policy and its effectiveness in boosting the incentive to innovate as competition in the product market increases.

The social welfare maximizing enforcement policy induces the patent-winner to choose a price that allows both buying and copying in equilibrium. That is, the optimal enforcement policy cannot deter copying in equilibrium. This optimal policy is increasing in the R&D efficiency. Thus an increase in the R&D efficiency has a direct effect and an indirect effect via the enforcement on the equilibrium R&D investment. The direct effect is a negative one because the increase in R&D efficiency requires less resource to innovate the product whose quality is normalized to one. The indirect effect is a positive one because of the following reason. The higher enforcement due to a higher R&D efficiency lowers copying and some of the consumers switch from copying to buying. Thus the demand increases. The higher enforcement also empowers the patent-winner to charge a higher price. The higher price and demand increases the

[3] Banerjee and Chatterjee (2010) and Banerjee (2013) also considers competition among innovating firms who faces illegal competition from a firm selling unlicensed copies of copyrighted products. However, these papers only address the private incentives for profitable innovation but do not consider public enforcement polices to counter such non-compliance of IPR laws.

profit, which increases the equilibrium R&D investment. The negative direct effect and the positive indirect effect lead to an ambiguous overall result.

A fall in product differentiation that intensifies the product market competition has non-monotonic effect on the optimal enforcement. Specifically, as product differentiation decreases from high levels, the optimal enforcement initially increases and beyond some critical level it decreases. However, for the entire range, a fall in product differentiation unambiguously reduces the incentive to innovate. That is, even a decrease in product differentiation in the range where the optimal enforcement is increasing it fails to prevent the fall in equilibrium R&D investment. This can be explained as follows.

A decrease in product differentiation has direct negative effect on the profit because higher competition reduces the price. Though the demand increases but the lower price has the dominant impact. This negative impact on profit in turn reduces the equilibrium R&D investment. The decrease in product differentiation has an indirect effect via its impact on the optimal enforcement. Consider the range where a decrease in product differentiation increases the optimal enforcement. The higher enforcement has an indirect positive impact on the profit for reasons explained previously, and hence, has a positive impact on the equilibrium R&D investment. However, this positive indirect effect is outweighed by the direct negative effect thereby causing the equilibrium R&D to decrease. In the range where a decrease in product differentiation decreases the optimal enforcement, both the direct and indirect effects on profit are negative. Thus the effect on R&D investment is also negative.

Our analysis shows that though in equilibrium there is always positive enforcement, which restricts IPR violation, but it fails to reverse the negative effect of competitive pressure on the incentive to innovate. However, the positive enforcement may restrict the lowering of R&D investment, which can be more drastic in the absence of any enforcement. This is true for the relatively high product differentiation scenario where a decrease in product differentiation increases the optimal enforcement. However, it is not true for the low product differentiation case where a decrease in product differentiation, which further adds to the already negative direct impact of low product differentiation on the incentive to innovate.

Thus our paper partially supports the repeated calls by organizations like the Business Software Alliance (BSA) for strengthening of copyright laws and effective public enforcement against copyright infringement to protect the interest of the software industry.[4] After all the information technology (IT) sector has been one of the dominant engines of growth in the 21st century contributing about $650 billion or 4.3% of the GDP in the U.S. in 2011.[5] However, the advent of this technology has also witnessed a growing infringement IPR in the form of illegal copying of digital products either for personal consumption or for retail purpose. For example, the BSA Global Software Survey (2013) mentions that *global* piracy rose from 42 percent in 2011 to 43 percent in 2013. The commercial value of unlicensed PC software installations totalled a staggering amount of $62.7 billion *globally* in 2013.[6] The most serious concern regarding such misappropriation of IPR is its impact on the incentive to innovate. Such infringement is not only restricted to the developing and the emerging economies but also quite prevalent in the OECD countries. A report by The Sydney Morning Herald (November 2, 2015) mentions that "Australia is one of the pirating capitals of the world…".[7]

This paper is arranged as follows. In Section 2 we provide the model and analyse the consumers' and firms' decisions. Section 3 contains the social welfare analysis and concluding remarks are given in Section 4.

2. The model

We consider a market for a copyrighted product like software that faces the threat of misappropriation of IPR in the form of end-user piracy. This means that some consumers can potentially make unlicensed copies of the licensed product for personal consumption. Consumers engaged in such illegal activities faces an expected fine (f), which constitutes the government's enforcement policy and is discussed later in this section.

The quality of the legitimate product is normalised to 1 and the copied product is an inferior substitute of the legitimate one, which is captured by the parameter $q \in (0,1)$.[8] It can be

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[4] BSA is the advocate for the global software industry. See www.bsa.org.

[5] Bureau of Economic Analysis, GDP-by-Industry (value added by industry; accessed December 12, 2012), http://www.bea.gov/iTable/index_industry.cfm.

[6] See www.bsa.org and http://globalstudy.bsa.org/2013/.

[7] See http://www.smh.com.au/digital-life/social-radar/what-now-for-piracy-in-australia-20151101-gkoet0.html.

[8] We set the bound $q \in (0,1)$ to ensure that the profits are not indeterminate.

interpreted as an exogenous index of the poor quality of the copied product. This can be viewed as the present discounted value of future service and updates that are available at a lower price and only come with the purchase of a legitimate product. The qualitative difference is intended to capture these aspects and is assumed to be common knowledge.[9] Higher values of q imply that the copied product is relatively closer to the legitimate one in terms of quality. This means that higher the value of q lower is the degree of product differentiation. So the degree of product differentiation is measured by $1-q$. We will use this measure for the rest of the analysis.

There are two firms involved in R&D competition to develop the product. The firms are asymmetric with respect to R&D efficiency. That is, firm 0 is relatively more efficient than firm 1 in the sense that the probability of success of firm 0 is higher than that of firm 1 when both firms invest the same amount. The outside option of each of these innovating firms is assumed to be 0. The patent winning firm becomes the monopolist.

We consider a sequential game of the following form.

Stage 1: Government chooses an enforcement policy f. The enforcement cost is $\frac{cf^2}{2}$ where $c>0$ measures the enforcement efficiency. The lower the value of c, the greater is the enforcement efficiency.

Stage 2: Firms 0 and 1 simultaneously choose R & D investments R_0 and R_1.

Stage 3: The firm that wins the patent chooses a price p for the product.

Stage 4: Consumer either buys the product or illegally copies it or consumes nothing.

We consider a continuum of consumers indexed by θ, which represents the consumers' valuation of the product. We assume that θ follows a uniform distribution and lies in the interval $\theta \in [0,1]$. Each consumer is assumed to consume at most one unit of the product. Either he buys the legitimate product or makes illegal copies or do not consume it. A consumer making illegal copies faces an expected penalty f, which is the product of the fine and the probability of getting detected. The the utility of a type-θ consumer is as follows.

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[9] See Besen and Kirby (1989), Takeyama (1994), Banerjee (2003), Lahiri and Dey (2013), Lu and Poddar (2012) for similar assumption.

$$U(\theta)=\begin{cases}\theta-p, & \text{if the consumer buys the original product,}\\ q\theta-f, & \text{if the consumer illegally downloads,}\\ 0, & \text{otherwise.}\end{cases} \tag{1}$$

Using this framework we now analyse each stages of the game.

2.1. Stage 4: Consumers' decision

Consumers choose to buy or illegally copy depending on the *individual rationality* (IR) and *incentive compatibility* (IC) constraints.[10] A consumer buys the product if the following IR and IC conditions are satisfied.

$$\theta-p\geq 0\Rightarrow\theta\geq\theta_1\equiv p \tag{IR-M}$$

$$\theta-p\geq q\theta-f\Rightarrow\theta\geq\theta_2\equiv\frac{p-f}{(1-q)} \tag{IC-M}$$

θ_1 is the marginal consumer who is indifferent between buying and not consuming and θ_2 is the marginal consumer who is indifferent between buying and copying,

Similarly, a consumer illegally copies if the following IR and IC conditions hold.

$$q\theta-f\geq 0\Rightarrow\theta\geq\theta_3\equiv\frac{f}{q} \tag{IR-C}$$

$$\theta\leq\theta_2\equiv\frac{p-f}{1-q} \tag{IC-C}$$

θ_3 is the marginal consumer who is indifferent between copying and not consuming.

Using the expressions for the different marginal consumers, Lemma 1 summarizes the different market configurations, that is, the conditions under which both buying and copying occurs and when there is only buying. The proof of Lemma 1 and other findings are given in the Appendix unless otherwise mentioned.

Lemma 1.

If $\theta_1\geq\theta_3$, then there is both buying and copying. If $\theta_1\leq\theta_3$, then there is only buying.

