REPUTATIONAL BIDDING*

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Abstract

We consider auctions where bidders care about the reputational effects of their bidding behavior since each bidder is in a sender-receiver signaling game where bids act as signals. However, not all auctions - Dutch auctions being a simple example - allow for the disclosure of all bids. Accordingly, we focus our analysis on alternative disclosure rules that capture most of the standard possibilities. In this setting, we characterize symmetric and monotone equilibria for standard auction formats, describe conditions for such equilibria to exist and show that what matters for expected revenues is not the price mechanism but the type of information about bids available after the auction. We then proceed to rank our disclosure rules according to their ability to deliver efficient equilibria or, conditional on achieving the latter, the revenues they imply. We find that these ranking can conflict in that a given disclosure rule can be better than another in terms of guaranteeing efficiency but be worse in terms of expected revenue to the seller. In particular, we find that under certain conditions, full disclosure of bids may be bettered by less disclosure in terms of either efficiency or revenues. We also find that first- and second-price sealed-bid auctions with disclosure of price are not equivalent. In fact, these case be ranked depending on the type of reputational effects.

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1. Introduction

It is often the case that auctions are conducted in an environment where bidding behavior has some implications for the bidders' future economic activity. In this paper, we focus on the case where bidders care about their publicly perceived type at the end of the auction, as revealed by their bidding behavior.

This can happen in many different scenarios. For example, consider the case where large corporate firms bid to acquire a technological start-up with a new product. Bidding firms know that the level of interest in this acquisition, as signalled by their bidding behavior, will tell the stockmarket something about the overall managerial strategy pursued by the bidders. Thus, their bidding behavior will influence the perceived opinion stockholders have of the management's competence.

A different example can be one where bidders are contemporary art experts who bid for themselves or on behalf of wealthy patrons, under a (linear) bonus-contract, for a given work of art. They know that their valuation for the work of art being auctioned will be interpreted as an indicator of expertise by potential future clients. These clients will therefore form expectations that will have an impact on the bidders' bidding strategy and those expectations will be affected by the bidder's behavior in the auction. A related example is when bidders have interdependent user-values and are themselves experts in estimating unknown user-values with statistically independent expertise-types.[1] Interdependencies in this set up will arise, for instance, if bidders face an unknown common user-value and bidders estimate prior to the auction this use-value.[2] In the presence of a perfectly competitive 'market for expertise', bidding behavior will, then, affect in general the reputational returns from this market.

Finally, in any given auction, it may simply be that bidders are concerned with the effect their perceived valuation of the object for sale will have on their reputation with their peers. At charity auctions, for example, it might be that bidders have some valuation for the object for sale. This will be affected by the bidder's use-value of that particular object, but also by the

[1] Alternatively, one can think of the bidders as bidding on behalf of their principal on a contract that rewards them at a fixed proportion of the true use-value, with the latter being revealed after the auction to the principal and the bidder.

[2] See for instance Milgrom and Weber [17]).

bidder's intrinsic generosity. Bidders know therefore that their bidding behavior in the auction will have an impact on their overall reputation for generosity. The literature (e.g. Andreoni and Petrie [1]) provides evidence that people care about their perceived generosity in charitable giving, particularly when their peers observe their behavior. It follows therefore that one should expect bidding behavior to be affected by the type of information about bidding that will be publicly available at the end of the auction. Conversely, one can also imagine other contexts, where being perceived to attach a high value to some object carries with it a certain amount of social stigma because, for example, the object is out of fashion.

The examples cited above have the common feature that bidding behavior does not just determine the chances of winning the auction and the price paid by the winner, but also sends signals that will influence the future utility of bidders. Indeed, the bidders care about the information potential future clients get *regardless* of whether they win the auction or not. In effect, we have a signaling game where each bidder is a sender, while the receivers do not participate in the auction but react to the information generated by it by taking actions that affect the bidders' utilities. We consider auctions where a single-indivisible object is for sale with bidders' types all being taken from the same distribution. We focus on the case where the action of a receiver that affects a given bidder only depends on the bidder's perceived type further assuming that reputational effects are ex-ante the same for all bidders. Thus, we have a model where every bidder is ex-ante identical and where a bidder's reputational returns only depend on her perceived type. We emphasize, however, that even with this assumption, a crucial difference with a standard signaling game is that beliefs about a bidder's type might not just depend on her bidding behavior, but also on that of others. For example, if the bidders are participating in an auction where only the winner's bid is disclosed, such as a Dutch auction, they know that their bid will be disclosed only if they win the auction. So, even if all bidders play a fully separating equilibrium by using monotone bidding strategies, for losers the information that will be revealed is simply that their valuation is lower than the winner's. To isolate the implications of this we will assume that actual types are statistically independent.

Throughout the paper, we focus on five different disclosure rules. We have disclosure rule T, where all the bids and the identity of the corresponding bidders are revealed; disclosure rule

S where only the identity of the winner, but none of the bids are disclosed; disclosure rule D where only the identity and the bid of the winner are disclosed as in Dutch auctions or first-price sealed-bid auctions where only the price is disclosed; disclosure rule P where only the identity of the winner, the highest losing bid and the identity of the corresponding bidder are disclosed, as in a second-price sealed-bid auction where the price is disclosed; and finally, disclosure rule E where only the losing bids and the identity of the corresponding bidders are revealed, as in English auctions.[3] These bidding rules cover most of the realistic cases one can imagine.

We also restrict attention to auctions where the mechanism implies that the price paid is the highest bid (Dutch or first-price sealed-bid auctions) or the second highest (English or second-price sealed-bid auctions). Our general analysis begins by characterizing bidding functions in pure strategy Perfect Bayesian Equilibria where bidding strategies are symmetric and strictly monotone, whenever such equilibria exists.

