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# TAXPAYERS'CHOICES UNDER STUDI DI SETTORE: WHAT DO WE KNOW AND HOW WE CAN INTERPRET IT?

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*Studi di settore* (*Sds*) can be seen as a procedure that is midway between mechanisms of audit selection and methods of normal taxation. Its impact depends both on the efficiency of the audit selection criteria as well as on the strength of the political compromise that is built-in. This paper has three purposes. The first is to construct a simple model of the firm's choice under *Sds*. The second is to use the model to discuss the stylized facts emerging from the implementation of *Sds* in the period 1998-2004. The third is to provide some policy indications to evaluate recent amendments to *Sds*.

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## 1. INTRODUCTION

"Studi di settore" (*Sds*, or 'business sector analyses') were introduced in Italy in 1998; however, their nature and interpretation is rather controversial (Arachi-Santoro, 2007). *Sds* are basically audit selection mechanisms: a small or medium sized firm (which is the taxpayer, TP, in this paper) can be audited if it reports a level of turnover (sales proceeds) that is lower than the presumed level. The latter depends primarily on the value of inputs as reported by the firm.[2] However, the presumed level also depends on the av-

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[2] Thus, *Sds* are based on an endogenous threshold, while the optimal audit scheme (with commitment), suggested in the literature, requires an entirely exogenous threshold (Andreoni et al., 1998; Sanchez and Sobel, 1993). It would be interesting to explore theoretically whether the differences between *Sds* and the optimal audit procedures suggested in the literature are actually justified by the specific context in which *Sds* are designed, such that *Sds* may in fact be conditionally optimal. In this paper, however, we propose that such a design, rather than being inspired by the search for an optimal auditing scheme, can be understood by considering it not just as an audit selection mechanism.

erage productivity of the firms within the same business sector. Thus, *Sds* can also be regarded as indirect methods of normal or presumptive taxation similar to those applied in many other countries, although no presumptive tax is formally levied. Finally, there is another interpretation of *Sds* that is grounded in political economy. Some organizations representing small and medium sized firms, which we will call 'TP's representatives', are deeply involved in the process of calculating the presumed level of turnover and in the implementation of *Sds*, as we describe in Section 2. Thus, many argue that *Sds* should be seen as the outcome of a political compromise between small and medium firms and the Tax Agency.

To sum up, *Sds* can be seen as being midway between application of audit selection mechanisms and methods of presumptive (normal) taxation whose impact depends on the efficiency of the audit selection criteria as well as on the strength of the built-in political compromise. In this paper we model the TP's choice based mainly on the first line of interpretation (*Sds* as audit selection mechanisms) but, in order to try to explain the stylized facts, we make reference in the last part of the paper to a more 'political' argument.

This paper has three main objectives. First, to construct a simple model of firm's choice under *Sds*; second to use this model to discuss the stylized facts emerging from the implementation of *Sds* in the period 1998-2004 and third, to formulate some policy suggestions.

The paper is structured as follows. In Section 2 we describe the main features of *Sds* and highlight the stylized facts emerging from their implementation in the period 1998-2004. In Section 3 we construct a simple model of the TP' choice under *Sds* on the basis of Scotchmer (1987) and Cowell (2003). In Section 4 we provide some theoretical findings, which are discussed in Section 5. Section 6 reports the results of a numerical simulation based on actual data in order to try to capture the main features of the 1998-2004 period. Section 7 provides some concluding remarks on the policy implications of the paper.

## 2. Description of and some stylized facts relating to Sds

*Sds* are audit selection mechanisms based on a sophisticated statistical procedure that signals firms that report an "implausibly low" level of turnover. *Sds* were introduced in 1998 after lengthy debate and, since then, have progressively grown in importance; in fiscal year 2004 70% of Italian firms, i.e. around 4 million taxpayers, were eligible to be audited on the basis of *Sds*.

We describe below a typical *Sds* for a given business sector. Initially, data are collected from all firms (corporated and unincorporated companies, individual entrepreneurs, self-employed people) reporting an annual turnover not greater than 5.164.569 euros. Data include structural variables- such as surface area of offices and warehouses, number of employees, type of customers and so on- and accounting variables, mainly costs. Principal components analysis (PCA) is applied in order to select from all those collected, the structural variables that are statistically the most significant. These variables are then used to construct the clusters, the key element of the statistical procedure. Specifically, all the firms belonging to a cluster are homogeneous with respect to the structural variables selected by PCA.

Having defined the concept of a cluster we now briefly illustrate how *Sds* work. *Sds* can generate two types of audits. A firm is liable for a type I audit if and only if it reports a turnover value that is lower than the presumed (normal) level. The presumed level of turnover is calculated as the product of a vector of values reported by the firm, and their corresponding parameters. The values refer to a set of independent variables, which are statistically associated with turnover; in this paper we define these variables as the relevant (independent) variables. Essentially, the relevant independent variables are physical and economic inputs. For example, for the real estate sector, the main relevant independent variables are square metreage and number of rented houses and buildings, the value of capital goods, the cost of services, and the cost of labour. The parameters, resulting from an econometric investigation, reflect the average relationship between the relevant independent variables and turnover, for a subset of firms belonging to the same cluster and satisfying a given 'consistency criterion'. This criterion, in turn, is based on the cumulative distribution of indicators such as value added per worker, inventory turnover and the ratio of sales to the book value of capital assets.

In this paper, the presumed level of turnover is denoted by $\beta\hat{X}_i$ where $\hat{X}_i$ is the vector of the relevant independent variables and $\beta$ is the associated vector of parameters. This value has to be compared with the reported turnover, which we denote with $\hat{R}_i$. If the firm reports a turnover value that is lower than the presumed value and it is actually audited, the penalty that can be applied is largely discretionary since it is the outcome of a sort of bargaining process between the Tax Agency and the TP (it belongs to the procedure known as *accertamento con adesione*). This is a notable change with respect to what is commonly assumed in theoretical models, i.e. that penalties are set by the law and known by every taxpayer. Setting aside the possibility of imperfect knowledge of the legal rules, the important point here is that the penalty in type I audits depends on a number of variables such as the attitude of the responsible tax officer, the ability of the TP to argue that $\hat{R}_i < \beta\hat{X}_i$, the local office's revenue target and so on.

In type II audits, the firm may be audited on the difference between reported values of relevant independent variables and their true value. Type II audits are the logical counterpart of type I audits. Clearly, if $\beta > 0$, firms can escape type I audits by simply underreporting $X$. Therefore, reports of $X$ should be audited. However, for some years, this simple reasoning has not prevailed. Tax authorities, taxpayers and their representatives have focused almost exclusively on type I audits. Until 2004 type II audit activity was virtually zero and this helps to explain TP's behaviour in the period 1998-2004 demonstrated in this paper.

The two types of audits are obviously intertwined, but intrinsically different. Type I audits are about the difference between TP's presumed and reported turnover. There can be a number of reasons for this difference, such as a temporary halt in production, a change in the structure of the market, indisposition of the entrepreneur (recall that we are dealing with small and medium sized enterprises – SMEs), and so on. Type II audits are simpler, since they are based exclusively on the difference between the actual and the reported value of , although if such the audit has a positive outcome the TP's entire tax liability must be recalculated.

What is the role of the TP's representatives in this process? Their role is a prominent one. These representatives are gathered together in an expert committee – the *Comitato degli Esperti* – which is required to pass opinion on the ability of every *Sds* to actually represent the economic reality of the business sector to which it is applied. Although this opinion does not represent a formal commitment, and a single *Sds* may be enacted even if the TP's representatives are not in agreement with it, it is politically very important and, so far, all *Sds* have been 'approved' by this committee. Moreover, it is well known that TP's representatives are asked to state their views about the different steps of the procedure, such as the choice of data being demanded, the selection of clusters and relevant (independent) variables and so on. In sum, TP's representatives are closely involved in the *Sds* calculation and in the implementation process.