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[10] These constraints are extensively used in the literature on copyright infringement, for example, Takeyama (1994), Banerjee (2003), Lahiri and De (2013).

Lemma 1 is diagrammatically represented in Figures 1 and 2 showing the different rankings of the marginal consumers and the resulting market configurations.

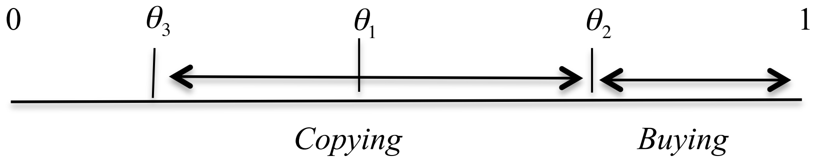

Figure 1: Both buying and copying

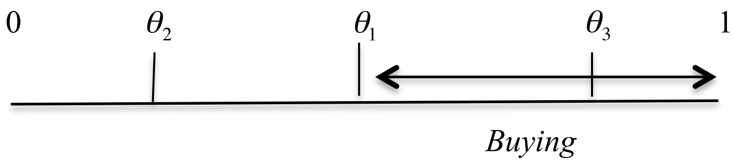

Figure 2: Only buying

Following Lemma 1 and using Figures 1 and 2 we get the demand for the patent-winner's product, which is given in equation (2).

$$D = \begin{cases} 1-\theta_2 = 1-\dfrac{p-f}{1-q}, & \textit{if } qp \geq f, \\ 1-\theta_1 = 1-p, & \textit{otherwise.} \end{cases} \tag{2}$$

We use the demand function to determine the patent-winner's choice of price in Stage 2.

2.2. Stage 3: Patent-winner's choice of price

From the demand function given in equation (2) we get the stage 3 revenue (r) of the patent-winner as follows.

$$r = \begin{cases} p\left(1-\dfrac{p-f}{1-q}\right), & \textit{if } qp \geq f \\ p(1-p), & \textit{otherwise.} \end{cases} \tag{3}$$

Let $f_1 \equiv \dfrac{q(1-q)}{2-q}$ and $f_2 \equiv \dfrac{q}{2}$. Now $f_2 > f_1$ because $\dfrac{q}{2} - \dfrac{q(1-q)}{2-q} = \dfrac{q^2}{2(2-q)} > 0$. Then using the revenue function in equation (3) we get the equilibrium price, which is summarized in Proposition 1.

Proposition 1.

The equilibrium price denoted by p^* *is as follows.*

$$p^* = \begin{cases} \dfrac{1-q+f}{2}, & if \quad f \le f_1, \\ \dfrac{f}{q}, & if \quad f_1 \le f \le f_2, \\ \dfrac{1}{2}, & if \quad f \ge f_2. \end{cases}$$

The implications of Proposition 1 are as follows. When enforcement is low ($f \le f_1$) then some of the consumers have the incentive to copy and it is not profitable for the patent winner to choose a price that prevents copying. That is, the equilibrium price is such that there is both copying and buying. If enforcement is in the intermediate range ($f_2 \ge f \ge f_1$) then the optimal price is $p^* = \dfrac{f}{q}$. Observe that this is the same as the valuation of the marginal consumer who is indifferent between copying and not copying as shown in the (IR-C) constraint. Thus, enforcement in the intermediate range induces the patent winner to choose the equilibrium price that deters copying and there is only buying. However, the threat of copying still exists which prevents the patent winner from charging the monopoly price. This threat disappears only when enforcement exceeds the critical level f_2 and this allows the patent holder to charge the monopoly price. That is, copying is blockaded and the monopoly outcome prevails only when $f \ge f_2$. These three cases are diagrammatically represented in Figure 3.

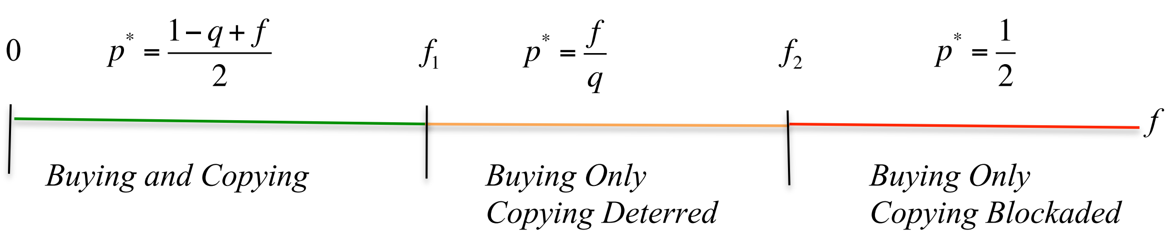

Figure 3: Diagrammatic representation of Proposition 1.

From Proposition 1 we get the patent winner's equilibrium revenue as follows.

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r^* = \begin{cases} \dfrac{(1-q+f)^2}{4(1-q)}, & if \quad f \le f_1, \\ \dfrac{(q-f)f}{q^2}, & if \quad f_1 \le f \le f_2, \\ \dfrac{1}{4}, & if \quad f \ge f_2. \end{cases} \tag{3}
$$

Proposition 2.

(i) An in increase in enforcement (f) increases r^* at an increasing rate in the interval $f \le f_1$, and increases r^* at a decreasing rate in the interval $f_1 \le f \le f_2$ rate.

(ii) A decrease in the degree of product differentiation (that is an increase in q) reduces r^*.

Part (i) of Proposition 2 follows from the fact that an increase in enforcement reduces the consumers' incentive to copy, which increases the patent winner's stage 3 revenue for the following reason. An increase in *f* increases $p^* = \dfrac{1-q+f}{2}$ because $\dfrac{dp^*}{df} = \dfrac{1}{2} > 0$. Further, some the consumers switch from copying to buying which increases the demand for the patent-winner's product $(1-\theta_2^*)$ because $\theta_2^* = \dfrac{1-q-f}{2(1-q)}$ is decreasing in *f* since $\dfrac{d\theta_2^*}{df} = -\dfrac{1}{2(1-q)} < 0$.

When enforcement is low ($f \le f_1$) then there is both buying and copying as shown in Figure 3 and the equilibrium price is low. Therefore, the revenue is also low. Thus an increase in enforcement has a large impact on the revenue and increases it at an increasing rate. However, when the enforcement reaches f_1 and is in the interval $f_1 \le f \le f_2$ then the equilibrium price deters copying (Figure 3). In this interval an increase in enforcement increases the revenue but at a decreasing rate because the revenue is approaching the monopoly level which is attained at f_2 since copying is blockaded. Any further increase in enforcement beyond f_2 does not alter the monopoly outcome. These properties of the patent winner's revenue with respect to enforcement are diagrammatically represented in Figure 4.

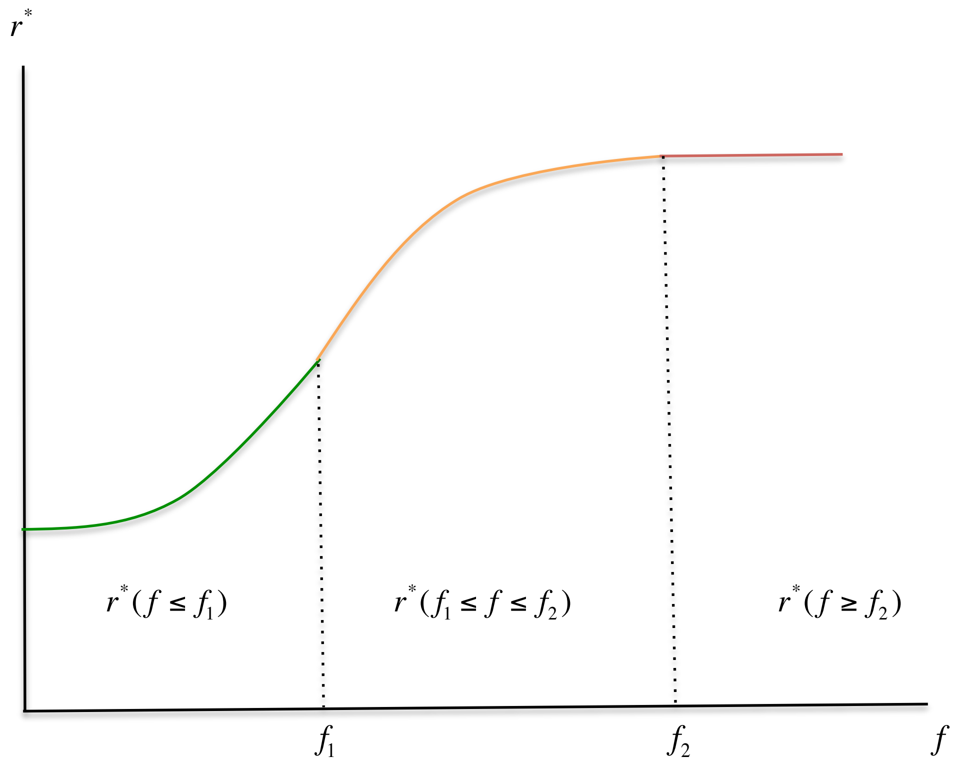

Figure 4: Properties of the revenue function with respect to enforcement.