We show that these bidding functions share a strong analogy with the bidding functions when there are no reputational concerns. Indeed, the bidding functions are analogous to the ones in the absence of external incentives, after using what we call the bidders' *effective valuations* in the place of their (expected) user-values. These effective valuations are directly related to user-values, but take into account the reputational effects. The most important thing to note about effective valuations is that while they are independent of the price mechanism used in the auction, they are dependent on the disclosure rule. We can show that whenever symmetric and monotone bidding functions exist, the expected revenue of any auction is simply the expected value of the effective valuation for the second-highest bidder. A direct consequence is that we have a revenue equivalence result for price mechanisms, *for a given disclosure rule.*

Our first result regarding the properties of symmetric and monotone bidding functions shows that whenever reputational returns are strictly increasing in perceived type then overbidding (compared to the case of no reputational effects) will occur, while underbidding will occur in the opposite scenario, *whatever* the disclosure rule. Bradley et al. [3], among others, provide evidence that shareholders often obtain little benefits or even losses from acquisitions. Several

[3] Whenever we refer to English auctions in the paper, we refer to the specific version where bids increase continuously until the next to last bidder drops out. The auction stops there and the remaining bidder is declared the winner. It is important in our context that the winning bidder is not allowed to bid against herself as is theoretically possible in other forms of English auctions.

explanations have been put forward for this, some of which (see, for example, Mørck et al. [20]) identify managerial incentives as possible source of overbidding. Our setting provides a further possible explanation along these lines, which puts emphasis on the reputational incentives that managers might face.

The above still leaves two issues open for analysis. The first issue is that symmetric monotone equilibria may not exist. This is important as for the special case in which reputational returns are strictly increasing in the perceived type, this analysis amounts to an efficiency analysis as symmetric and monotone equilibria are the only ones that guarantee ex-ante efficiency. In our set-up, whenever such equilibria exist for second-price auctions, then they also exist for first-price auctions. The reason is that, as in the absence of reputational effects, bids in the former auctions "shade" the bids in the latter auctions in a monotone way. Given this observation, we then focus on second-price auctions and study how different disclosure rules affect the likelihood that such equilibria exist.

We find that S auctions are the ones that give most guarantees for existence of symmetric monotone equilibria since they give rise to bidding functions that have the same slope as the bidding functions under no reputational effects, while additional conditions are required for the other disclosure rules. Moreover, when reputational returns are strictly increasing in perceived type, we can show that under a usual log concavity condition D auctions are more likely than T auctions to give rise to monotone and symmetric equilibria. When reputational returns are strictly increasing in perceived type, there is an incentive to overbid. However, in D auctions low types have a relatively weaker incentive to overbid than T auctions because in such auctions the losers' types are not revealed. Analogously, when reputational returns are strictly decreasing in perceived type, we can show that under a usual log concavity condition E auctions are more likely than T auctions to give rise to monotone and symmetric equilibria. When reputational returns are strictly decreasing in perceived type, there is an incentive to underbid. However, in E auctions, high types have a relatively weaker incentive to underbid than T auctions because in such auctions the winner's type is not revealed. These results imply directly that full disclosure of bids is dominated in terms of guaranteeing that the object will be sold to the bidder with the highest user-value by confidentiality or partial disclosure of bids.

The second issue, is that conditional on the existence of symmetric and monotone equilibria, expected revenues do depend on disclosure rules. Among others, we show that in our set up:

1. E and T auctions are (expected) revenue equivalent. While the two disclosure rules generate very different bidding functions, we have that the different reputational incentives for low types in E versus T auctions are exactly compensated, in expectation, by the different reputational incentives for high types.

2. D auctions are less susceptible to reputational incentives in the sense that in the case of overbidding (resp. underbidding) they will provide smaller (resp. larger) expected revenues than E and T auctions. The difference is all due to the fact that when bidders care about their reputation, a mechanism that reveals types for winners only, as a D auction does, provides smaller reputational incentives for low types to overbid than a mechanism, like a T auction, where all types are announced.

3. In the case of overbidding (resp. underbidding), T auctions will provide more (resp. less) expected revenue than S auctions, as long as the number of bidders is large enough. To understand this, note that in the former (resp. latter) case T auctions provide stronger incentives to high (resp. low) types, who have a high (resp. low) probability of winning, to overbid (resp. underbid). The reason is that in T auctions the winner's (resp. any loser's) bid is revealed, while in S auctions the winner's (resp. any loser's) perceived type is (resp. is bounded from above by) the first order statistic. This divergence in incentives will become more amplified if bidders increase in their numbers.

4. In the case of overbidding (resp. underbidding) P auctions provide smaller (resp. larger) expected revenue than T (and E) auctions[4] and larger (resp. smaller) revenue than D auctions. The first result is due to the fact that in P auctions, low types have lower (resp. stronger) reputational incentives than in E auctions, as in the latter all losing bids are revealed. The second result is more subtle: in both P and D auctions only one bidder's bid is revealed, but low types have a significantly higher chance of being the highest loser than

[4]This holds for more than two bidders. One can very easily see that with only two bidders P and E auctions are equivalent.

the winner while the difference between the probability of being the winner or the highest loser is not so significant for high types. Thus, the greater reputational incentives for low types in P auctions are stronger overall than the greater reputational incentives for high types in D auctions.

These results have three particularly interesting implications. The first is that there might be a trade-off between being able to guarantee ex-ante that a symmetric and monotone equilibrium exists and, conditional on existence of such equilibria, attaining high expected revenues. For example, if reputational effects are strictly increasing in perceived type, then D auctions are preferable to T auctions if one wants to have the better shot at guaranteeing efficient allocation of the object whereas, conditional on existence of symmetric and monotone equilibrium, T auctions are preferable to D auctions in terms of expected revenues. Thus, a government might choose a different disclosure rule from a profit-maximizing seller, when bidders are involved in reputational bidding.

The second implication is that if the disclosure rule is to reveal the price (and the identities of the corresponding bidder and the winner), then second-price auctions will generate higher (resp. lower) expected revenues than first-price auctions in the case of overbidding (resp. underbidding). This is due to the above revenue comparison of P and D auctions for given price mechanism, the revenue equivalence of first- and second-price auctions that deploy the P (or D) disclosure rule and the fact that, with price-disclosure, first-price auctions are de facto D auctions while second-price auctions are de facto P auctions. This is of particular interest given the celebrated revenue equivalence, when reputational incentives are assumed away, between first- and second-price auctions when bidders' types are statistically independent and the fact that price-disclosure is a common practice.

The third implication is that we show that T auctions are indeed revenue dominant when being perceived to be of a high type is good as in the charitable auctions case. This conforms with the evidence that charitable giving increases when contributions are not anonymous (Andreoni and Petrie [1]).