We next present some stylized facts (hereafter referred to as SFs). Table 1, which is derived from Agenzia delle Entrate (2007), shows that the percentage of firms liable for a type I audit has declined in the period 1998-2004.

This means that a growing percentage of firms have reported a turnover not lower than the presumed one (SF #1). At first sight this trend would seem to confirm the effectiveness of *Sds* in inducing compliance with the tax system since it should mean that, over time, firms have increased their average reported levels of turnover. However, there are some additional facts that need to be considered. These include evidence supporting the idea that firms are inaccurately reporting the values of the rel-

TABLE 1 – *Percentage of Italian firms liable for type I audit* *

| YEAR | PERCENTAGE OF AUDITABLE FIRMS^ |
|---|---|
| 1998 | 51% (51%) |
| 1999 | 47% (47%) |
| 2000 | 40% (42%) |
| 2001 | 37% (40%) |
| 2002 | 33% (36%) |
| 2003 | 29% (33%) |
| 2004 | 31% (35%) |

* *Source:* Agenzia delle Entrate (2007), Figura 3 p.5.
^ In parentheses the % of firms auditable among those for which the *Sds* was enacted since 1998 (approx. 40% of total).

evant independent variables, thus manipulating (lowering) the presumed level of turnover. Table 2 (Pisani, 2004, p.13) shows the average presumed level of turnover for a subset of 1,5 million firms (i.e 40% of the total) where *Sds* have been applied since 1998.

TABLE 2 – *Average presumed level of turnover for a subset of 1,5 millions of firms* *

| YEAR | AVERAGE PRESUMED LEVEL OF TURNOVER (IN EUROS) |
|---|---|
| 1998 | 125.529 |
| 1999 | 128.883 |
| 2000 | 123.779 |
| 2001 | 127.715 |

* *Source:* Pisani (2004), Tabella 2.2 p. 13. The presumed level of turnover is calculated as the product of a vector of values reported by the firm, and of their corresponding parameters.

According to Pisani (2004) these figures cannot be explained on the grounds of economic cycles. For given values of the parameters[3], it seems likely that the values of the relevant independent variables have been massively manipulated in order to keep the presumed level constant (in real terms) or even to reduce it over time. This idea is supported by strong anecdoctical evidence, with many tax practitioners admitting to having "played with Gerico", the software used to apply *Sds*, as well as by recent results of

[3] As explained above, the parameters are actually derived from econometric estimation, i.e. OLS regressions for the subset of firms satisfying the 'consistency criterion'. Strictly speaking, therefore, these parameters are endogenous and could be manipulated within collusive agreements. It is however reasonable to assume that the subset of 'consistent firms' is so large as to prevent collusion so that parameters are treated as exogenous.

a statistical analysis (whose main results are available on the Italian government's website). The manipulation of the relevant independent variables can thus be looked on as the second stylized fact (SF #2).

The third SF is that the percentage of firms that have actually been subjected to type II audits was negligible in the period 1998-2004. In particular, no specific monitoring activity on the values of *X*'s reported by taxpayers was either announced or planned by the Tax Agency up to 2004. As a consequence, we can assume that the expected penalty was perceived as being very low for the entire period 1998-2004 (SF #3).

## 3. The model

The model used is based on a combination of the models proposed by Scotchmer (1987) and Cowell (2003), adapted to take account of the legal and institutional framework of the design and implementation of *Sds*.

The TP is a risk-neutral firm which aims at minimizing the amount of its expected tax liability (as in Scotchmer, 1987) gross of the concealment cost *G* generated by tax evasion. The idea (Cowell, 2003) is that tax-evasion is a costly activity since it entails organizational costs (manipulation of current accounts, implementation of a collusion agreement between employers and employees) and possibly also psychological costs. In Cowell (2003) the crucial feature *G* of is its convexity with respect to the amount of tax evasion. The assumption of increasing marginal concealment cost enables some interesting results even when risk-aversion is not explicitly accounted for. Also, we note that the sign of the second derivative of *G* plays an important role in our model.

We follow the literature on tax evasion by firms (see Myles, 1997 for a summary) by assuming proportional taxes. This implies that our model may apply to taxes such as Ires (the Italian tax on corporations) and Irap (the Italian tax on value added) but generally not to Irpef (the Italian tax on individuals, including unincorporated businesses and self-employed people). However, the analysis does apply to Irpef if the change in the tax base holds the TP within the same bracket.

We depart from the literature on tax evasion to specify the audit function. The usual assumption in the literature on optimal audits (Andreoni et al., 1998, Sanchez and Sobel, 1993) is that audits are aimed at detecting the true level of profits, but this is not the case in the *Sds* legal structure.

As explained in Section 2, there are two possible types of audits based on *Sds*.

A type I audit may be applied when the turnover reported by the TP is lower than the presumed (normal) level of turnover. This latter depends in part on a set of relevant variables as reported by the TP and in part on the features of the economic sector to which the TP belongs. These features are captured by a set of parameters. The application of these parameters to the corresponding set of reported independent variables generates the presumed level of turnover for the TP. To simplify the notations, and without loss of generality, we consider only one reported variable and the value of its corresponding parameter. The type I audit function can then be expressed by $q(\hat{R}_i < \beta\hat{X}_i)$ where $\hat{R}_i$ is the reported turnover, $\beta > 0$[4] is the parameter and $\hat{X}_i$ is the reported level of the relevant variable. We now briefly describe the main properties of $q(.)$.

There are two main legal and institutional constraints concerning type I audits. First, when the turnover is (at least) equal to the presumed level, the TP is not liable for a type I audit. Second, the probability of being audited is "small" when the difference between $\hat{R}_i$ and $\beta\hat{X}_i$ is also not large.[5] To model these constraints in a proper manner, we assume that, from the viewpoint of the TP, the audit function takes the following specification

$$\begin{gathered} q(\hat{R}_i / \beta\hat{X}_i) = \frac{1}{\delta} - \frac{1}{\delta}\frac{\hat{R}}{\beta\hat{X}_i}, \hat{R} \leq \beta\hat{X}_i \\ q = 0, \hat{R}_i > \beta\hat{X}_i \end{gathered} \tag{1}$$

In other words, we assume a linear decreasing audit function satisfying $q(1) = 0$ where $\delta$ is inversely related to the steepness of the type I audit function: a smaller $\delta$ means a steeper type I audit function, and viceversa.

Type II audits may be based on the difference between the true and the reported levels of the relevant variables.[6] Since there are no explicit legal constraints, we just assume that there is a nonnegative constant probability $p$ of

[4] The assumption $\beta > 0$ is plausible since the overwhelming majority of relevant variables are positively correlated with turnover.

[5] To be more precise, the law states that a type I audit can be conducted only if the difference between $\hat{R}_i$ and $\beta\hat{X}_i$ is serious ("grave"). In practice this criterion has lead to differentiate between taxpayers reporting $\hat{R}_i < \beta\hat{X}_i - CV$ and taxpayers reporting $\beta\hat{X}_i > \hat{R}_i > \beta\hat{X}_i - CV$, where $CV > 0$ is a confidence value associated to the econometric estimation of $\beta$ (see Section 2). The probability to be audited is higher in the the former case. Strictly speaking, this assumption would lead to a bracket-structure of $q(.)$. However, we assume here that $q(.)$ is continuous since the Tax Agency is believed to audit more frequently TPs reporting a larger value of $\beta\hat{X}_i - \hat{R}_i$. This is plausible given that the Tax Agency has an incentive to focus on more productive audits to meet its revenue targets.