Part (ii) of Proposition 2, which shows the effect of product differentiation on the revenue, can be intuitively explained as follows. In the interval $0 \leq f \leq f_1$ an increase in the quality of the copied product (reduction in product differentiation) increases the equilibrium quantity because $\theta_2^* = \dfrac{1-q-f}{2(1-q)}$ is decreasing in q. However, the equilibrium price $p^* = \dfrac{1-q+f}{2}$ falls. The negative impact on price dominates the positive impact on demand and revenue declines. In the interval $f_1 \leq f \leq f_2$ the equilibrium price $p^* = \dfrac{f}{q}$ that deters copying is decreasing in q. However, the equilibrium quantity $1-\theta_1^*$ is increasing in q because $\theta_1^* = p^* = \dfrac{f}{q}$ is decreasing in q. The negative impact on price dominates the positive impact on demand and revenue declines.

The properties of the equilibrium revenue with respect to enforcement and product differentiation will be later used for the social welfare analysis. We next proceed to Stage 2 of the game where the competing firms choose their R&D investments.

2.3. Stage 2: Firms' R&D choice

Recall that firm 0 is the relatively more efficient firm in the sense that the probability of success is higher than that of firm 1 when both firms invest the same amount. Let α ($\alpha > 1$) be the relative efficiency of firm 0 over firm 1. The *ex ante* probability of the more efficient firm (firm 0) winning the patent is given by the contest success function $\frac{\alpha R_o}{\alpha R_o + R_1}$ and that of the relatively inefficient firm (firm 1) is given by $\frac{R_1}{\alpha R_o + R_1}$.[11] The patent winner earns the revenue r^* in Stage 2 as given by equation (3). Therefore the expected profit functions of the firms are as follows.

$$\begin{aligned} \pi_o &= \frac{\alpha R_o}{\alpha R_o + R_1} r^* - R_o \\ \pi_1 &= \frac{R_1}{\alpha R_o + R_1} r^* - R_1 \end{aligned} \qquad (4)$$

Using these expected profit functions we derive the equilibrium R&D investment of the two firms and their properties with respect to product differentiation, fine and the efficiency parameter. These results are stated in Proposition 3.

Proposition 3.

The Nash Equilibrium R&D investments are $R = R_0 = R_1 = \frac{\alpha r^*}{(\alpha+1)^2}$. *R decreases if:*

(i) *product differentiation decreases (q increases);*

(ii) *enforcement (f) decreases;*

(iii) *R&D efficiency (α) increases.*

Observe that though the firms differ in terms of their R&D inefficiency, in equilibrium they invest the same amount. Furthermore, as the relative R&D efficiency increases (α increases) firm 0's equilibrium R&D investment decreases and the same is true for firm 1

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[11] For a comprehensive review of contest functions see Jia, Hao & Skaperdas, Stergios & Vaidya, Samarth, 2013.

because R&D investments are strategic complements. However, the efficient firm (firm 0) wins the patent race with probability $\frac{\alpha}{\alpha+1}$, which is increasing in α. This probability exceeds the inefficient firm's (firm 1) probability of winning the patent $\frac{1}{\alpha+1}$, which is decreasing in α. Consequently, the efficient firm's equilibrium expected profit $\pi_0^* = \frac{\alpha^2 r^*}{(\alpha+1)^2}$ is increasing in the relative R&D efficiency and the inefficient firm's equilibrium expected profit $\pi_1^* = \frac{r^*}{(\alpha+1)^2}$ is decreasing in the same. Thus change in the relative R&D efficiency has asymmetric effects on the expected equilibrium profits of the two firms. In contrast, any change in enforcement and product differentiation affects the revenue r^*, which is the patent winner's prize, and hence affects the equilibrium expected profits of the two firms symmetrically.

3. Stage 1: Social welfare analysis and government's choice of enforcement

In this section we discuss stage 1 of the game where the government chooses its enforcement policy that maximizes social welfare. Social welfare is defined as the sum of the surplus of every agent in the society including the consumer surplus of the copiers. We later show that excluding this surplus from the social welfare function do not qualitatively alter the result. We adopt the utilitarian approach and also include the consumer surplus of the agents who copies. The social welfare function is given in equation (5) and the different components are explained below.

$$SW = CS_b + CS_c + E + \Pi \qquad (5)$$

CS_b and CS_c are the consumer surpluses of the buyers and the copiers. E is the net expected enforcement revenue, which is defined later in the section. $\Pi = \pi_0 + \pi_1$ is the total expected profit of the two firms. Using equation (4) we get $\Pi = r^* - 2R$ where $R = \frac{\alpha r^*}{(\alpha+1)^2}$ which we know from Proposition 3. Therefore, $\Pi = \frac{r^*(\alpha^2+1)}{(\alpha+1)^2} = r^* A$, where $A = \frac{(\alpha^2+1)}{(\alpha+1)^2}$ and its properties are summarised in Lemma 2.

Lemma 2. *A is increasing in* α *and* $A \in \left(\frac{1}{2}, 1\right)$.

We will restrict our attention to the range $f \le f_2$ because enforcement beyond f_2 only adds to the enforcement cost without changing the monopoly outcome. Within this range of enforcement the equilibrium price depends on whether the expected fine is in the interval $0 \le f \le f_1$ where there is both buying and copying or in the interval $f_1 \le f \le f_2$ where there is only buying, (see Proposition 1 and Figure 3). Consequently, the social welfare function consists of two functions, one for each of two enforcement ranges as shown in equation (6).

$$SW = \begin{cases} SW^1 \equiv SW(0 \le f \le f_1) \\ SW^2 \equiv SW(f_1 \le f \le f_2) \end{cases} \tag{6}$$

We analyse each of these two functions that comprises the social welfare function.

3.1. Analysis of $SW^1 \equiv SW(0 \le f \le f_1)$

In the interval $0 \le f \le f_1$ the equilibrium price is $p^* = \frac{1-q+f}{2}$ and there is both buying and copying because the relationship between the marginal consumers in equilibrium satisfies $\theta_2^* > \theta_1^* > \theta_3^*$ (as shown in Figure 1) where $\theta_1^* = p^*$, $\theta_2^* = \frac{p^* - f}{1-q} = \frac{1-q-f}{2(1-q)}$, and $\theta_3^* = \frac{f}{q}$. The social welfare function is,

$$SW^1 = CS_b^1 + CS_c^1 + E^1 + \Pi^1. \tag{7}$$

The components of this social welfare function are as follows.

$$\begin{aligned} CS_b^1 &= \int_{\theta_2}^{1} (\theta - p^*)\, d\theta = \frac{p^*(1+2q) - 2p^* f}{2(1-q)^2} \\ CS_c^1 &= \int_{\theta_3}^{\theta_2} (q\theta - f)\, d\theta = \frac{(qp^* - f)^2}{2q(1-q)^2} \\ E^1 &= \int_{\theta_3}^{\theta_2} f\, d\theta - \frac{cf^2}{2} = \frac{(qp^* - f)f}{2q(1-q)} - \frac{cf^2}{2} \\ \Pi^1 &= r^* - 2R = \frac{(\alpha^2+1)p^{*2}}{(1-q)(\alpha+1)^2} = \frac{Ap^{*2}}{(1-q)} \end{aligned} \tag{8}$$

The term $\int_{\theta_3}^{\theta_2} f\,d\theta$ in the expression E^1 is the total expected fine collected by the government from the copiers. Therefore, the government's net expected enforcement revenue is $E^1 = \int_{\theta_3}^{\theta_2} f\,d\theta - \frac{cf^2}{2} = \frac{(qp^* - f)f}{2q(1-q)} - \frac{cf^2}{2}$.

The properties of each of these components with respect to the enforcement f are summarized in Lemma 3. These properties, which are intuitively explained below, will be used for discussing the enforcement that maximizes SW^1.

Lemma 3.