However, T auctions are dominated by S (when bidders are sufficiently many), D and P auctions when there is underbidding. This, in conjunction with our earlier discussion of existence, provides another instance in which transparency may not be optimal, in particular if there is underbidding. So, for instance when out of fashion items, or items perceived as "guilty pleasures" are sold, S auctions will then dominate T auctions both in terms of expected revenues (when there are sufficiently many bidders) and guaranteeing that the object will be sold to the bidder with the highest user-value. Likewise, if in addition a usual log concavity condition is satisfied, then T auctions will be dominated by D auctions.

Holmström [9] has shown, in a principal-agent model, that more information improves the principal's inference about the agent's effort, and this, in turn, results in a Pareto improvement. Prat [22] shows that more information is not necessarily optimal when the principal cannot commit ex-ante to an incentive scheme. Transparency may also not be beneficial in a common agency model, where the decentralized incentive schemes offered by the principals are strategic substitutes (Maier and Ottaviani [15]).[5] In our case, transparency is not optimal, neither in terms of revenues nor in terms of allocating the object to the bidder with the highest user-value, because of the reputational effects on bidders' behavior when low types are associated with high reputational returns.

There is a literature that deals with cases where reputational effects distort bidding behavior: Goeree [6], Haile [7], Das Varma [4], Salmon and Wilson [23] and Katzman and Rhodes-Kropf [11].[6] There are however two main differences with this literature. First, here, bidding has reputational effects regardless of whether a bidder has won or lost the auction.[7] Second, in most of these papers it is only the T disclosure rule that is investigated; the focus is on the comparison of various price mechanisms. The only exception that we know is Katzman and Rhodes-Kropf [11], where second-price E auctions are also explored, and they highlight that, for their set-up, revenue comparisons between T and E auctions, with the same price mechanism, do not lead to

[5] For more related literature on the value of transparency see references in Prat [22] and Maier and Ottaviani [15].

[6] See also Molnar and Virag [19] who study the revenue-maximizing selling mechanism, that includes also the type of information that a seller releases at the end of the mechanism, when buyers compete for the right to enter an "outside" market.

[7] A related theme appears also in the literature on sequential auctions when bidders with correlated types do not have a unit-demand (see Ortega [21], Weber [24] and Hausch [8]). These models deal with the case of two bidders, and Weber [24] discusses also the case of bidders being initially asymmetrically informed. More importantly for our purposes, all these models deal with the case of all initial bids being disclosed prior to the sale of the remaining units, like our T disclosure rule. We, however, investigate a wide range of disclosure rules. We also focus on the case of independently and identically distributed types. We leave the extension of our set-up to the case of affiliated types and/or initially asymmetrically informed bidders for future research.

unambiguous results. In our context, however, we show that these disclosure rules are revenue equivalent.[8]

Our main contribution to the literature on auctions is therefore to isolate the implications of pure reputational concerns (which are relevant in many real life situations) on bidding behavior, and center the analysis on the revenue and efficiency properties of various informational linkages, for *given* price mechanisms. We see also our analysis as belonging to a new trend that seeks to study auctions in more complex environments. Several have been emphasizing for some time the importance of understanding the surrounding environment (see, for instance, Milgrom [17]). For example, Klemperer [12] and [13] observes that collusion, entry issues and political issues are crucial to the performance of actual auctions. Indeed, Maldoom [14] emphasizes how agency relationship between experts, management and shareholders might have an impact on the outcomes. In this paper, we introduce reputational effects and show how they can critically change the analysis even in the simplest auction environment.

The organization of the paper is as follows. Section 2 introduces the model. Section 3 characterizes bidding functions and discusses expected revenues for given disclosure rules. Section 4 focuses on a comparative analysis of disclosure rules. There, to develop further understanding of reputational incentives, we also explore the case where only winners have reputational concerns. Section 5 summarizes and discusses future research. Most of our proofs and our figures are relegated to an appendix.

2. The Model

There are several ways to think of our setup. One is that of several parallel sender-receiver signaling games with $N \geq 2$ sender-receiver pairs. In these games, the senders use a standard sealed-bid auction for a single, indivisible object, alongside a given disclosure rule, as a costly signaling device. Crucially, however, depending on the disclosure rule, the signal each receiver gets may not just depend only on her sender's bid (recall our discussion of Dutch auctions in the Introduction). Another possibility, at the other extreme, is that of N senders and just one receiver. As long as, conditional on the receiver knowing the senders' types, the receiver's action

[8] Note that in some of these papers it is in fact D auctions that are analyzed, but, because of particular assumptions they make, D auctions there are strategically equivalent to T auctions.

that affects a particular sender's utility only depends on that sender's type, our analysis goes through.

So, we have a game between N bidders and K receivers with $1 \leq K \leq N$. Bidder i (he) is characterized by a type $x_i \in [\underline{\omega}, \overline{\omega}] \equiv X$. The type is an attribute that is directly related to the actual user-value of bidder i from consuming the object. Types are private and iid. with cdf F. The latter is assumed to be twice differentiable with $f \equiv F' > 0$. Let $\mathbf{x} \equiv (x_1, ..., x_N)$ and $\mathbf{x}_{-i} \equiv (x_1, ..., x_{i-1}, x_{i+1}, ..., x_N)$. Also, let $y = \max_{j \neq i} \{x_j\}$ be the highest type amongst i's competitors, which is distributed according to $G \equiv F^{N-1}$. Furthermore, $y_2 = \max_{j \neq i} (\{x_j\} / y)$ is the second highest type amongst i's competitors. Define $L(y_2|y) \equiv \Pr(Y_2 \leq y_2 | Y = y)$. For these variables we use capital letters whenever we wish to emphasize they are to be interpreted as random variables. We describe bids with the notation $\mathbf{b} = (b_1, ..., b_N)$ where b_i denotes the bid of bidder $i = 1, ..., N$. Finally, $a_i \in A_i$ is the action taken by some receiver that only affects bidder i. Assume that, while a receiver can influence the payoffs of many bidders, each and every one bidder is affected by one and only one receiver. We refer to such receiver as "the relevant receiver" (she).