[6] We take the relative rather than the absolute difference since we have assumed $\beta > 0$ so that reporting $\hat{X}_i > X_i$ is never profitable (it would increase both the expected penalty of a type II audit and the difference $\beta\hat{X}_i - \hat{R}_i$).

a type II audit and that the corresponding penalty applies to the weighted difference between the true and the reported level of the relevant variable, i.e to $\beta(X_i - \hat{X}_i)$.

Finally, we embody the concealment cost in the analysis as $G = G(X_i - \hat{X}_i)$ where $G' > 0$ since the TP has to modify its current accounts (if $X_i$ is an accounting variable) or the structure of its firm (if $X_i$ is a structural variable which we assume can be measured) in the event of a type II audit. We also adopt Cowell's (2003) assumption of convexity, thus $G'' > 0$.

To sum up, the TP minimizes his total expected payment (EP) defined as

$$EP = \tau\left(\hat{R}_i - \hat{C}_i\right) + q(.)(1+f_1)\tau\left(\beta\hat{X}_i - \hat{R}_i\right) + p(1+f_2)\tau\beta\left(X_i - \hat{X}_i\right) + G(X_i - \hat{X}_i) \quad (2)$$

with respect to $\hat{R}_i$ and $\hat{X}_i$, where $\hat{C}_i$ denotes the reported costs that are different from the variable $X_i$,[7] $f$'s are unitary penalties for the two types of audits and $\tau$ is the proportional tax rate.

Some comments concerning (2) are in order. First, one might think that the assumption of convexity of $G$(.), although common in the literature, is not appropriate here and, more specifically, that there may be increasing returns to scale in tax evasion. Second, and relatedly, it could be argued that underreporting turnover is also costly, so that concealment costs would depend also on the difference between $\hat{R}_i$ and $\beta\hat{X}_i$. Let us tackle these two issues together.

In a conventional audit procedure, based on the difference between reported turnover $\hat{R}_i$ and true turnover $R_i$ economies of scale would almost certainly emerge. To conceal turnover it is necessary to instruct workers appropriately and to keep 'black' accounting. Once initiated, these activities are likely to have low or zero marginal cost. However, type I audits are not focused on the true turnover $R_i$, but rather on the difference between $\hat{R}_i$ and $\beta\hat{X}_i$. In other words, choosing a given level of turnover to report is not associated with the concealment of actual turnover, and, within some limits, there is no difference in the concealment cost if a high or a low level of $\hat{R}_i$. is chosen. This explains why  does not depend on the difference between $\hat{R}_i$ and $\beta\hat{X}_i$.

We now need to explain why we retain the assumption of convexity of $G$(.). This is associated with the nature of $X$'s as either accounting or structural variable. The manipulation of an accounting variable, in order to be credible, should be accompanied by manipulation of other variables. For ex-

[7] If the variable $X_i$ is a cost, then this cost is not included in the vector $\hat{C}_i$. Note also that $\hat{C}_i$, strictly speaking, is a choice variable. However it is clear from the expression of EP that the TP will select the highest possible value of $\hat{C}_i$.

ample, to credibly manipulate (underreport) the value of capital goods, the value of depreciation must be decreased. However, the accounting structure is, to some extent, rigid, so that the cost of manipulation, although perhaps not continuously, is increasing in the amount of manipulation. On the other hand, some structural variables are physical inputs whose concealment may also be increasingly costly (although again not necessarily in a continuous manner). Take, for example, a restaurant. Here, an important structural variable would be the number of tables used. It is plausible to assume that concealing the first $n$ tables is less costly than concealing the $n + 1th$, when, for example, there is no available space to hide the $n + 1th$ table in the event of an audit.

Finally, the penalty in the case of type II audit is calculated on the basis of $\tau\beta$ rather than on the difference $(X_i - \hat{X}_i)$ to reflect the fact that when type II audit is conducted tax officers are required to recalculate the whole tax liability.

## 4. THE RESULTS

Differentiating (2) with respect to $\hat{R}_i$ and $\hat{X}_i$ yields

$$\frac{\partial EP}{\partial \hat{R}_i} = \tau\left[1 - q(1+f_1)\right] + q'_R(1+f_1)\tau\left(\beta\hat{X}_i - \hat{R}_i\right) \tag{3}$$

$$\frac{\partial EP}{\partial \hat{X}_i} = \tau\beta\left[q(1+f_1) - p(1+f_2)\right] + q'_X(1+f_1)\tau\left(\beta\hat{X}_i - \hat{R}_i\right) - G'(X_i - \hat{X}_i) \tag{4}$$

where the partial derivatives of $q$ with respect to $\hat{R}_i$ and $\hat{X}_i$ are denoted respectively by $q'_R$ and $q'_X$.

Using (1) as the specification for $q(.)$ we obtain (see Appendix)

$$\hat{R}_i = \beta\hat{X}_i\left[1 - \frac{\delta}{2(1+f_1)}\right] \tag{5}$$

which is the necessary and sufficient condition for the tax-minimizing $\hat{R}_i$, given $\hat{X}_i$. In accordance with the intuition, for a given $f_1$, the steeper the $q(.)$, i.e. the smaller the $\delta$, and the closer $\hat{R}_i$ would be to $\beta\hat{X}_i$. This means that, if $\hat{X}_i$ were not manipulable, (5) would provide the solution to the TP's problem.

However, $\hat{X}_i$ is manipulable and, using (1), its optimal value satisfies (see Appendix)

$$G'(X_i - \hat{X}_i) = \tau\left[\beta\left(\frac{1}{\delta}(1+f_1) - p(1+f_2)\right) - \frac{1}{\delta\beta}\tau(1+f_1)\left(\frac{\hat{R}_i}{\hat{X}_i}\right)^2\right] \tag{6}$$

The interpretation of (6) hinges on the convexity of $G$. If $G$ is convex, as postulated by Cowell (2003) then $\hat{X}_i$ decreases when the value of $G'$ increases. In turn, according to (6), this value depends on two terms.

The expression within square brackets is related positively to the weighted difference between the unitary penalties for the two types of audits. The weights are given, for the penalty of type I audit, by $1/\delta$, i.e. a measure of steepness of the audit function, and for the penalty of type II audit, by the constant probability $p$. For a given $p$, the steeper the type I audit function, the higher the optimal value of $G'$ and the lower the optimal value of $\hat{X}_i$. In other words, for a given $\hat{R}_i$, this term captures the manipulation of $\hat{X}_i$: in response to a steeper $q(.)$, the TP simply lowers $\hat{X}_i$ in order to decrease the probability of a type I audit (for a given $\hat{R}_i$) until the marginal reduction in expected taxation is equal to the marginal increase in concealment costs.

The interpretation of the second term on the RHS of (6) is less immediate, since it is the weighted square of the ratio between $\hat{R}_i$ and $\hat{X}_i$. To find the optimal value of $\hat{X}_i$ it is necessary to substitute (5) in (6). The conditions for an internal optimal solution are the following (see Appendix)

$$G'(X_i - \hat{X}_i) = \tau\beta\left(1 - \frac{\delta}{4(1+f_1)} - p\,(1+f_2)\right) \tag{7}$$

$$1 - \frac{\delta}{4(1+f_1)} > p\,(1+f_2) \tag{8}$$

In (7) we find again a direct relationship between $\delta$, i.e the steepness of the type I audit function, and the optimal value of $\hat{X}_i$: the smaller the $\delta$, i.e the steeper the type I audit function, the higher is the optimal value of $G'$ and thus the lower (under convexity of $G$) is the optimal value of $\hat{X}_i$. If, on the contrary, the value of $p(1 + f_2)$ is so high that (8) does not hold, then the TP would find it optimal to report $\hat{X}_i = X_i$ regardless of $\delta$ and provided the marginal concealment cost is also non-negative.