(i) CS_b^1 *is concave in f.*

(ii) CS_c^1 *is decreasing and convex in f.*

(iii) E^1 *is concave in f.*

(iv) Π^1 *is increasing and convex in f.*

To understand Lemma 3 we have to analyse the effect of a change in *f* on the equilibrium price because it will affect the position of the marginal consumers in equilibrium. This in turn will affect the consumer surpluses and the demand thereby affecting the revenue of the patent winner. The comparative static analysis of change in *f* and on the equilibrium price and the marginal consumers are provided in Table 1. We will use Table 1 to discuss the effect of an increase in enforcement on the various components of SW^1.

Table 1: Effect of an increase in *f* on equilibrium price and marginal consumers

Variables	Increase in *f*
$p^* = \frac{1-q+f}{2}$	Increases p^* because $\frac{dp^*}{df} = \frac{1}{2} > 0$
$\theta_1^* = p^*$	Increases θ_1^* because $\frac{d\theta_1^*}{df} = \frac{1}{2} > 0$
$\theta_2^* = \frac{1-q-f}{2(1-q)}$	Decreases θ_2^* because $\frac{d\theta_2^*}{df} = -\frac{1}{2(1-q)} < 0$
$\theta_3^* = \frac{f}{q}$	Increases θ_3^* because $\frac{d\theta_3^*}{df} = \frac{1}{q} > 0$

Let us now explain the concavity of CS_b^1 with respect to *f*. From Table 1 we know that an increase in f increases the equilibrium price but reduces θ_2^* which means that the buyers' demand $(1-\theta_2^*)$ increases. This is because the higher enforcement increases the copying cost and hence, some of the copiers shift from copying to buying. While the higher price reduces CS_b^1, the higher demand raises it.

These effects are shown in Figure 5 where we draw the buyers' utility function $U_b(\theta)=\theta-p^*$, which is linear with respect to θ with a slope 1. Since $\theta_1^*=p^*$, hence $U_b(\theta_1^*)=0$ and the consumers with valuation in the range $\theta\in[\theta_2^*,1]$ constitute the buyers.

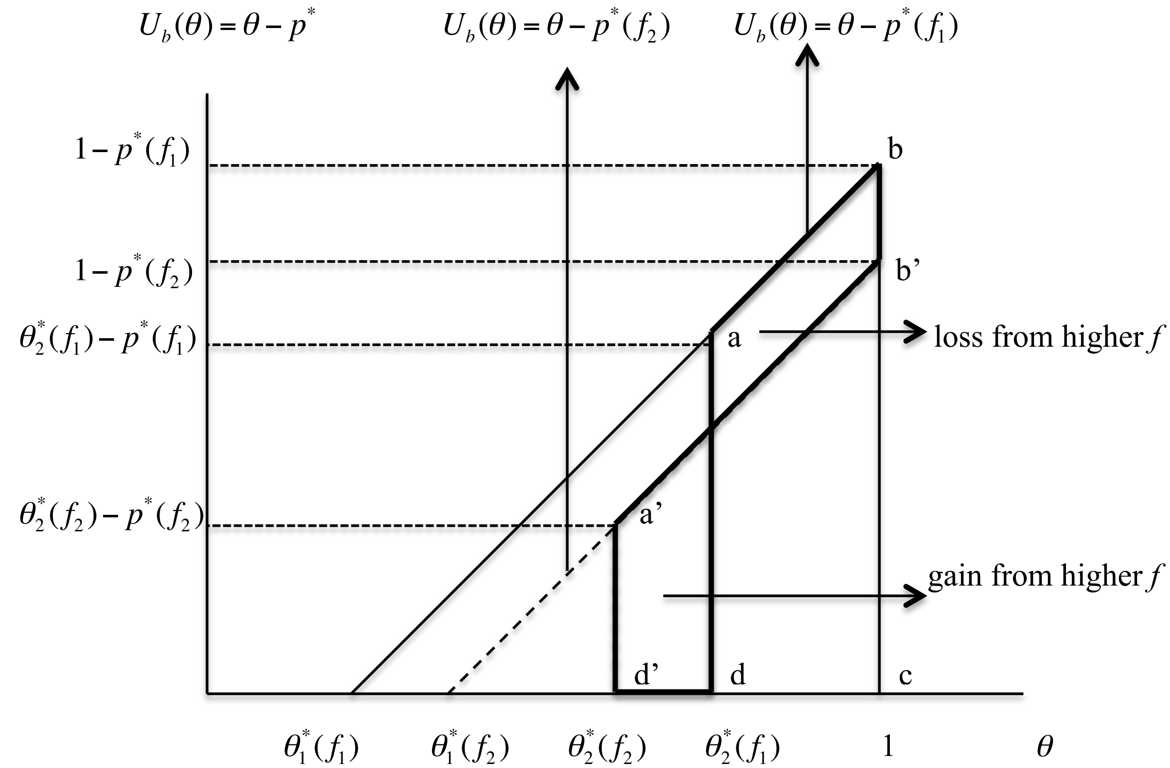

Figure 5: Effect of higher enforcement on buyers' consumer surplus

Suppose the initial enforcement is f_1. The utility function is the line $U_b(\theta)=\theta-p^*(f_1)$ and the buyers are in the interval $\theta\in[\theta_2^*(f_1),1]$. Hence the initial consumer surplus is the area of the quadrilateral abcd. Consider an increase in enforcement from f_1 to f_2. As shown in Table 1, this increase in enforcement increases θ_1^* from $\theta_1^*(f_1)$ to $\theta_1^*(f_2)$ and decreases θ_2^* from $\theta_2^*(f_1)$ to $\theta_2^*(f_2)$. So the utility function shifts down to $U_b(\theta)=\theta-p^*(f_2)$. The buyers' valuation lies in the interval $\theta \in [\theta_2^*(f_2),1]$ and the consumer surplus is the area of the quadrilateral a'b'cd'. The gain in the buyers' consumer surplus due to an increase in demand and the loss due to a higher price are as shown in Figure 5, which explains the concavity of CS_b^1.

Let us now consider the consumer surplus of the copiers whose valuations are in the range $[\theta_3^*,\theta_2^*]$. An increase in enforcement increases the copying cost. So some of consumers switch from copying to buying. This is also evident from the fact that θ_2^* is decreasing in *f* as shown in Table 1 The increase in *f* also requires a marginal consumer with a higher valuation to become indifferent between copying and not consuming, that is, $\theta_3^* = \dfrac{f}{q}$ is increasing in *f*. Thus an increase in *f* reduces the interval $[\theta_3^*,\theta_2^*]$ and therefore, CS_c^1 is decreasing in *f*. We show this diagrammatically in Figure 6.