Specifically, payoffs for bidder i, *gross of the price paid* in the auction, are defined as follows:

$$\begin{aligned} U^w(x_i, a_i) &= x_i + a_i \\ U^l(a_i) &= a_i \end{aligned}$$

where the superscript $o = w, l$ captures the fact that payoffs, gross of price paid, depend on whether a bidder has won or not the auction. Note, however, that actual bids, apart from determining the price paid, only play an informational role. Our additively separable formulation allows us to separate bidders' incentives between reputational incentives and those that derive from the auction itself. Note that extending most of our results to the case of interdependent (expected) user-values is straightforward, insofar the (expected) user-value is increasing in types (e.g. the private estimates of the values).[9]

[9] In other words, our setting can easily accommodate the Milgrom and Weber [18] set up as long as one maintains the additive separability between user-values and reputational returns and restricts attention to non-ascending auctions. Our results change qualitatively to the same extent that they would change even without reputational concerns: revenue equivalence between first and second price auctions for a given disclosure rules would no longer obtain.

As for receivers, we assume that the relevant receiver for bidder i has a utility function such that her choice only depends on her beliefs about some characteristic θ_i of bidder i. We allow for the case in which $\theta_i = x_i$, of course, but we also wish to allow for the case in which the auction-relevant type x_i simply functions as a signal on i's characteristic that the relevant receiver really cares about.

We denote with $a(\theta_i)$ the $i-$bidder's relevant receiver's choice of action if she knew θ_i. We assume, however, that θ_i is unknown to the relevant receiver. Assume also that the receiver's preferences are such she has a unique optimal action for any given beliefs she might have over the true value of θ_i.[10] In addition, assume that $a(\theta_i)$ is twice differentiable, bounded and with bounded derivatives. Let us use the convention hereafter that $a_\theta(\theta_i) > 0$. As mentioned above, the bidder's type x_i is correlated with θ_i. In particular, for the case in which $\theta_i \neq x_i$, we assume that for all i, the θ_i's are (conditionally) iid. Moreover, it is common knowledge that θ_i and x_i are related according to the common cdf $T(\theta_i \mid x_i)$, which is assumed to be twice continuously differentiable with associated density $t(\theta_i \mid x_i)$ and bounded derivatives. We denote with

$$V(x_i) \equiv E_T \left[a\left(\theta_i\right)|x_i\right]$$

the action taken by the relevant receiver for bidder i, conditional on x_i. Note, by the properties of $T(\theta_i \mid x_i)$, that $V(x_i)$ is differentiable. Also, given $a_\theta(\theta_i) > 0$, we have that $V_x(x_i) > 0$ if $\frac{\partial}{\partial x}T(\theta_i \mid x_i) < 0$ and vice versa.

Note that conditional on type x being known by the relevant receiver, the reputational returns, $V(x)$, and thereby gross payoff, $x + V(x)$, are the same for every bidder with type x. This symmetry assumption facilitates comparisons with the standard case of symmetric auctions in the absence of reputational effects and allows us to focus on symmetric and monotone equilibria.

We can provide several examples of our setup.

Example 1 *Bidders are publicly listed firms attempting a takeover of another firm. In this case,*

[10] For instance, one can think of the relevant receiver's payoff from interacting with bidder i as $-(a_i - \theta_i)^2$, where here $a(\theta_i) = \theta_i$. Alternatively, one can think of a perfectly competitive market for the purchase of services of individuals with relevant attribute θ and zero reservation wage, with firms having profits of $a(\theta) - w$ per-individual they hire at a given wage $w \geq 0$. Zero profit with θ being individuals' private information would then imply that the perfectly competitive wage is $w = E_\mu[a(\theta)]$ where μ denotes the beliefs of firms about the $\theta-$attribute of the typical individual in the market. This set up, coupled all individuals being bidders with gross payoffs $x + w$, will give rise to a model like the one in the main text.

θ_i is the underlying attribute for bidder/firm i that the stock market cares about such as managerial efficiency. Moreover, $a(\theta_i)$ is the fundamental stock-value if firm's attribute was publicly known. $T(\theta_i \mid x_i)$ could then be the beliefs distribution over bidder i's attribute given that i's user-value for the acquisition is x_i. Clearly, $V(x_i)$ would be a strictly increasing function whenever the stock market perceives a high willingness to pay for the firm for sale as a signal of high managerial ability, while $V(x_i)$ would be strictly decreasing whenever the stock market makes the opposite inference. The former case would arise if the higher the acquisition's profitability, the higher the managerial efficiency is likely to be. Conversely, for the latter case.

Example 2 *In a different interpretation, the attribute θ_i could be the inherent generosity of bidder i who participates in a charity auction. With such interpretation, i cares about the reputation as a philanthropist he can establish with his peers (who are then his relevant receiver). We denote with $a(\theta_i)$ the reputational returns he would get if θ_i became known to his peers, as a result of social status, public relations etc. Finally, x_i represents the value i attributes to the object for sale in this particular (charity) auction. In particular, this will be a function of his private use-value, the particular charity being donated to and his inherent generosity. This allows for the possibility that i may not intrinsically value the object for sale very much but still attaches a large value to donating to this particular charity. In this setting, it is reasonable to assume that $V(x_i)$ is a strictly increasing function.*

Example 3 *Finally, a third possible interpretation is that bidder i is interested in establishing a reputation for expertise in estimating the (unknown) user-values of objects that may become available in the future. This might be because, for example, the bidder is a collector who is interested in her recognition as an expert with his peers or, the bidder might actually be offering his expertise in a market for experts where the reward/wage is increasing in perceived expertise. In this context, imagine that c_i is the true value to i of the object for sale. The object for sale is an experience good, and so c_i is unknown to i himself. These valuations are private and iid. across bidders. Bidder i receives a private signal σ_i which is affiliated with c_i. The extend of affiliation is dependent on his unknown expertise θ_i. We then have that $x_i = E[C_i|\sigma_i]$, with $\frac{\partial x_i}{\partial \sigma_i} > 0$, is the estimated user-value that i has given the signal he receives. In this scenario, $a(\theta_i)$ could either represent the wage for experts of ability θ_i in a perfectly competitive market or the reputational* *returns to i from being perceived of being an expert of ability θ_i from his peers. An increasing (resp. decreasing) $V(x_i)$ would capture the case of priors in the market for experts being such that higher (resp. lower) value-estimates are associated with better experts. To complete the description of this environment one would then need to postulate an unconditional cdf for c_i and an "expertise technology", in the form of a conditional cdf $H(x_i \mid c_i, \theta_i)$, where we assume that expertise types θ_i are independent of the true user-values, that is consistent with $F(x_i)$ and $T(\theta_i \mid x_i)$.*