These relationships can be grasped more easily by obtaining explicit solutions for $\hat{X}_i$ and $\hat{R}_i$ for a particular case. Suppose that the concealment cost function has the following specification:

$$G(X_i - \hat{X}_i) = (X_i - \hat{X}_i)^{\alpha}, \alpha > 1 \tag{9}$$

Differentiating (9) and using the result in (7) the optimal value of $\hat{X}_i$ that we denote by $\hat{X}_i^*$, is

$$\hat{X}_i^* = X_i - \left[z(\delta)/\alpha\right]^{1/(\alpha-1)}, z(\delta) \equiv \tau\beta\left(1 - \frac{\delta}{4(1+f_1)} - p(1+f_2)\right) \tag{10}$$

Since $\hat{X}_i^*$ is positively correlated with $\delta$, the TP tends to underreport more (less) the value of $X_i$ when $\delta$ is smaller, i.e. when the type I audit function is steeper (flatter).[8]

Using (10) in (5), the optimal value of $\hat{R}_i$ that we denote by $\hat{R}_i^*$, is

$$\hat{R}_i^* = \beta\left[X_i - \left[z(\delta)/\alpha\right]^{1/(\alpha-1)}\right]\left[1 - \frac{\delta}{2(1+f_1)}\right] \tag{11}$$

provided that (8) and $\hat{R}_i \leq \beta\hat{X}_i$ hold, i.e that

$$\delta < \min\left[\theta, 2(1+f_1)\right],\ \theta \equiv 4(1+f_1)\left[1 - p(1+f_2)\right]. \tag{12}$$

In sum, because of the convexity of a steeper type I audit function, i.e. a smaller $\delta$, has two opposite effects when $p(1+f_2)$ is sufficiently low: it tends to increase the ratio $\hat{R}_i/\beta\hat{X}_i$ and to decrease the absolute value of $\hat{X}_i$. The hypothesis of the perception by the TP of a type I audit function getting steeper over time provides an interesting insight into the interpretation of the stylized facts reported in Section 2. Under this hypothesis, the increase in the ratio $\hat{R}_i/\beta\hat{X}_i$ can help explain SF#1 (growing percentage of firms reporting $\hat{R}_i \geq \beta\hat{X}_i$), while the decrease in absolute values of $\hat{X}_i$ helps to explain SF#2 (manipulation of presumed level of turnover) and the assumption of a low value of $p(1+f_2)$ seems to correspond to SF#3. We pursue this interpretation in the following Section.

## 5. A TENTATIVE EXPLANATION OF THE STYLIZED FACTS

We can now suggest an interpretation of the stylized facts described in Section 2. Consider two types of TP, an optimist and a pessimist, who have to make their choices in two periods. In period 1, the optimistic type is characterized by $(\delta^0, f_1^0)$ while the pessimistic type is characterized by $(\delta^p, f_1^p)$ with $\delta^0 > \delta^p$ and $f_1^0 < f_1^p$. The inequality $\delta^0 > \delta^p$ reflects the fact that the pessimistic type expects the type I audit function to decrease more rapidly in the ratio $\hat{R}_i/\beta\hat{X}_i$ than what is believed by the optimistic type. The inequality

[8] On the other hand, for a given $\delta$ a higher value of $\alpha$ implies a steeper marginal concealment cost and this pushes the TP to moderate underreporting. See Section 6 for more details on these results.

$f_1^0 < f_1^p$, on the other hand, reflects different expectations about the magnitude of the penalty, with the pessimistic type expecting a higher penalty (see Section 2 on the interpretation of the type I penalty as a bargain between the Tax Agency and the TP). Using (5) it is easy to see that if the pessimistic view (with respect to the previous period) is revealed to be the correct one, the optimistic type may be induced to change his expectations and thus his choices of $\hat{R}_i$ and of $\hat{X}_i$ for the second period.

We are now provided with a possible explanation for SF#1 (growing percentage of firms reporting $\hat{R}_i \geq \beta\hat{X}_i$) and SF#2 (manipulation of presumed level of turnover): over time, increased pessimism among TPs has persuaded an increasing number of them to expect a "tougher attitude" from the Tax Agency, i.e. a smaller $\delta$ and a steeper type I audit function. This, in turn, according to the results of our model, might have prompted TPs to react by reducing $\hat{X}_i$ given the low value of $p(1 + f_2)$ (SF#3), and increase the ratio $\hat{R}_i/\beta\hat{X}_i$.

One problem with this explanation is that it is not clear why and how the pessimistic expectation could have been revealed to be the correct one in the period observed (1998-2004), therefore it is not clear why and how it should have been endorsed by a growing number of TPs. The most natural channel of ex-post revelation of the correct value of $\delta$ would be the percentage of type I audits actually conducted by the Tax Agency. But this percentage was not revealed until recently and we now know that it has not increased over time[9] so that it is difficult to make a case for a simple process of rational learning by taxpayers.

The increased pessimism might, however, be the outcome of a coordination game between taxpayers. Suppose that the Tax Agency in a given period can run a fixed number of audits, possibly defined at the local level. If every TP expects other TPs to report a turnover at least equal to the presumed one, every taxpayer may find it profitable to decrease the probability of a type I audit by increasing the ratio $\hat{R}_i/\beta\hat{X}_i$.

An alternative channel of the dynamics of $\delta$ may be associated with the role of tax practitioners and tax consultants, with particular emphasis on TP's representatives (see the Introduction) since these organizations act as tax consultants for their members. To understand why TP's representatives may find it reasonable to suggest 'pessimism' we shall recall that TP's representatives are actively involved in the elaboration of *Sds*. This involvement is a source of political power and status, so that it is plausible that TP's representatives have made an effort to convince their members of the desir-

[9] Data provided by the Tax Agency to the Italian Parliament in June 2007 show that the % of audits has remained approximately constant at around 5% from 1999 to 2002.

ability of complying with the *Sds* by reporting $\hat{R}_i = \beta\hat{X}_i$. An emphasis on the 'toughness' of the Tax Agency, and thus $\delta = \delta^p$ may serve this purpose.[10] This suggests that the business sectors where the majority of TPs are members of the organizations acting as TP's representatives, should have shown higher rates of compliance with respect to less cohesive sectors. On the other hand, in these highly unionized sectors a stronger lobbying effort displayed by TP's representatives could be associated with a lower value of the threshold $\beta\hat{X}_i$ and/or with greater manipulability of $X$'s. Although no data on differences in compliance and thresholds among business sectors have been disclosed so far, the idea that the built-in flexibility of the *Sds* procedure may led to discrimination among taxpayers belonging to different sectors has been indirectly acknowledged by the Ministry itself.[11] Thanks to the publication of a tax file reporting microdata on *Sds*, it should be possible, in the very near future, to test empirically these hypotheses.

## 6. Numerical simulation

In this Section we present a numerical simulation for the real estate sector. We assume that $G(.)$ is specified as in (9) so that $\hat{X}_i^*$ and $\hat{R}_i^*$ are given respectively by (10) and (11), under (12). We take the values of $\beta$ and of $X_i$ from the study of sector SG40U (*attività immobiliari*). More precisely, we identify $X_i$ as the variable 'square metreage of rented buildings' since the 'regional analysis' *(analisi di territorialità*) indicates that this is the most important regressor among those selected in the study. This implies $\beta$ =7,04 while we assume $X_i$ =1000, so that we are applying our approach to a firm which rents 1000 squared metres.