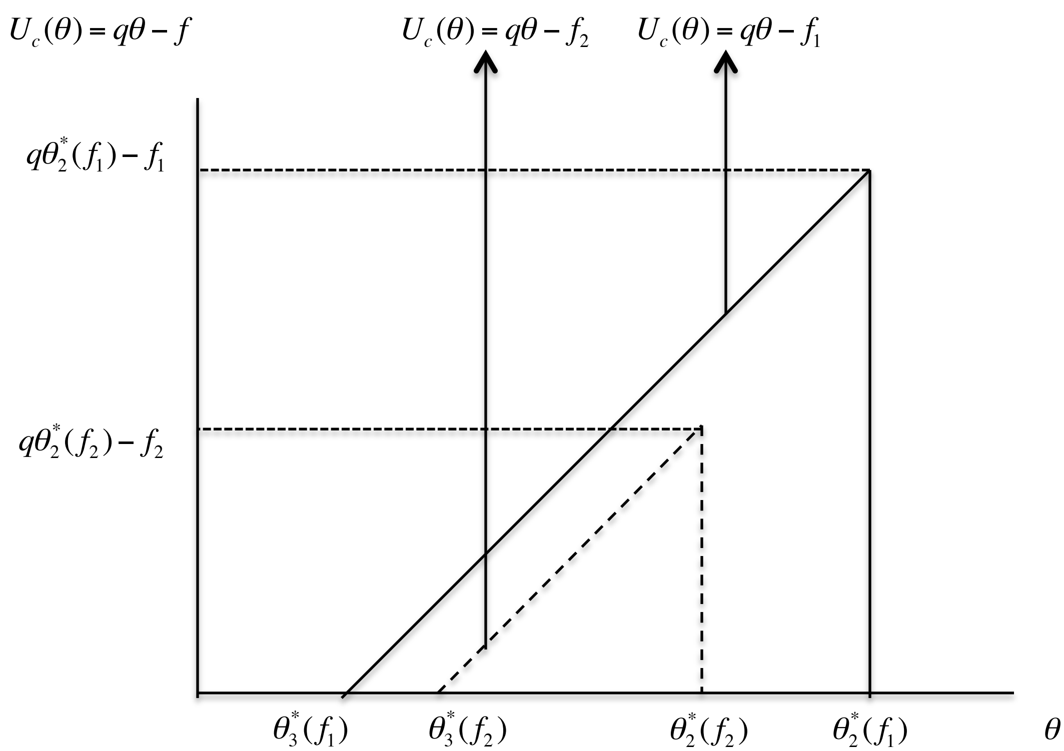

Figure 6: Effect of higher enforcement on copiers' consumer surplus

The copiers' utility function is $U_c(\theta) = q\theta - f$, which is linear with respect to θ with a slope q. Since $\theta_3^* = \frac{f}{q}$, hence $U_c(\theta_3^*) = 0$ and the consumers with valuation in the range $\theta \in [\theta_3^*, \theta_2^*]$ constitute the copiers. Suppose the initial enforcement is f_1 and the copiers' consumer surplus is the area of the triangle drawn in bold. Suppose the enforcement increases to f_2. Then reduces θ_2^* from $\theta_2^*(f_1)$ to $\theta_2^*(f_2)$ and increases θ_3^* from $\theta_3^*(f_1)$ to $\theta_3^*(f_2)$. So the new consumer surplus of the copiers' is the area of the triangle drawn in dashed lines. This area is less than the area of the triangle drawn in bold showing the reduction in the copiers' consumer surplus due an increase in enforcement.

The concavity of E^1 with respect to *f* can be intuitively explained as follows. An increase in *f* has a direct positive effect on the penal revenue $\int_{\theta_3}^{\theta_2} f \, d\theta$. However, as discussed earlier the increase in *f* also reduces the interval $[\theta_3^*, \theta_2^*]$, which generates a negative impact on the penal revenue. Now at $f = 0$, $\frac{dE^1}{df} = \frac{2qp^*}{2q(1-q)} > 0$. This means that at low levels of enforcement the positive effect dominates and therefore, E^1 increases. However, beyond a certain level the combined negative effect of a reduced interval $[\theta_3^*, \theta_2^*]$ and the higher enforcement cost dominates the positive effect.

An increase in *f* increases the revenue r^* at an increasing rate in the interval $0 \le f \le f_1$ as shown in Figure 4. This explains the convexity of Π^1 with respect to *f*.

We now use the properties of the elements of the social welfare function SW^1 to determine the enforcement that maximizes it. The result is summarized in Proposition 4.

Proposition 4.

SW^1 is concave in the enforcement f. $\bar{f} = \frac{q(1-q)(2A-1)}{2(2-q) + 4cq(1-q) - q(2A-1)}$ maximizes SW^1 and $\bar{f}$ is an interior solution, that is, $\bar{f} \in (0, f_1)$.

Proposition 4 is diagrammatically represented in Figure 7.

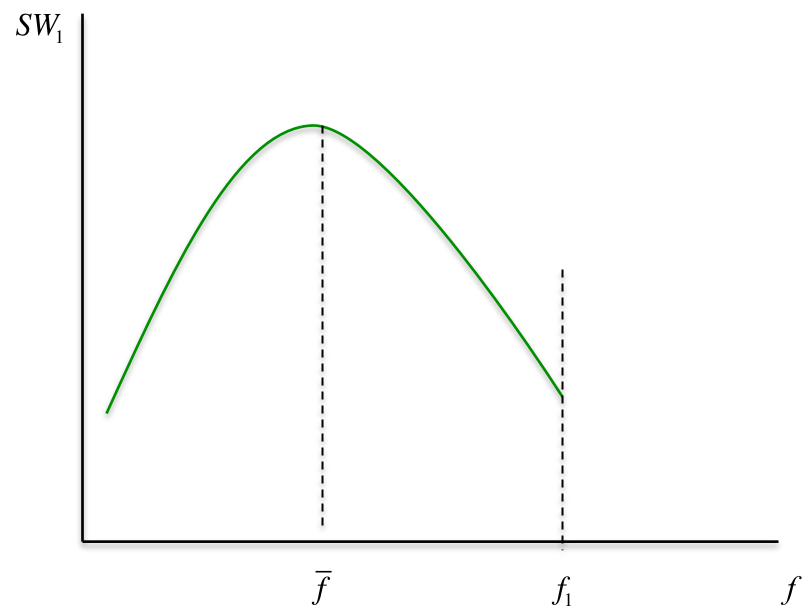

Figure 7: A diagrammatic representation of SW_1 and the existence of $\bar{f}$ as interior solution

It implies, that initially all the positive effects of an increase in enforcement dominates all the negative effects and therefore, social welfare increases. However, any increase in enforcement beyond $\bar{f}$, causes the negative effects to dominate which causes the social welfare to decrease. The positive and negative effects are discussed in detail in the analyses of the consumer surpluses, expected net enforcement revenue and the firms' expected profits. An important implication of Proposition 4 is that, at the corner point $f = f_1 = \frac{q(1-q)}{(2-q)}$, SW^1 has a negative slope, that is $\frac{dSW^1}{df} < 0$ at $f = f_1 = \frac{q(1-q)}{(2-q)}$.

3.2 Analysis of $SW_2 \equiv SW(f_1 \le f \le f_2)$

In the interval $f_1 \le f \le f_2$ the equilibrium price $p^* = \frac{f}{q}$ prevents copying (Proposition 1) and there is only buying though the threat of copying exists as shown in Figure 3. This is because the relationship between the marginal consumers in equilibrium satisfies $\theta_2^* \le \theta_1^* \le \theta_3^*$. Thus in this case all consumers with valuation in the interval $\theta \in [\theta_1^*, 1]$ buy the product where $\theta_1^* = p^* = \frac{f}{q}$. Further, since there is no copying there are no fines. Therefore, the consumer surplus, net expected enforcement revenue and the firms joint expected profits are given in equation (9).

$$CS_b^2 = \int_{\theta_1}^{1} (\theta - p^*)\,d\theta = \frac{1}{2}\left(1 - \frac{f}{q}\right)^2$$
$$E^2 = -\frac{cf^2}{2} \qquad (9)$$
$$\Pi^2 = r^* A = \frac{(q-f)fA}{q^2}$$

We get $r^* = \frac{(q-f)f}{q^2}$ from equation (3). The properties of these three components of the social welfare function are summarized in Lemma 4.

Lemma 4.

(i) CS_b^2 *is decreasing in f at an increasing rate.*

(ii) E^2 *is decreasing in f at a decreasing rate.*

(iii) Π^2 *is non-decreasing and concave in f.*

Lemma 4 can intuitively explained as follows. An increase in enforcement raises the price and the marginal valuation of the consumer who is indifferent between buying and not buying, because $\theta_1^* = p^* = \frac{f}{q}$. Thus the demand shrinks and coupled with the rise in price reduces the consumer surplus. The increase in enforcement also raises the enforcement cost which therefore reduces E^2. That Π^2 is non-decreasing and concave in *f* follows from the same property of r^* in the interval $f_1 \le f \le f_2$ as shown in Figure 4.

The social welfare function in the interval $f_1 \le f \le f_2$ is,

$$SW^2 = CS_b^2 + E^2 + \Pi^2 \qquad (10)$$

The result for the enforcement that maximizes SW^2 is summarized in Proposition 5 and diagrammatically represented in Figure 8.

Proposition 5.

SW^2 is decreasing in f and is maximized at $f_1 = \dfrac{q(1-q)}{(2-q)}$.

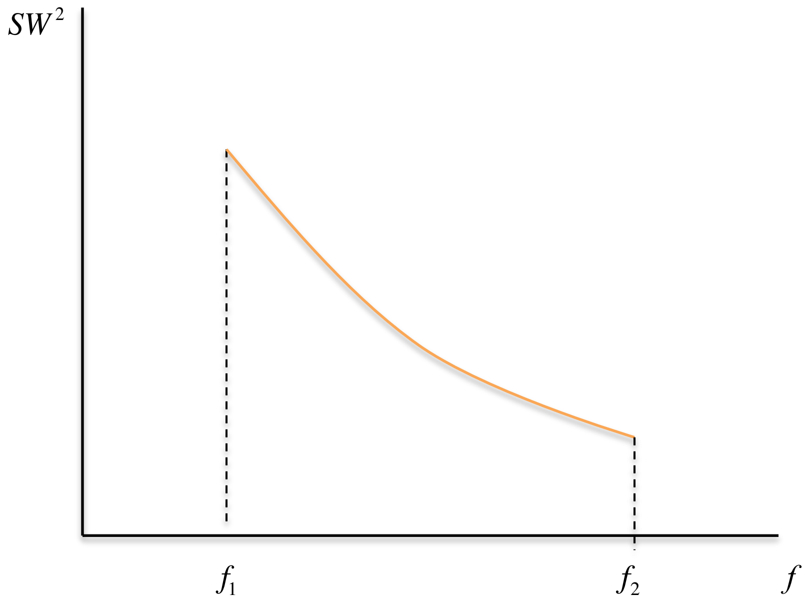

Figure 8: A diagrammatic representation of SW^2 with respect to f

From Lemma 3 we know that an increase in enforcement reduces the consumer surplus at an increasing rate but increases the firms' expected profit at a decreasing rate. Such an increase also increases the enforcement cost. The negative effects of an increase in enforcement dominate the positive effect thereby causing SW^2 to be a decreasing function of the enforcement. Therefore, the optimal solution is a corner solution, which is f_1, which is the lower bound of the interval over which SW_2 is defined.