Given the postulated ex ante symmetry of bidders, we will focus, throughout the paper, on symmetric and monotone Perfect Bayesian Equilibria (PBE) in pure strategies of the auctions under consideration. We will refer to these simply as PBE, with the understanding that we always refer to symmetric and monotone pure strategy equilibria. Restricting attention to PBE follows the usual practice in the literature when the auction is symmetric as in our setup. A PBE will be represented by a strictly increasing bidding function $\beta(x)$, with its inverse be denoted by β^{-1}.

In any PBE the relevant receiver for bidder i will choose her optimal action given her beliefs about the bidder's type. These beliefs, and thereby the (expected) reputational returns and the corresponding gross payoff of bidder i, will depend on the bidding function $\beta(x)$ and the information that is publicly available at the end of the auction (and hence on the outcome of the auction). To define the reputational returns properly, we need to describe the information available to receivers at the end of the auction. In the paper, we will focus on a few special cases which cover most of the disclosure rules in practice.

We begin by defining with $E[V(X_i) \mid \mathbf{b}, \phi, o]$ the expected reputational return for bidder i given the vector of bids $\mathbf{b}$, disclosure rule ϕ and having won $(o = w)$ or lost $(o = l)$ the auction. Moreover, for a PBE β we also define a function $v^{\phi o}(y, z_i)$ as the expected return for bidder i given a bid $b_i = \beta(z_i)$ and that the highest competing bid is $\beta(y)$. Finally, we define

$$E_F[V(X_i) \quad | \quad X_i < y] \equiv Z(y),$$
$$E_F[V(X_i) \quad | \quad X_i > y] \equiv T(y).$$

- T auctions. These are auctions where all bids are publicly revealed, alongside the identities of the corresponding bidders. Any sealed-bid auction where all bids are disclosed at the end of the auction is a T auction. In such auctions, the expected reputational return in a PBE of a bidder who bids $b_i \in \beta(X)$ is

$$E[V(X_i) \mid \mathbf{b}, T, o] = V(\beta^{-1}(b_i)) = v^{Tw}(y, \beta^{-1}(b_i)) = v^{Tl}(y, \beta^{-1}(b_i)),$$

regardless of whether the bidder has won or lost the auction.

- D auctions. These are auctions where only the identity of the winner and her bid are publicly revealed. Dutch auctions are necessarily D auctions, but sealed-bid auctions where only the winning bid is announced by the auctioneer are also D auctions. Also, in our context, a first-price sealed-bid auction with D disclosure rule, a Dutch auction and a first-price sealed-bid auction where only the winner's identity and the price are disclosed are all strategically equivalent. In such auctions, the expected reputational return in a PBE of a bidder who bids $b_i \in \beta(X)$ is

$$E[V(X_i) \mid \mathbf{b}, D, w] = V(\beta^{-1}(b_i)) = v^{Dw}(y, \beta^{-1}(b_i)),$$

for a winner (i.e. $b_i > \beta(y)$), and

$$E[V(X_i) \mid \mathbf{b}, D, l] = Z(y) = v^{Dl}(y, \beta^{-1}(b_i)),$$

for a loser (i.e. $b_i < \beta(y)$).

- E auctions. These are auctions where all bids, except the winner's, are publicly revealed alongside the identities of the corresponding bidders and the winner. English auctions where bidding stops when the next-to-last bidder withdraws are necessarily E auctions. In such auctions, the expected reputational return in a PBE of a bidder who bids $b_i \in \beta(X)$ is

$$E[V(X_i) \mid \mathbf{b}, E, w] = T(y) = v^{Ew}(y, \beta^{-1}(b_i)),$$

for a winner, and

$$E[V(X_i) \mid \mathbf{b}, E, l] = V(\beta^{-1}(b_i)) = v^{El}(y, \beta^{-1}(b_i)),$$

for a loser.

- P auctions. These are auctions where only the second-highest bid and the identities of the two highest bidders are publicly revealed. A second-price sealed-bid auction where the price is announced, alongside the identities of the two highest bidders, by the auctioneer is a P auction. Clearly, if there are only two bidders, these auctions are equivalent to E auctions. For this reason when we will be referring to P auctions hereafter we will be assuming that $N > 2$. In such auctions, the expected reputational return in a PBE of a bidder who bids $b_i \in \beta(X)$ is

$$E[V(X_i) \mid \mathbf{b}, P, w] = T(y) = v^{Pw}(y, \beta^{-1}(b_i)),$$

for a winner, and

$$E[V(X_i) \mid \mathbf{b}, P, l] = V(\beta^{-1}(b_i))L\left(\beta^{-1}(b_i)|y\right) + \int_{\beta^{-1}(b_i)}^{\overline{\omega}} Z(s)dL\left(s|y\right) = v^{Pl}(y, \beta^{-1}(b_i)),$$

for a loser.

- S auctions. These are auctions where only the identity of the winner is announced[11] In such auctions, the expected reputational return in a PBE of a bidder who bids $b_i \in \beta(X)$ is

$$E[V(X_i) \mid \mathbf{b}, S, w] = E_G[T(Y)] = v^{Sw}(y, \beta^{-1}(b_i)),$$

for a winner, and

$$E[V(X_i) \mid \mathbf{b}, S, l] = E_G[Z(Y)] = v^{Sl}(y, \beta^{-1}(b_i)),$$

for a loser.