For reasons that will shortly become apparent, we first discuss plausible values for the policy variables concerning type II audits, i.e $f_2$ and $p$.

If we consider that, in general, the probability of a substantial audit (i.e of an audit concerning the accuracy of the data reported by the TP) is around 5%, it seems reasonable to assume that

$$1\% \leq p \leq 10\% \tag{13}$$

[10] This pessimism could have gradually spread to the entire population of tax consultants through a sort of 'contagion process' (Morris, 2000).

[11] The Commission for the revision of the Sds (so called Commissione Rey from its President'name)has just released a report that stresses that the arbitrariness in the definition of the 'consistency criterion' (see Section 2) may have led to discrimination among different economic sectors (and among clusters).

With regard to $f_2$ we should remember that a type II audit is sanctioned, in general, with unitary sanctions going from 100% to 200% of the tax evaded as a consequence of the manipulation (see dlgs 471/1997 article 5, c.4). Thus, in general, we can assume

$$1 \le f_2 \le 2 \tag{14}$$

We now turn to a discussion of the plausible values of the policy variables concerning type I audit, i.e. $f_1$ and $\delta$.

A type I audit is analysed taking into account the rules of the so-called *accertamento con adesione*. This is a sort of bargaining process between the TP and the Tax Agency which: i) ensures a discount of 25% on the ordinary sanctions (ranging between100% and 200% of the amount due); ii) allows the Tax Agency to grant additional 'discounts' that may even generate a 'negative sanction'. In other words, the procedure may end up with a TP paying less than the amount originally due. To take this into account we assume that

$$-0{,}1 \le f_1 \le 1 \tag{15}$$

On the other hand, there are reasonable constraints concerning $\delta$:

i) $0 \le q \le 1$
ii) $\hat{R}_i^* \ge 0$
iii) $\hat{X}_i^* \in [0, X_i]$.

The Appendix shows that, given equations (13), (14) and (15), i), ii) and iii) are jointly equivalent to

$$1 \le \delta \le 2(1+f_1) \tag{16}$$

The last variable to be simulated is $\alpha$. We assumed above that $\alpha > 1$. We will retain the two values of $\alpha$ : $\alpha = 1{,}5$ (steep marginal concealment cost function) and $\alpha = 1{,}1$ ( flat marginal concealment cost function).[12]

To sum up, our simulations for the real estate sector are characterized by the following features:

i) policy variables are defined by equations (13)-(16);
ii) $\tau = 35\%$; $\beta = 7{,}04$; $X_i = 1000$;
iii) $\alpha$ either equals 1,1 (flat marginal concealment cost) or 1,5 (steep marginal concealment cost).

[12] Although these values are similar they produce remarkable differences in equilibrium values, as we shall see.

To obtain policy insights we consider two scenarios.

i) a scenario in which $p$ and $f_2$ are given while $f_1$ and $\delta$ vary such that the resulting picture is close to the period 1998-2004, which we call the 'past scenario';
ii) a scenario in which $f_1$ and $\delta$ are given while $p$ and $f_2$ vary such that the resulting picture resembles the period beginning 2006, which we call the 'future scenario'.

To characterize the 'past scenario' we need to consider that until 2004 the probability of type II audits was very low. No specific monitoring activity on the values of $X_i$'s reported by taxpayers was either announced or planned by the Tax Agency until 2004. Therefore, for the purposes of the 'past scenario', we can assume that $p = 1\%$. Regarding $f_2$, we assume only an intermediate value, i.e $f_2 = 1{,}5$. As a consequence in the 'past scenario' we have $p(1 + f_2) = 2{,}5\%$.

The values of $\hat{R}_i^*$, for the case of a steep marginal concealment cost ($\alpha = 1{,}5$) are reported in Table 3 for different values of $\delta$ and of $f_1$.

Table 3 outlines the standard picture.[13] In policy terms, expected revenues can be raised by increasing either the probability of a type I audit or associated sanctions. As a consequence, the maximum revenue (5.270 euros)

TABLE 3 – *Values of $\hat{R}_i^*$ for the case of a steep marginal concealment cost ($\alpha = 1{,}5$) in the 'past scenario'**

| $\delta\downarrow f_1\rightarrow$ | 1 | 0,9 | 0,8 | 0,7 | 0,6 | 0,5 | 0,4 | 0,3 | 0,2 | 0,1 | 0 | –0,1 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 5.270 | 5.177 | 5.075 | 4.960 | 4.831 | 4.685 | 4.518 | 4.325 | 4.100 | 3.834 | 3.515 | 3.125 |
| 1,1 | 5.094 | 4.993 | 4.880 | 4.754 | 4.612 | 4.451 | 4.267 | 4.055 | 3.808 | 3.515 | 3.164 | 2.734 |
| 1,2 | 4.919 | 4.808 | 4.685 | 4.547 | 4.393 | 4.217 | 4.017 | 3.785 | 3.515 | 3.196 | 2.813 | 2.344 |
| 1,3 | 4.744 | 4.624 | 4.490 | 4.341 | 4.173 | 3.983 | 3.766 | 3.515 | 3.222 | 2.876 | 2.461 | 1.954 |
| 1,4 | 4.568 | 4.439 | 4.295 | 4.135 | 3.954 | 3.749 | 3.515 | 3.245 | 2.930 | 2.557 | 2.110 | 1.563 |
| 1,5 | 4.393 | 4.254 | 4.100 | 3.928 | 3.734 | 3.515 | 3.264 | 2.975 | 2.637 | 2.238 | 1.758 | 1.172 |
| 1,6 | 4.217 | 4.069 | 3.905 | 3.722 | 3.515 | 3.281 | 3.013 | 2.704 | 2.344 | 1.918 | 1.407 | 782 |
| 1,7 | 4.042 | 3.885 | 3.710 | 3.515 | 3.296 | 3.047 | 2.762 | 2.434 | 2.051 | 1.599 | 1.055 | 391 |
| 1,8 | 3.866 | 3.700 | 3.515 | 3.308 | 3.076 | 2.813 | 2.511 | 2.164 | 1.758 | 1.279 | 703 | 0 |

* *Source:* author's simulation for the real estate sector assuming $X_i = 1000$, $\beta = 7{,}04$, $\tau = 35\%$, $p(1+f_2) = 2{,}5\%$ and $G(.)$ specified as in (9).

[13] Note that, to simplify results, we have taken $\delta = 1{,}8$ as the maximum value since we have $min(2(1 + f_1)) = 1{,}8$. In theory higher values of $\delta$ would be admissible for all $f_1 > -0{,}1$.

is obtained when $\delta = 1$ and $f_1 = 1$ (north-west corner of the matrix) and revenue is linearly increasing in $f_1$ and decreasing in $\delta$ (i.e increasing in the steepness of $q$). The 'growing pessimism' dynamics referred to in previous sections, i.e. a generalized belief that the Tax Agency is increasing the probability of an audit so that $\delta$ is getting lower, should have induced taxpayers with a steep marginal concealment cost to pay more taxes, even if $p(1 + f_2)$ was set at low values.

Things change quite dramatically if the marginal concealment cost becomes flatter, so that $\alpha = 1{,}1$.