3.3. Socially optimal enforcement

We now combine the analyses of SW^1 and SW_2 as presented in the previous two subsections to determine the socially optimal monitoring rate denoted by f^*. The choice is between $\overline{f}$ that maximizes SW^1 and f_1 that maximizes SW^2. The result is summarized in Proposition 6 and we discuss the proof in the main text because it is instructional.

Proposition 6.

$f^* = \bar{f} = \dfrac{q(1-q)(2A-1)}{2(2-q)+4cq(1-q)-q(2A-1)}$ is the unique enforcement that maximizes the social welfare $SW = \begin{cases} SW^1 \equiv SW(0 \le f \le f_1) \\ SW^2 \equiv SW(f_1 \le f \le f_2) \end{cases}$ as given in equation (6). The equilibrium price is $p^* = \dfrac{1-q+\bar{f}}{2}$ which allows both buying and copying.

Proof of Proposition 6.

f_1 is the upper bound for SW^1 and lower bound for SW^2. At $f = f_1 = \dfrac{q(1-q)}{(2-q)}$, $p^* = \dfrac{1-q+f_1}{2} = \dfrac{f_1}{q}$ which prevents copying. Thus at $f = f_1$, the two social welfare functions are identical, that is, $SW^1(f_1) = SW^2(f_1)$. This is because, both social welfare functions have only buyers, there is no revenue from the penalty because there is no copying and the expected profits are the same. From Proposition 4 we know that $\bar{f}$ is an interior solution, that is, $\bar{f} \in (0, f_1)$. This means that the slope of SW^1 at f_1 is negative which is shown in the proof of Proposition 4. This means the social welfare at $\bar{f}$ is the highest and hence, it is the social optimum. **Q.E.D.**

Proposition 6 is diagrammatically represented in Figure 9. The intuition is the same as that mentioned after Proposition 4, which follows from the analyses of the consumer surpluses, net expected enforcement revenue and the firms' expected profit with respect to the enforcement as provided in Section 3.1.

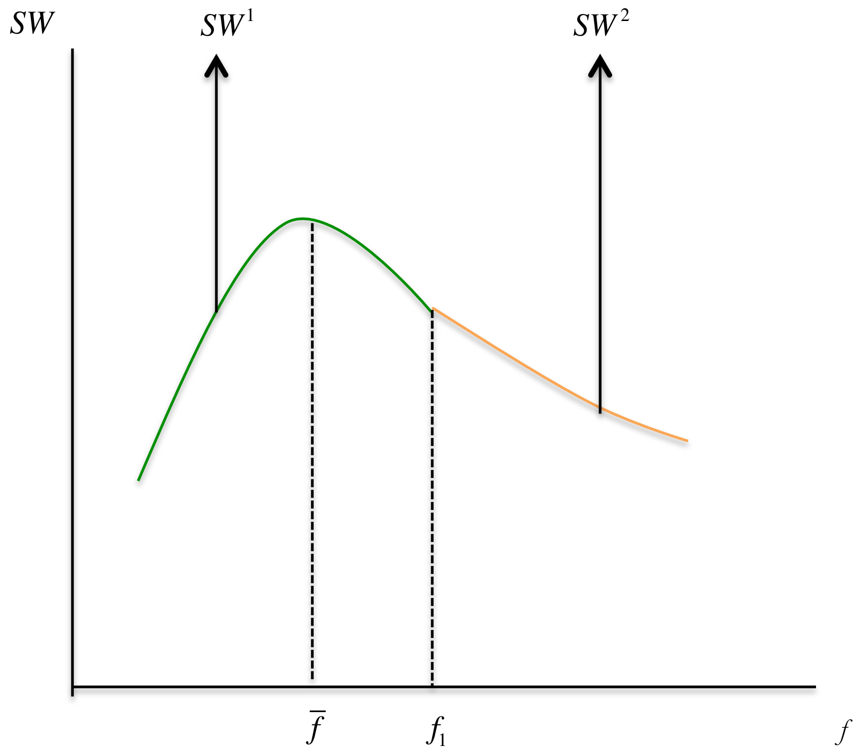

Figure 9: The socially optimal enforcement policy is $f^* = \overline{f}$.

It follows from Proposition 6 and Figure 9 that socially optimal enforcement policy $\overline{f}$ always lies in the interval $(0, f_1)$ for finite values of the enforcement efficiency parameter *c*. This result qualitatively the same for an alternative social welfare function that do not include the copiers' consumer surplus. This can be explained as follows. SW^1 is the relevant social welfare function that had the copiers' consumer surplus. Non-inclusion of this thus affects only SW^1 because copying is deterred in the enforcement range for which SW^2 is relevant and therefore, the later remains unaffected. The concavity of SW^1 continues to hold because the buyers' consumer is concave with respect to the enforcement as discussed previously. Recall that at f_1, SW^1 equals SW^2 as discussed in the proof of Proposition 6. At this point none of these social welfare functions contain the copiers' consumer surplus because copying is deterred. We have shown in the proof of Proposition 6 that at f_1, SW^1 has a negative slope. Hence, f_1 cannot maximize SW^1 that do not contain the copiers' consumer surplus. Thus the enforcement that maximizes the alternative SW^1 is an interior solution lying in the interval $(0, f_1)$ and this will also be the socially optimal outcome. This explains the robustness of our result.

Since $f^* = \overline{f} \in (0, f_1)$ is the socially optimal enforcement, hence, the relevant social welfare function to be considered for the comparative static analysis is SW^1. Proposition 7 summarizes the comparative static analysis of the socially optimal enforcement with respect to the relative R&D efficiency parameter (α), the degree of product differentiation captured by the parameter q, and the enforcement efficiency parameter (c).

Proposition 7.

(i) An increase in the relative R&D efficiency (α) increases the socially optimal enforcement policy $\overline{f}$.

(ii) There exists a degree of product differentiation $\hat{q} = \frac{4 - 2\sqrt{3 - 2A}}{2A + 1}$, where $\hat{q} \in \left(0.5858, \frac{2}{3}\right)$, such that $\overline{f}$ is non-decreasing in q for $0 < q \le \hat{q}$ and $\overline{f}$ is decreasing in q for $\hat{q} < q < 1$. $\hat{q}$ is increasing in α.

(iii) An increase in the enforcement efficiency (that is, a decrease in c) increases $\overline{f}$.

Proposition 7 can be explained as follows. An increase in the relative R&D efficiency reduces the R&D investment, which being a cost in the social welfare function increases the latter. Consequently, the socially optimal enforcement consequently increases. A fall in product differentiation that intensifies the product market competition has non-monotonic effect on the optimal enforcement. Specifically, as product differentiation increases from low levels, the optimal enforcement initially increases and beyond some critical level it decreases. This non-monotonic effect is due to the non-monotonic effect of a decrease in product differentiation on SW_1. An improvement in the enforcement technology reduces the enforcement cost, which increases SW_1 and hence, increases the socially optimal enforcement.

We use the comparative static results of the socially optimal enforcement to discuss the effect of change in product differentiation and relative R&D efficiency on the optimal R&D investment, which is $R = \frac{\alpha r^*}{(\alpha+1)^2} = \frac{\alpha}{(\alpha+1)^2} \frac{(1 - q + \overline{f})^2}{4(1-q)}$, as given in Proposition 3. Any change in product differentiation (q) or relative R&D efficiency (α) will have a direct effect on the equilibrium R&D investment and an indirect effect via the socially optimal enforcement as shown in equation (11).

$$\frac{dR}{d\alpha}=\frac{dR}{d\overline{f}}\frac{d\overline{f}}{d\alpha}+\frac{dR}{d\alpha}$$
$$\frac{dR}{dq}=\frac{dR}{d\overline{f}}\frac{d\overline{f}}{dq}+\frac{dR}{dq} \tag{11}$$

The first term in each of these equations represent the indirect effect and the second one represents the indirect effect. The result is summarized in Proposition 8.