The above reputational returns in a PBE, $v^{\phi o}(y, \beta^{-1}(b_i))$, capture the beliefs of the relevant receiver. These beliefs follow Bayes rule, wherever it is possible, given publicly available information at the end of the auction and the expected equilibrium bidding behavior of the bidders in the auction. The most important aspect to underline here is that these beliefs do not necessarily depend on b_i alone, but potentially on all other bids, as the disclosure rule may imply that b_i is not revealed. For example, with reference to the D auctions case described above, a losing

[11] Clearly, for T, D and E auctions the information on the identity of the winner is redundant as it can be recovered from the available information on bids, but for P and S auctions it is not redundant.

bidder i's bid will not be known to the relevant receiver, but the latter does have the information that i's bid was lower than the winning bidder's. This makes the model one where signaling is noisy but in a way that is potentially endogenous: in the D, P and E auctions, a bidder does not know ex-ante whether her bid will be disclosed. Simple inspection shows that for all our disclosure rules, the functions $v^{\phi o}$ are twice differentiable in each argument. Moreover, $v^{\phi o}$ and their derivative in the second argument are all bounded.

We restrict attention to PBE that are supported by the following off-the-equilibrium-path beliefs:

Assumption A (Beliefs) Let $\beta(\bullet)$ be a symmetric and monotone bidding strategy in a PBE of the auction under consideration. We assume that in any such equilibrium, any bid lower than $\beta(\underline{\omega})$ is believed to come from type $x_i = \underline{\omega}$ and any bid higher than $\beta(\overline{\omega})$ is believed to come from a type $x_i = \overline{\omega}$. Further, if there is a bid b and a type $\widehat{x}$ such that $b \in \left(\lim_{x_i \to \widehat{x}^-} \beta(x_i), \lim_{x_i \to \widehat{x}^+} \beta(x_i)\right)$ then b is believed to come from type $x_i = \widehat{x}$.

The focus on PBE with off-the-equilibrium-path beliefs that satisfy Assumption A, allows us to associate to each vector of bids $\mathbf{b}$ a corresponding vector $\mathbf{z}$ of types.[12] Given our focus on such equilibria, it will be convenient to work with the vector $\mathbf{z}$ rather than with $\mathbf{b}$. We can also think of $z_i \equiv \beta^{-1}(b_i)$ as the report to the seller by bidder i of her type (via her bid). We will therefore refer to z_i as the *announcement* of bidder i (to the seller).

In our set up, given one of the disclosure rules ϕ described above, we therefore have that if bidder i announces z_i, the other bidders announce their true types and it is commonly expected that every bidder deploys the bidding function β, then the expected payoff for bidder i - gross of price paid - is equal to

$$\int_{\underline{\omega}}^{z_i} (x_i + v^{\phi w}(s, z_i))dG(s) + \int_{z_i}^{\bar{\omega}} v^{\phi l}(s, z_i)dG(s).$$

It would help understanding to note that in the private independent values model we would have $v^{\phi l} = v^{\phi w} \equiv 0$. Thus, in our framework, announcements/bids affect both the chances of winning (and the price paid) and the payoffs conditional on winning or losing the auction.

[12] That is, with z_i such that $\beta(b_i) = z_i$ if $b_i \in \beta(X_i)$, $z_i \equiv \underline{\omega}$ if $b_i < \beta(\underline{\omega})$, $z_i \equiv \overline{\omega}$ if $b_i > \beta(\overline{\omega})$ and $z_i \equiv \widehat{x}$ if $b_i \in (\lim_{x \to \widehat{x}^-} \beta(x), \lim_{x \to \widehat{x}^+} \beta(x))$.

3. Bidding Functions

We begin with an important definition:

Definition 1 *Let*

$$\Psi^{\phi}(x_i, z_i) \equiv$$
$$\equiv x_i + v^{\phi w}(z_i, z_i) - v^{\phi l}(z_i, z_i) + \frac{1}{g(z_i)} \left\{ \int_{\underline{\omega}}^{z_i} v_z^{\phi w}(s, z_i) dG(s) + \int_{z_i}^{\overline{\omega}} v_z^{\phi l}(s, z_i) dG(s) \right\}.$$

We then have that $\psi^{\phi}(x_i) \equiv \Psi^{\phi}(x_i, x_i)$ is the effective valuation *for bidder i with type x_i who faces a disclosure rule ϕ.*

$\Psi^{\phi}(x_i, z_i)$ is the net welfare gain to bidder of type x_i from winning (gross of payments) relative to the marginal increase in the probability of winning, after increasing the announcement marginally over z_i, given that the disclosure rule is ϕ. An effective valuation is such net welfare gain when $z_i = x_i$. In what follows we will sometimes refer to $\widetilde{\psi}^{\phi}(x_i) \equiv \psi^{\phi}(x_i) - x_i$ as the reputational component of the effective valuation.

For our chosen disclosure rules, we then have that[13]

$$\begin{aligned}
\psi^{T}(x_i) &= x_i + \frac{1}{g(x_i)} V_x(x_i), \\
\psi^{D}(x_i) &= x_i + V(x_i) - Z(x_i) + \frac{G(x_i)}{g(x_i)} V_x(x_i), \\
\psi^{E}(x_i) &= x_i + T(x_i) - V(x_i) + \frac{1 - G(x_i)}{g(x_i)} V_x(x_i), \\
\psi^{P}(x_i) &= x_i + T(x_i) - V(x_i) \\
&\quad + \frac{1 - F(x_i)}{f(x_i)} \left[V_x(x_i) + (N-2) Z_x(x_i) \right] \\
\psi^{S}(x_i) &= x_i + E_G[T(Y)] - E_G[Z(Y)]
\end{aligned}$$

It will help intuition to note the following. First, in the absence of reputational concerns, we have $V(x_i) \equiv 0$. In the benchmark case where reputational concerns exist but the types are revealed to everyone at the end of the auction we have, with some abuse of notation, $v^{\phi o}(y, z_i) \equiv V(x_i)$ for any y and z_i. In this case, $\psi^{\phi}(x_i) = x_i$, as in the standard case of private and independent

[13] Calculating effective valuations given our definitions of $v^{\phi w}(y, z_i)$ and $v^{\phi l}(y, z_i)$ is immediate except for the case $\phi = P$ which is detailed in the appendix.

values. It follows that, in the case of reputational bidding, $\psi^{\phi}(x_i) - x_i \equiv \tilde{\psi}^{\phi}(x_i)$ captures the reputational effect on bidders' effective valuations. Second, $\frac{V_x(x_i)}{g(x_i)}$ captures, in a monotone and symmetric equilibrium, the reputational gain (or loss) relative to the increase in the likelihood of winning the auction from increasing marginally the perception of the receiver about one's type by means of increasing one's bid marginally. This relative gain is relevant only when one's bid is revealed. So, by the nature of the disclosure rules under scrutiny here, this relative gain/loss is always relevant for $T-$auctions, never relevant for $S-$auctions, while it is only relevant for losers in E or $P-$auctions and for the winners of $D-$auctions. Third, when some bids may not be revealed, the event of winning itself carries a reputational net gain (or loss). This is captured by $V(x_i) - Z(x_i)$ for $D-$auctions, by $T(x_i) - V(x_i)$ for E and $P-$auctions, and by $E_G[T(Y)] - E_G[Z(Y)]$ for $S-$auctions. Fourth, it follows directly that, with monotone $V(x)$, the various relative reputational gains/losses discussed above have the same sign as V_x.