TABLE 4 – *Values of $\hat{R}_i^*$ for the case of a flat marginal concealment cost ($\alpha = 1{,}1$) in the 'past scenario'**

| $\delta\downarrow f_1\rightarrow$ | 1 | 0,9 | 0,8 | 0,7 | 0,6 | 0,5 | 0,4 | 0,3 | 0,2 | 0,1 | 0 | –0,1 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1.974 | 2.182 | 2.384 | 2.577 | 2.756 | 2.916 | 3.048 | 3.144 | *3.190* | 3.173 | 3.071 | 2.859 |
| 1,1 | 2.348 | 2.526 | 2.693 | 2.846 | 2.980 | 3.087 | 3.162 | *3.192* | 3.167 | 3.071 | 2.883 | 2.580 |
| 1,2 | 2.639 | 2.784 | 2.916 | 3.028 | 3.116 | 3.174 | *3.192* | 3.162 | 3.071 | 2.903 | 2.640 | 2.258 |
| 1,3 | 2.857 | 2.970 | 3.065 | 3.138 | 3.182 | *3.191* | 3.158 | 3.071 | 2.919 | 2.688 | 2.358 | 1.908 |
| 1,4 | 3.013 | 3.094 | 3.154 | 3.187 | *3.189* | 3.153 | 3.071 | 2.932 | 2.726 | 2.438 | 2.051 | 1.541 |
| 1,5 | 3.116 | 3.165 | *3.190* | 3.187 | 3.149 | 3.071 | 2.944 | 2.758 | 2.503 | 2.165 | 1.726 | 1.162 |
| 1,6 | 3.174 | *3.192* | 3.184 | 3.146 | 3.071 | 2.953 | 2.785 | 2.557 | 2.258 | 1.875 | 1.390 | 778 |
| 1,7 | *3.193* | 3.181 | 3.142 | 3.071 | 2.961 | 2.808 | 2.602 | 2.336 | 1.998 | 1.575 | 1.047 | 390 |
| 1,8 | 3.179 | 3.139 | 3.071 | 2.969 | 2.827 | 2.640 | 2.401 | 2.100 | 1.726 | 1.266 | 700 | 0 |

* *Source:* author's simulation for the real estate sector assuming $X_i = 1000$, $\beta = 7{,}04$, $\tau = 35\%$, $p(1+f_2) = 2{,}5\%$ and $G(.)$ specified as in (9).

Various observations can be made.

First, the behaviour of $\hat{R}_i^*$ depicted in Table 4 is neither linear in $\delta$ nor in $f_1$. We can identify many 'local maxima' i.e values of $\hat{R}_i^*$ which are the maxima for the row and for the column to which they belong. They are reported in italics in Table 4 and they form a sort of diagonal in the matrix. Now, the south-west corner of the matrix is the global maximum (corresponding to approximately 3.193 euros), but all the other local maxima, i.e. numbers in italics along the diagonal, are very close to this value.

In policy terms, this means that to maximize reported revenues when $p(1 + f_2)$ is set at a low value and the marginal concealment cost is flat, the Tax Agency should either announce a high probability of audit or a high sanction (i.e adopt a tough attitude in the bargaining process). In other words, we can say that the two traditional instruments of anti-evasion policies, sanctions and audits, should not be both increased in this case, i.e when the marginal concealment cost is flat and $p(1+f_2)$ is small. The reason for this is that increasing the probability of an audit (or increasing the sanction) has opposite effects on $\hat{R}_i^*$: it decreases $X_i^*$ but it also increases the ratio between $\hat{R}_i^*$ and a given $\beta X_i^*$. Therefore an appropriate anti-evasion policy targeted towards taxpayers who can easily manipulate $X_i$'s (i.e who have flat marginal concealment cost), under a low value of $p(1+f_2)$, should be based on a balance between $\delta$ and $f_1$, since increasing both may be counterproductive. From a different perspective, the results in Table 4 show that the 'growing pessimism' emerging from the analysis in the previous sections, i.e. a generalized belief that the Tax Agency was increasing the probability of an audit which was lowering $\delta$, may have produced the reverse effects in terms of revenues. Such a dynamic would almost inevitably decrease $\hat{R}_i^*$ in this case (flat marginal concealment cost and low value of $p(1+f_2)$) if the value of $f_1$ is high (in the range between 70% and 100%) while the reverse would be true if $f_1$ were low or negative. In other words, increased pessimism may have reduced the taxes paid by TPs who at the same time believed that the Tax Agency was maintaining a tough stance in the bargaining process generating $f_1$. However, it might also have worked to increase the taxes paid by those who believed that a small $f_1$ would emerge.

To sum things up, the 'past scenario' suggests that the seemingly disappointing results of the *Sds* in terms of revenues (see Pisani, 2004, Santoro 2006) may depend, at least in part, on the fact that many TPs who were in a position to easily manipulate $X_i$'s have believed that the probability of a type I audit was increasing and that the Tax Agency was adopting a tough stance in the bargaining process generating $f_1$.

Scenario 2, where $p$ and $f_2$ can vary while $f_1$ and $\delta$ are fixed, can be characterized to describe the period that beginning 2006. The 2007 budget law included a specific rule for the application of $f_2$ to the case of manipulation of $X_i$'s and a special type II audit campaign was announced. This may have caused the values of both $p$ and of $f_2$ to increase. On the other hand, the Tax Agency has revealed that the probability of a type I audit is not particularly high and has given a number of signals that local taxation offices will be likely to be fairly lenient in the bargaining process leading to $f_1$. To put these changes into the context of our framework, let us suppose that $\delta = 1{,}8$ and that $f_1 = 0{,}5$. Finally, measures were adopted to make the manipulation of $X_i$'s more difficult and thus more costly.[14] In Table 5 we report values of $\hat{R}_i^*$ for the case $\alpha = 1{,}5$ (flat marginal concealment cost).

[14] We refer here to *indicatori di normalità economica* (economic normality indicators) whose purpose is, for example, to detect taxpayers altering the values of the cost of inventories and of the values of capital goods.

TABLE 5 – *Values of* $\hat{R}_i^*$ *for the case of a steep marginal concealment cost* ($\alpha$ = 1,5) *in the 'past scenario'**

| $p\downarrow f_2\rightarrow$ | 1 | 1,1 | 1,2 | 1,3 | 1,4 | 1,5 | 1,6 | 1,7 | 1,8 | 1,9 | 2 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1% | 2.812 | 2.812 | 2.813 | 2.813 | 2.813 | 2.813 | 2.813 | 2.813 | 2.813 | 2.813 | 2.813 |
| 2% | 2.813 | 2.813 | 2.813 | 2.813 | 2.813 | 2.813 | 2.813 | 2.813 | 2.813 | 2.813 | 2.813 |
| 3% | 2.813 | 2.813 | 2.813 | 2.813 | 2.813 | 2.813 | 2.813 | 2.813 | 2.813 | 2.813 | 2.813 |
| 4% | 2.813 | 2.813 | 2.813 | 2.813 | 2.813 | 2.813 | 2.813 | 2.813 | 2.813 | 2.813 | 2.813 |
| 5% | 2.813 | 2.813 | 2.813 | 2.813 | 2.813 | 2.813 | 2.814 | 2.814 | 2.814 | 2.814 | 2.814 |
| 6% | 2.813 | 2.813 | 2.814 | 2.814 | 2.814 | 2.814 | 2.814 | 2.814 | 2.814 | 2.814 | 2.814 |
| 7% | 2.814 | 2.814 | 2.814 | 2.814 | 2.814 | 2.814 | 2.814 | 2.814 | 2.814 | 2.814 | 2.814 |
| 8% | 2.814 | 2.814 | 2.814 | 2.814 | 2.814 | 2.814 | 2.814 | 2.814 | 2.814 | 2.814 | 2.814 |
| 9% | 2.814 | 2.814 | 2.814 | 2.814 | 2.814 | 2.814 | 2.814 | 2.814 | 2.814 | 2.815 | 2.815 |
| 10% | 2.814 | 2.814 | 2.814 | 2.814 | 2.814 | 2.814 | 2.815 | 2.815 | 2.815 | 2.815 | 2.816 |

* *Source:* author's simulation for the real estate sector assuming $X_i$ = 1000, $\beta$ = 7,04, $\tau$ = 35%, $\delta$ = 1,8, $f_1$ = 0,5 and $G$(.) specified as in (9).