Proposition 8.

(i) As R&D competition becomes more asymmetric (α increases) then the equilibrium R&D investment increases if the indirect effect dominates the direct effect. Otherwise, the reverse is true.

(ii) Decrease in product differentiation (increase in q) unambiguously reduces the equilibrium R&D investment.

Proposition 8(i) can be explained as follows. From Proposition 7 we know that the socially optimal enforcement. Thus an increase in the R&D efficiency has a direct effect and an indirect effect via the enforcement on the equilibrium R&D investment. The direct effect is a negative one because the increase in R&D efficiency requires less resource to innovate the product whose quality is normalized to one as shown in Proposition 3. The indirect effect is a positive one because of the following reason. The higher enforcement due to a higher R&D efficiency lowers copying and some of the consumers switch from copying to buying. Thus the demand increases. The higher enforcement also empowers the patent-winner to charge a higher price. The higher price and demand increases the profit, which increases the equilibrium R&D investment. The negative direct effect and the positive indirect effect lead to an ambiguous overall result.

An intuitive explanation of Proposition 8(ii) is as follows. Proposition 7 shows that a fall in product differentiation that intensifies the product market competition has non-monotonic effect on the optimal enforcement. A decrease in product differentiation has direct negative effect on the profit because higher competition reduces the price. Though the demand increases but the lower price has the dominant impact. This negative impact on profit in turn reduces the equilibrium R&D investment. The decrease in product differentiation has an indirect effect via its impact on the optimal enforcement. Consider the range where a decrease in product differentiation increases the optimal enforcement. The higher enforcement has an indirect positive impact on the profit for reasons explained previously, and hence, has a positive impact on the equilibrium R&D investment. However, this positive indirect effect is outweighed by the direct negative effect thereby causing the equilibrium R&D to decrease. In the range where a decrease in product differentiation decreases the optimal enforcement, both the direct and indirect effects on profit are negative. Thus the effect on R&D investment is also negative.

Our analysis shows that though in equilibrium there is always positive enforcement, which restricts IPR violation, but it fails to reverse the negative effect of competitive pressure on the incentive to innovate. However, the positive enforcement may restrict the lowering of R&D investment, which can be more drastic in the absence of any enforcement. This is true for the relatively high product differentiation scenario where a decrease in product differentiation increases the optimal enforcement. This increase partially offsets the direct negative effect. Without enforcement only the direct negative effect of a lowering of product differentiation exists and therefore, it is larger than overall effect with enforcement. However, it is not true for the low product differentiation case where a decrease in product differentiation decreases the socially optimal enforcement, which further adds to the already negative direct impact of low product differentiation on the incentive to innovate.

5. Conclusion

In this paper we analysed optimal enforcement policies in an environment where there is competition in R&D and the patent winner faces IPR violation by consumers who makes illegal copies of the patented or copyrighted product for personal consumption. The expected penalty for such malfeasant activities constituted the government's enforcement policy. The firms competing in R&D differed with respect to their R&D efficiency and we adopted a contest function approach to model the R&D race.

We showed that the socially optimal enforcement policy induces the patent-winner to choose a price that allows both buying and copying. That is, the optimal policy cannot deter copying. An increase in the relative R&D efficiency unambiguously increases the optimal enforcement but has ambiguous effects on the incentive to innovate. However, a lowering of product differentiation has a non-monotonic effect on the optimal enforcement and reduces the incentive to innovate.

Our finding thus suggests that while the optimal enforcement reduces copying but cannot reverse the negative impact that lowering of product differentiation or higher product market competition has on the incentive to innovate. When product differentiation is high (low product market competition), the presence of enforcement restricts the fall in the incentive to innovate that could have occurred in the absence of any enforcement. However, when product differentiation is low (high product market competition), then the presence of enforcement exacerbates the fall in the incentive. That is, the fall in the incentive to innovate is higher in the presence of enforcement than in its absence.

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Appendix

Proof of Lemma 1. If $\theta_1 \geq \theta_3 \Rightarrow qp \geq f$, then $\theta_2 - \theta_1 = \dfrac{p-f}{(1-q)} - p = \dfrac{qp-f}{(1-q)} \geq 0$. Therefore, $\theta_2 \geq \theta_1 \geq \theta_3$. From the above expressions we see that if $\theta_1 \leq \theta_3$ holds then $\theta_2 \leq \theta_1$. Now $\theta_2 - \theta_3 = \dfrac{p-f}{(1-q)} - \dfrac{f}{q} = \dfrac{qp-f}{q(1-q)} \leq 0$. Thus we have $\theta_2 \leq \theta_1 \leq \theta_3$. **Q.E.D**.

Proof of Proposition 1. Suppose $\theta_1 > \theta_3$ holds in which case there is buying and copying. Then the relevant revenue function is $r = \left(1 - \frac{p-f}{(1-q)}\right)p$ and the first order condition yields $p^* = \frac{1-q+f}{2}$. This price must satisfy the inequality $\theta_1 \geq \theta_3 \Rightarrow qp^* \geq f$ which gives $f \leq f_1 \equiv \frac{q(1-q)}{2-q}$. Next suppose that the inequality $\theta_1 \leq \theta_3$ hold. In this case there is no copying and the relevant revenue function is $r = (1-p)p$ which yields the equilibrium price as $p^* = \frac{1}{2}$. This price must satisfy the inequality $\theta_1 \leq \theta_3 \Rightarrow p^* \leq \frac{f}{q}$ which on rearrangement gives $f \geq f_2 \equiv \frac{q}{2}$. That is, in this range of penalty the monopoly outcome is restored. Any fine in excess of f_2 cannot improve on the monopoly outcome and given that enforcement is costly we will restrict our attention for the rest of the analysis to $f \leq f_2 \equiv \frac{q}{2}$. We know that $f_2 > f_1$. So let us consider the interval $f_2 \geq f \geq f_1$. Let us consider the boundary points. At f_1, $p^* = \frac{1-q+f_1}{2} = \frac{1-q}{2-q} = \frac{f_1}{q}$. Similarly at f_2, $p^* = \frac{1}{2} = \frac{f_2}{q}$. Recall that $\theta_3 = \frac{f}{q}$ which means that by charging $p^* = \frac{f_1}{q}$ at f_1 and $p^* = \frac{f_2}{q}$ at f_2 the patent winning firm deters copying. Next we show that charging $p = \frac{f}{q}$ for any *f* in the interval $f_2 \geq f \geq f_1$ is optimal. Now $\frac{f}{q} \geq \frac{1-q+f}{2}$ for $f \geq f_1$. So for any *f*, deterring copying by charging the boundary price $\frac{f}{q}$ is superior to charging a lower price and serving the same size market. Now let us compare the revenue from charging the price $\frac{f}{q}$ to that from charging $\frac{1}{2}$, that is, compare $r\left(\frac{f}{q}\right) = \frac{f}{q} - \left(\frac{f}{q}\right)^2$ to $r\left(\frac{1}{2}\right) = \frac{1}{4}$. Now $r\left(\frac{f}{q}\right) \geq r\left(\frac{1}{2}\right)$ whenever $\frac{f}{q} \leq \frac{1}{2}$, which is the same as the condition $f \leq f_2$. **Q.E.D.**

Proof of Proposition 2. (i) In the interval $f \le f_1$, $\frac{dr^*}{df} = \frac{1-q+f}{2(1-q)} > 0$ and $\frac{d^2r^*}{df^2} = \frac{1}{2(1-q)} > 0$. In the interval $f_1 \le f \le f_2$, $\frac{dr^*}{df} = \frac{q-2f}{q^2} \ge 0$. This is because of the following reason. The maximum penalty is $f_2 = p^* q$ and at f_2, the equilibrium price is $p^* = \frac{1}{2}$. This means that the maximum penalty is $f_2 = \frac{q}{2}$ and thus $\frac{dr^*}{df} = \frac{q-2f}{q^2} \ge 0$. $\frac{d^2r^*}{df^2} = \frac{-2}{q^2} < 0$.