Our first result characterizes bidding functions for first-price (FP) and second-price (SP) sealed-bid auctions. We reemphasize, however, that in our setting, Dutch auctions are strategically equivalent to a FP sealed-bid auction with disclosure rule D, while English auctions where bidding stops when the next-to-last bidder withdraws are strategically equivalent to a SP sealed-bid auction with disclosure rule E. From now on, we slightly abuse notation by denoting $\lim_{x_i \to \underline{\omega}^+} \psi^{\phi}(x_i)$ with $\psi^{\phi}(\underline{\omega})$.[14]

Proposition 1 *Assume* $\boldsymbol{A}$ *holds,* $\psi^{\phi}(\underline{\omega}) \geq 0$ *and that* $\psi^{\phi}(x_i)$ *is strictly increasing.*

1. *The PBE in second-price auctions with a disclosure rule* ϕ*,* $\beta^{SP-\phi}$ *is given by*

$$\beta^{SP-\phi}(x_i) = \psi^{\phi}(x_i)$$

2. *The PBE in first-price auctions with a disclosure rule* ϕ*,* $\beta^{FP-\phi}$ *is given by*

$$\beta^{FP-\phi}(x_i) = E_G\left[\psi^{\phi}(Y) | Y < x_i\right] .$$

Proof Follows familiar steps. For completeness the proof is in an appendix which is available upon request.■

[14]For $N = 2$ then $\psi^{\phi}(\underline{\omega})$ is necessarily well defined, given our assumptions, in particular that $f(\underline{\omega}) > 0$. However, for $N > 2$, we might have that $g(\underline{\omega}) = (N-1)F(\underline{\omega})^{N-2} f(\underline{\omega}) = 0$. It is in those cases that $\psi^{\phi}(\underline{\omega})$ should be interpreted as $\lim_{x_i \to \underline{\omega}^+} \psi^{\phi}(x_i)$.

It is immediate to see the similarity between this result and the bidding functions for the basic, private and independent values set-up. The only difference is that instead of bidding according to x_i bidders use their *effective valuations* $\psi^{\phi}(x_i)$.

Two more issues arise from the proposition above. The first is that $\psi^{\phi}(x_i)$ or $E_G\left[\psi^{\phi}(Y)|Y < x_i\right]$ are not guaranteed to be strictly increasing. The second is whether, assuming symmetric and monotone bidding functions are well defined in each case, revenue equivalence still obtains in our set-up.

We take up the first issue again in the next section, but with respect to the second issue, we can show that indeed revenue equivalence applies to any m price auctions whenever symmetric and monotone PBE exist for such auctions:

Proposition 2 *Consider a disclosure rule and any m price auction such that symmetric and monotone bidding functions exists for the given disclosure rule ϕ. Then, the auction provides an expected revenue to the seller equal to*

$$E_{F_2^{(N)}}\left[\psi^{\phi}\left(Y_2^{(N)}\right)\right]$$

where $F_2^{(N)}$ is the cdf of the random variable $Y_2^{(N)}$ that represents the second-highest type amongst all bidders.

Proof Follows familiar steps. For completeness the proof is in an appendix which is available upon request.■

One crucial aspect of proposition 2 is that effective valuations depend on the publicly available information at the end of the auction represented by the disclosure rule ϕ. So, auctions with the same price mechanism but different disclosure rules will in general have different expected revenues. Below, we investigate further issues of existence and revenue ranking that deal specifically with the disclosure rules S, T, D, P or E we identified above.

4. Comparing Disclosure Rules

4.1. Existence of PBE

We begin with some preliminary observations. The first observation is that $E_G\left[\psi^{\phi}(Y)|Y < x_i\right]$ is strictly increasing whenever $\psi^{\phi}(x_i)$ is strictly increasing[15]; in this sense first-price auctions are more likely to have a PBE than second-price auctions. Our results below can then be interpreted as characterizing the minimum conditions for existence of a PBE under any of the price mechanisms under scrutiny.

A second observation is that when V is strictly increasing, PBE are the only ones that can guarantee efficiency in that only in these equilibria does the highest type always win the auction and, in this case, the highest type is also the type that values winning the most.[16] We will therefore put particular emphasis on the case where V is strictly increasing.

The final observation is that simple inspection of the relevant effective valuations shows that a sufficient condition for $\psi^{\phi}(\underline{\omega}) \geq 0$, whatever the disclosure rule, is that $\lim_{x_i \to \underline{\omega}^+} \frac{V_x(x_i)}{g(x_i)}$ is bounded from below and $\underline{\omega}$ is sufficiently high. Assume hereafter that these are true. Conditional on these conditions being satisfied, then one can focus on conditions for the putative bidding functions to be strictly increasing.