Again, as in Table 3, the results in Table 5 are in line with the conventional wisdom. Increasing the probability of a type II audit, $p$, and/or increasing the associated sanction, $f_2$, for given values of $\delta$ and $f_1$, always increases expected revenues. The global maximum (approximately 2.815 euros) is at the south-east corner of the matrix, and $\hat{R}_i^*$ is linearly increasing in both $p$ and $f_2$. Unlike the 'past scenario', things do not change substantially if $\alpha$ = 1,1 (steep marginal concealment cost) so these results are omitted here. What happens is simply that the TPs manipulate $X_i$ more intensively so that $\hat{R}_i^*$ is slightly lower than in Table 5.

## 7. CONCLUDING REMARKS

The simple model of the TP's decision under *Sds* adopted here, suggests that a misperception of the probability of a type I audit, possibly generated by the organizations of SMEs involved in the construction of *Sds*, may explain why a growing number of firms in the period 1998-2004 manipulated the presumed level of turnover that they reported to the Tax Agency, thus lowering the number of firms liable for a type I audit. This explanation is based on the assumption of concealment cost function convexity, and on the very low levels of probability of a type II audit (whose object are the values of the independent variables relevant to the calculation of the presumed level of turnover).

Since misperception is a market failure, the first, obvious, policy prescription would be to give TPs a 'correct perception' of the risk by means of an appropriate auditing policy. Is a policy where the Tax Agency adopts a "tough" stance, the most appropriate? Our model suggests that it may not be. We have shown, in particular, that the seemingly disappointing results of the *Sds* in terms of revenues (see Pisani, 2004, Santoro 2006) may depend, at least in part, on the fact that many TPs in a position to easily manipulate $X_i$'s believed that the probability of a type I audit was increasing and/or that the Tax Agency was taking a tough line in the bargaining process generating the penalty $f_1$.

This result is conditional upon the very low probability of a type II audit. Increasing the probabilities of and possibly also the sanctions related to type II audits would change the picture quite dramatically, reducing the scope for the manipulation of $X_i$. It is difficult to understand why such a seemingly simple policy was not implemented until recently. The answer does not seem to be related to the legal and institutional features of this type of audit. As we have seen, there is a legal framework on which type II audits can be based, and a special type of audit (*accesso breve*) exists, which is particularly suitable. One explanation is related to the nature of *Sds* as a political compromise: the Tax Agency was somehow persuaded that the true values of the independent variables would be spontaneously revealed based on the fact that the TPs' representatives were involved in the elaboration of *Sds*. This belief, however, is not consistent with the evidence. The 2007 Italian Budget Law seems to be moving along new lines. On the one hand, it introduces new parameters (so-called *indicatori di normalità economica*) which can be seen as an attempt to make it more difficult (costly) to manipulate $X$. On the other hand, it dictates a sharp increase in the number of both type I and type II audits. It will be interesting to watch the outcome of these policies in future years.

Another, less obvious, policy prescription arising from our analysis is that the type I audit policy and thus the value of $\delta$ should vary according to the behaviour of the marginal concealment cost function. The marginal concealment cost is likely to be positively related to the size of the firm since bigger (in relative terms) companies need accurate accounting for internal auditing, and to the degree of 'aversion to evasion' in the sector/region in which the firm operates. The nature of the variable $X$ may also matter since there are some variables that are more easy to manipulate than others (usually accounting variables, and costs, which are easier to manipulate than structural variables, especially tangibles). There are, or were, lines of evolution within the *Sds* that try, or tried, to take similar factors into account. For example, until recently, the rules under which corporations were audited were different from those applied to unincorporated firms which usually adopt a simplified accounting regime. Also, in the immediate future the role of local differences will be heightened.

The most promising lines of future research are mainly empirical. First, when the data become available, it would be interesting to see whether the hypothesis of inter-sector variability in auditability rates is associated, on the one hand, with different threshold values and, on the other hand, with the 'political strength' of the TP's representatives in the sectors involved. Second, it might be possible to evaluate the impact of *Sds* on the firm structure, especially in sectors where a slight change in $\hat{X}_i$ may have significant consequences in terms of the presumed turnover (e.g. by inducing a firm to shift from one cluster to another).

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## APPENDIX

Note that, given (1) we have

$$q'_R = -\frac{1}{\delta\beta\hat{X}_i} < 0, \hat{R}_i \le \beta\hat{X}_i$$
$$q'_X = -\frac{\hat{R}_i}{\delta\beta\,\hat{X}_i^{\,2}} > 0, \hat{R}_i \le \beta\hat{X}_i$$

We first show how (5) is derived. Using (1) in (3) we obtain

$$\frac{\partial EP}{\partial \hat{R}_i} = \tau\left[1-\left(\frac{1}{\delta}-\frac{1}{\delta}\frac{\hat{R}_i}{\beta\hat{X}_i}\right)(1+f_1)\right]-\frac{1}{\delta\beta\hat{X}_i}(1+f_1)\tau\left(\beta\hat{X}_i-\hat{R}_i\right)$$

so that

$$\frac{\partial EP}{\partial \hat{R}_i}=0 \Leftrightarrow \tau\left\{1-(1+f_1)\left[\left(\frac{1}{\delta}-\frac{1}{\delta}\frac{\hat{R}_i}{\beta\hat{X}_i}\right)+\frac{1}{\delta\beta\hat{X}_i}\left(\beta\hat{X}_i-\hat{R}_i\right)\right]\right\}=0$$
$$\frac{\partial EP}{\partial \hat{R}_i}=0 \Leftrightarrow \tau\left[1-\frac{1}{\delta}(1+f_1)\right]+\frac{2\hat{R}_i}{\delta\beta\hat{X}_i}(1+f_1)\tau=\tau\frac{1}{\delta}(1+f_1)$$
$$\frac{\partial EP}{\partial \hat{R}_i}=0 \Leftrightarrow \frac{2\hat{R}_i}{\delta\beta\hat{X}_i}(1+f_1)\tau=\tau\left[\frac{2}{\delta}(1+f_1)-1\right]$$
$$\frac{\partial EP}{\partial \hat{R}_i}=0 \Leftrightarrow \frac{\hat{R}_i}{\hat{X}_i}=\beta\left[1-\frac{\delta}{2(1+f_1)}\right]$$

Note that this is a sufficient condition for a minimum since

$$\frac{\partial^2 EP}{\partial \hat{R}_i^{\,2}}=\frac{2(1+f_1)\tau}{\delta\beta\hat{X}_i}>0$$

We now show how (6) is derived. From (4) we have:

$$\frac{\partial EP}{\partial \hat{X}_i}=0 \Leftrightarrow G'(X_i-\hat{X}_i)=\tau\beta\left[q(1+f_1)-p(1+f_2)\right]+q'_X(1+f_1)\tau\left(\beta\hat{X}_i-\hat{R}_i\right)$$