(ii) Consider the interval $f \le f_1$. $\frac{dr^*}{dq} = \frac{(1-q+f)(q+f-1)}{4(1-q)^2} < 0$ because. $f \le f_1$ implies $f \le \frac{q(1-q)}{2-q}$ since $f_1 \equiv \frac{q(1-q)}{2-q}$. Now $(1-q) - \frac{q(1-q)}{2-q} = \frac{2(1-q)^2}{2-q} > 0 \Rightarrow (1-q) > f_1$. Thus $f \le (1-q)$ which means $(q+f-1) < 0$. In the interval $f_1 \le f \le f_2$, $\frac{dr^*}{dq} = \frac{q^2 f - 2qf(q-f)}{q^4} = \frac{f(2f-q)}{q^3} \le 0$ because the fine is $f_2 = \frac{q}{2}$. **Q.E.D.**

Proof of Proposition 3. The expressions for the R&D investments follow from the first order maximization condition of the profit functions given in equation (4) with respect to the R&D investments. Since R is directly related to the second stage revenue r^* hence, the properties of r^* with respect to q and f also hold for R. $\frac{dR}{d\alpha} = \frac{r^*(1-\alpha)}{(\alpha+1)^3} < 0$ since by assumption $\alpha > 1$. **Q.E.D.**

Proof of Lemma 2. $\frac{dA}{d\alpha} = \frac{2(\alpha-1)}{(\alpha+1)^3} > 0$ because by assumption $\alpha > 1$. $A = \frac{\alpha^2+1}{(\alpha+1)^2} = \frac{\alpha^2\left(1+\frac{1}{\alpha^2}\right)}{\alpha^2\left(1+\frac{2}{\alpha}+\frac{1}{\alpha^2}\right)}$. $Limit_{\alpha\to 1}\, A = \frac{1}{2}$ and $Limit_{\alpha\to\infty}\, A = 1$. **Q.E.D.**

Proof of Lemma 3. (i) $\frac{dCS_b^1}{df} = \frac{(qp^* - f) - p^*(1-q)}{2(1-q)^2}$ and $\frac{d^2CS_b^1}{df^2} = \frac{-(1.5-q)}{2(1-q)^2} < 0$.

(ii) $\frac{dCS_c^1}{df}=\frac{-(qp^*-f)(2-q)}{2q(1-q)^2}<0$ and $\frac{d^2CS_c^1}{df^2}=\frac{(2-q)(1-0.5q)}{2q(1-q)^2}>0$.

(iii) $\frac{dE^1}{df}=\frac{2(qp^*-f)-f(2-q)}{2q(1-q)}-cf$ and $\frac{d^2E^1}{df^2}=\frac{2\left(\frac{q}{2}-1\right)-(2-q)}{2q(1-q)}-f<0$.

(iv) $\frac{d\Pi^1}{df}=\frac{(\alpha^2+1)p^*}{(1-q)(\alpha+1)^2}>0$ and $\frac{d^2\Pi^1}{df^2}=\frac{(\alpha^2+1)}{2(1-q)(\alpha+1)^2}>0$. **Q.E.D.**

Proof of Proposition 4.

$\frac{dSW^1}{df}=\frac{dCS_b^1}{df}+\frac{dCS_c^1}{df}+\frac{dE^1}{df}+\frac{d\Pi^1}{df}=\frac{q(1-q)(2A-1)-f[2(2-q)+4cq(1-q)-q(2A-1)]}{4q(1-q)}$. The term $[2(2-q)+4cq(1-q)-q(2A-1)]$ is positive because $q(2A-1)\in(0,1)$ and $2(2-q)+4cq(1-q)>1$. $\frac{d^2SW^1}{df^2}=\frac{-[2(2-q)+4cq(1-q)-q(2A-1)]}{4q(1-q)}<0$ implying that SW^1 is concave in *f*. At $f=f_1=\frac{q(1-q)}{(2-q)}$, $\frac{dSW^1}{df}\Big|_{f=f_1}=\frac{-[(3-2A-q)+4cq(1-q)]}{4(2-q)}<0$ because $(3-2A-q)>0$ since $2A+q<2$ (from Lemma 2 we know that $A<1$ and by assumption $0<q<1$). Then solving $\frac{dSW^1}{df}=0$ we get $\bar{f}=\frac{q(1-q)(2A-1)}{2(2-q)+4cq(1-q)-q(2A-1)}$ which is an interior solution, that is, $\bar{f}\in(0,f_1)$. **Q.E.D.**

Proof of Lemma 4.

(i) $\frac{dCS_b^2}{df}=-\frac{1}{q}\left(1-\frac{f}{q}\right)<0$ and $\frac{d^2CS_b^2}{df^2}=\frac{1}{q^2}>0$. (ii) $\frac{dE^2}{df}=-cf<0$ and $\frac{d^2E^2}{df^2}=-c<0$. (iii) $\frac{d\Pi^2}{df}=\frac{(q-2f)A}{q^2}\geq 0$ since $f\leq\frac{q}{2}$ and $\frac{d^2\Pi^2}{df^2}=\frac{-2A}{q^2}<0$ **Q.E.D**

Proof of Proposition 5

$\frac{dSW^2}{df} = \frac{dCS_b^2}{df} + \frac{dE^2}{df} + \frac{d\Pi^2}{df} = -\frac{1+A}{q} - \frac{f(2A-1)}{q^2} - cf < 0$ because from Lemma 2 we know that $\frac{1}{2} < A < 1 \Rightarrow 1 < 2A < 2$. **Q.E.D**

Proof of Proposition 7.

(i) $\frac{d\bar{f}}{d\alpha} = \frac{d\bar{f}}{dA}\frac{dA}{d\alpha} = \frac{4q(1-q)[(2-q)+2cq(1-q)]}{\left(2(2-q)+4cq(1-q)-q(2A-1)\right)^2}\frac{dA}{d\alpha} > 0$ because from Lemma 2 we know that $\frac{dA}{d\alpha} > 0$.

(ii) $\frac{d\bar{f}}{dq} = \frac{d\bar{f}}{dq} = \frac{(2A-1)\left((2A+1)q^2 - 8q + 4\right)}{\left(2(2-q)+4cq(1-q)-q(2A-1)\right)^2}$. The denominator is positive and the numerator is decreasing in q because $\frac{d\left((2A+1)q^2-8q+4\right)}{dq} = 2(2A+1)-8 < 0$. Solving $(2A+1)q^2 - 8q + 4 = 0$ we get $\hat{q} = \frac{4-2\sqrt{3-2A}}{2A+1}$. Now $\frac{d\hat{q}}{dA} = \frac{6-4A}{(2A+1)^2\sqrt{3-2A}} > 0$. Hence $\hat{q}$ is increasing in α because $\frac{dA}{d\alpha} > 0$. Recall from Lemma 2 that $A \in \left(\frac{1}{2}, 1\right)$. Substituting $A = \frac{1}{2}$ and $A = 1$ in $\hat{q} = \frac{4-2\sqrt{3-2A}}{2A+1}$ we get $\hat{q} = \frac{2}{3}$ and $\hat{q} = 2-\sqrt{2} = 0.5858$ respectively. Since $\hat{q}$ is increasing in *A*, it follows that for $A \in \left(\frac{1}{2}, 1\right)$, $\hat{q}$ lies in the interval $\hat{q} \in \left(0.5858, \frac{2}{3}\right)$ which implies that $\hat{q}$ satisfies the assumption $q \in (0,1)$ and therefore, $\hat{q}$ exists.

(iii) From the expression for $\bar{f}$ it is evident that $\bar{f}$ and *c* are inversely related. This implies that an increase in enforcement efficiency, which means a decrease in *c*, increases $\bar{f}$. **Q.E.D.**

Proof of Proposition 8.

(i) From Proposition 3 we know that $\frac{dR}{d\bar{f}} > 0$ and from Proposition 7 we know that $\frac{d\bar{f}}{d\alpha} > 0$. So the indirect effect of an increase in α is positive. From Proposition 3 we know that $\frac{dR}{d\alpha} < 0$. So the overall effect depends up on which effect is dominant.

(ii) From Proposition 7 we know that $\frac{d\bar{f}}{dq} \le 0$ for $\hat{q} \le q < 1$. From Proposition 3 we know that $\frac{dR}{d\bar{f}} > 0$. Hence the indirect effect is negative. From Proposition 3 we also know that $\frac{dR}{dq} < 0$ which means that the direct effect is also negative. Thus an increase in *q* in the interval $\hat{q} \le q < 1$ reduces *R*. Let us consider the interval $q < \hat{q}$. From Proposition 7 we know that $\bar{f}$ is concave in q, which that $\bar{f}$ has the highest slope in the neighbourhood of $q = 0$. Let us evaluate $\frac{dR}{dq}$ in the neighbourhood of $q = 0$. In this neighbourhood $\frac{d\bar{f}}{dq} = \frac{2A-1}{4}$ and $\bar{f} = 0$. Thus we get $\frac{dR}{dq} = \frac{dR}{d\bar{f}}\frac{d\bar{f}}{dq} + \frac{dR}{dq} = \frac{\alpha(2A-3)}{8(\alpha+1)^2} < 0$ because $A < 1$ which we know from Lemma 1. **Q.E.D.**