Using (1) we can write:

$$\frac{\partial EP}{\partial \hat{X}_i}=0 \Leftrightarrow G'(X_i-\hat{X}_i)=\tau\beta\left[\left(\frac{1}{\delta}-\frac{1}{\delta}\frac{\hat{R}_i}{\beta\hat{X}_i}\right)(1+f_1)-p(1+f_2)\right]$$
$$+\frac{\hat{R}_i}{\delta\beta\,\hat{X}_i^{\,2}}(1+f_1)\tau\left(\beta\hat{X}_i-\hat{R}_i\right)$$
$$\frac{\partial EP}{\partial \hat{X}_i}=0 \Leftrightarrow G'(X_i-\hat{X}_i)=\tau\beta\left[\frac{1}{\delta}(1+f_1)-p(1+f_2))\right]$$
$$-\tau\beta\frac{1}{\delta}\frac{\hat{R}_i}{\beta\hat{X}_i}(1+f_1)+\frac{\hat{R}_i}{\delta\beta\,\hat{X}_i^{\,2}}(1+f_1)\tau\left(\beta\hat{X}_i-\hat{R}_i\right)$$

$$\frac{\partial EP}{\partial \hat{X}_i}=0 \Leftrightarrow G'(X_i-\hat{X}_i)=\tau\beta\left[\left(\frac{1}{\delta}-\frac{1}{\delta}\frac{\hat{R}_i}{\beta\hat{X}_i}\right)(1+f_1)-p(1+f_2)\right]$$
$$+\frac{\hat{R}_i}{\delta\beta\,\hat{X}_i^{\,2}}(1+f_1)\tau\left(\beta\hat{X}_i-\hat{R}_i\right)$$

$$\frac{\partial EP}{\partial \hat{X}_i}=0 \Leftrightarrow G'(X_i-\hat{X}_i)=\tau\beta\left[\frac{1}{\delta}(1+f_1)-p(1+f_2)\right]$$
$$-\tau\beta\frac{1}{\delta}\frac{\hat{R}_i}{\beta\hat{X}_i}(1+f_1)+\frac{\hat{R}_i}{\delta\beta\,\hat{X}_i^{\,2}}(1+f_1)\tau\left(\beta\hat{X}_i-\hat{R}_i\right)$$

$$\frac{\partial EP}{\partial \hat{X}_i}=0 \Leftrightarrow G'(X_i-\hat{X}_i)=\tau\beta\left[\frac{1}{\delta}(1+f_1)-p(1+f_2)\right]$$
$$+(1+f_1)\left[-\tau\beta\frac{1}{\delta}\frac{\hat{R}_i}{\beta\hat{X}_i}+\frac{\tau\hat{R}_i\beta\hat{X}_i}{\delta\beta\,\hat{X}_i^{\,2}}-\tau\frac{1}{\delta\beta}\left(\frac{\hat{R}_i}{\hat{X}_i}\right)^2\right]\Leftrightarrow$$

$$G'(X_i-\hat{X}_i)=\tau\left[\beta\left(\frac{1}{\delta}(1+f_1)-p(1+f_2)\right)-\frac{1}{\delta\beta}(1+f_1)\left(\frac{\hat{R}_i}{\hat{X}_i}\right)^2\right]$$

Finally, note that $\frac{\partial EP}{\partial \hat{X}_i}=0$ is necessary and sufficient for an optimal $\hat{X}_i$ when $G$ is convex since since

$$\frac{\partial^2 EP}{\partial \hat{X}_i^{\,2}}=\frac{2}{\delta\beta}\tau(1+f_1)\left(\frac{\hat{R}_i}{\hat{X}_i}\right)^2\frac{1}{\hat{X}_i}+G''(X_i-\hat{X}_i)>0$$

To derive (7), let us first rewrite (6) as follows:

$$G'(X_i-\hat{X}_i)=\Psi\beta-\frac{\Psi}{\beta}\left(\frac{\hat{R}_i}{\hat{X}_i}\right)^2-p(1+f_2)\tau\beta,\Psi\equiv\tau\frac{1}{\delta}(1+f_1)$$

Then we substitute (5) in (6) and obtain

$$G'(X_i-\hat{X}_i)=\Psi\beta-\frac{\Psi}{\beta}\beta^2\left[1-\frac{\delta}{2(1+f_1)}\right]^2-p(1+f_2)\tau\beta,\Psi\equiv\tau\frac{1}{\delta}(1+f_1)$$
$$G'(X_i-\hat{X}_i)=\Psi\beta\left(1-\left[1-\frac{\delta}{2(1+f_1)}\right]^2\right)-p(1+f_2)\tau\beta,$$
$$G'(X_i-\hat{X}_i)=\Psi\beta\left(1-\left[1+\frac{\delta^2}{4(1+f_1)^2}-\frac{\delta}{(1+f_1)}\right]\right)-p(1+f_2)\tau\beta,$$
$$G'(X_i-\hat{X}_i)=\Psi\beta\left(\left[\frac{\delta}{(1+f_1)}-\frac{\delta^2}{4(1+f_1)^2}\right]\right)-p(1+f_2)\tau\beta,$$
$$G'(X_i-\hat{X}_i)=\tau\beta\left(1-\frac{\delta}{4(1+f_1)}-p(1+f_2)\right)$$

Note that to have an internal solution a positive value for the RHS of this expression is required, i.e.

$$1-\frac{\delta}{4(1+f_1)} > p(1+f_2)$$

If the opposite holds, no manipulation is the optimal strategy for the TP

$$p(1+f_2) > 1-\frac{\delta}{4(1+f_1)} \Rightarrow \frac{\partial EP}{\partial \hat{X}_i} < 0 \Rightarrow \hat{X}_i = X_i$$

Now, we show that the following set of constraints:

$$\begin{gathered} \text{i) } 0 \le q \le 1 \\ \text{ii) } \hat{R}_i \ge 0 \\ \text{iii) } \hat{X}_i \in [0, X_i] \end{gathered}$$

which, adopted in the simulation of Section 6, is equivalent to the following constraint on $\delta$:

$$1 \le \delta \le 2(1+f_1)$$

Given (1) we can write

$$0 \le q \le 1 \Leftrightarrow \frac{\hat{R}_i}{\beta \hat{X}_i} \le 1 \;\&\; \delta \ge \left[1-\min\left(\frac{\hat{R}_i}{\beta \hat{X}_i}\right)\right].$$

Now we note that

$$\left[\hat{R}_i \ge 0 \;\&\; \beta \hat{X}_i \ge 0\right] \Leftrightarrow \min\left(\frac{\hat{R}_i}{\beta \hat{X}_i}\right) = 0$$

so that we can write

$$0 \le q \le 1 \Leftrightarrow \frac{\hat{R}_i}{\beta \hat{X}_i} \le 1 \;\&\; \delta \ge 1$$

Now using (5) we note that

$$\delta \ge 1 \Rightarrow \frac{\hat{R}_i}{\beta \hat{X}_i} \le 1$$

and that

$$\min\left(\frac{\hat{R}_i}{\beta \hat{X}_i}\right) = 0 \Leftrightarrow \delta \le 2(1+f_1)$$

so that constraints i) and ii) can be rewritten jointly as

$$1 \leq \delta \leq 2(1+f_1)$$

We also know that constraint iii) can be rewritten as follows (see above and equation (8) in text):

$$\hat{X}_i(\delta) \in [0, X_i] \Leftrightarrow \left(1 - \frac{\delta}{4(1+f_1)}\right) \geq p(1+f_2) \Leftrightarrow \delta \leq 4(1+f_1)[1 - p(1+f_2)].$$

Then, we can rewrite constraints i)-iii) jointly as

$$1 \leq \delta \leq \min[\theta, 2(1+f_1)], \theta \equiv 4(1+f_1)[1 - p(1+f_2)].$$

Finally, we note that

$$\min[\theta, 2(1+f_1)] = 2(1+f_1) \Leftrightarrow p(1+f_2) < \frac{1}{2}$$

where the last inequality is always verified under(13) and (14).