In what follows, it will help to denote by F_V the cdf of the random variable $V = V(X)$. Clearly, with a strictly monotone $V(x)$, we have

$$f_V(v) = \frac{f\left(V^{-1}(v)\right)}{\left|V_x\left(V^{-1}(v)\right)\right|}.$$

We can now have our first result of this section:

Proposition 3 *Whenever $V_x > 0$ for all $x > \underline{\omega}$ then $\psi^{\phi}(x) > x$ for all $x \in (\underline{\omega}, \bar{\omega})$, plus $\psi^{\phi}(\underline{\omega}) \geq \underline{\omega}$ and $\psi^{\phi}(\bar{\omega}) \geq \bar{\omega}$ for all disclosure rules $\phi \in \{S, T, D, E, P\}$. Conversely if $V_x < 0$ for all $x > \underline{\omega}$.*

[15] We have that

$$\begin{aligned} & \frac{d\left(E_G\left[\psi^{\phi}(Y)|Y < x_i\right]\right)}{dx_i} \\ = & \frac{g(x_i)}{G(x_i)}\left(\psi^{\phi}(x_i) - E_G\left[\psi^{\phi}(Y)|Y < x_i\right]\right) \end{aligned}$$

which is then positive for all x_i if $\psi^{\phi}(x_i)$ is increasing in x_i (but the reverse is not necessarily true).

[16] If V is not strictly increasing, then all we can argue is that PBE are the only equilibria that allocate the object to the bidder with the highest user-value/type, not necessarily the bidder who values winning the most.

Proof. It follows directly from the definitions of effective valuations and $T(x)$ and $Z(x)$ above.■

That is, if the reputational returns when one's bid is revealed in a symmetric and monotone equilibrium, $V(x_i)$, are strictly increasing (resp. decreasing) in x_i, we have that with any of our disclosure rules here, there is almost everywhere overbidding (resp. underbidding) due to the positive (resp. negative) reputational effect.[17]

The figure below shows the reputational component of the equilibrium SP bidding functions for our given disclosure rules in a parametrization that guarantees existence: F is uniform on $[0,1]$, $V(x_i) = \frac{1}{10}x_i^4$ and $N = 4$:[18]

[FIGURE 1 HERE]

The figure emphasizes the overbidding result when $V_x > 0$ since all components are strictly positive for $x_i > \underline{\omega}$. We will refer to Figure 1 below when we focus on ranking disclosure rules according to their propensity to generate strictly increasing first-price bidding functions. The following Lemma will be used extensively in proving the forthcoming propositions.

Lemma If $V_x(x) > 0$ for any $x > \underline{\omega}$, then $Z_x(x) > 0$ and $T_x(x) > 0$, and vice versa.

Proof. Differentiating $T(x)$ we have

$$T_x(x) = \frac{f(x)}{1 - F(x)}[T(x) - V(x)].$$

Differentiating $Z(x)$ we have

$$Z_x(x) = \frac{f(x)}{F(x)}[V(x) - Z(x)].$$

The lemma then follows directly from the fact that if $V_x(x) > 0$ for any $x > \underline{\omega}$, then $T(x) > V(x) > Z(x)$, and vice versa.■

[17] The only cases when there is no over/under-bidding for a type x_i are when (i) $x_i = \underline{\omega}$, $\phi = T$ and $\lim_{x_i \to \underline{\omega}^+} \frac{V_x(x_i)}{g(x_i)} = 0$, or (ii) $x_i = \underline{\omega}$, $\phi = D$ and $\lim_{x_i \to \underline{\omega}^+} \frac{V_x(x_i)}{g(x_i)}$ is well-defined.

[18] In other words, the figure shows $\widetilde{\psi}^{\phi}(x_i)$ for different disclosure rules. For $\phi = P$ and $\phi = E$, this reputational component is decreasing at some point but even in these cases, the whole bidding function $\psi^{\phi}(x_i)$ is strictly increasing.

We then have the following results regarding the relative efficiency properties of the disclosure rules under scrutiny here:

Proposition 4 *I* $\psi^S(x_i)$ *is strictly increasing.*

II Suppose f_V *is log-concave. Then*

a. *whenever* $V_x > 0$ *for all* $x > \underline{\omega}$, $\psi^D(x_i)$ *is strictly increasing if* $\psi^T(x_i)$ *is strictly increasing, and*

b. *whenever* $V_x < 0$ *for all* $x > \underline{\omega}$, $\psi^T(x_i)$ *is strictly increasing if* $\psi^D(x_i)$ *is strictly increasing.*

III Suppose that f_V *is log-concave. Then*

a. *whenever* $V_x > 0$ *for all* $x > \underline{\omega}$, $\tilde{\psi}^T(x_i)$ *is strictly increasing if* $\tilde{\psi}^E(x_i)$ *is strictly increasing, and*

b. *whenever* $V_x < 0$ *for all* $x > \underline{\omega}$, $\tilde{\psi}^E(x_i)$ *is strictly increasing if* $\tilde{\psi}^T(x_i)$ *is strictly increasing.*

Proof. See appendix.■

The first result in the proposition is very easy to interpret because when the disclosure rule is $\phi = S$ then:

$$\begin{aligned} \psi^S(x_i) &= x_i + \lambda, \\ \text{where } \lambda &\equiv E_G[T(Y) - Z(Y)] \end{aligned}$$

In other words, we have the bidding function from the standard independent private values framework with the addition of a constant, which reflects the net reputational gain from winning the auction with a secret bid. The latter captures that with the $\phi = S$ disclosure rule, only the identity of the winner is revealed, but no actual bids. Whether this additional term is positive or negative depends on whether being of a higher type is beneficial or detrimental vis-a-vis the reputational returns. Thus, for sufficiently high value of $\underline{\omega}$, a PBE is guaranteed to exist for this disclosure rule.

This is a very interesting property of S actions: even if $V(x_i)$ is a strictly increasing function, this is not sufficient in itself to guarantee that a PBE exists when $\phi = D, T, E, P$. The reason is that increasing a bid also increases the expected payment that a certain bidder expects to make and the reputational effect may not be sufficient to compensate for that. If monotonicity cannot be guaranteed, Proposition 4 IIa and IIb allow us to provide a ranking between these three disclosure rules in terms of their likelihood of generating a PBE whenever f_V is log-concave (which is satisfied by most common distributions).[19]

Under this log-concavity property, the proposition tells us that if V is strictly increasing then whenever a strictly increasing second-price bidding function exists when the disclosure rule is $\phi = T$, we are guaranteed that the corresponding second-price bidding function with disclosure rule $\phi = D$ also exists (see part IIa). In that sense, the disclosure rule $\phi = D$ is more likely to be efficient than the disclosure rule $\phi = T$. The intuition is easy to grasp by referring to figure 1, where we have that reputational effects are increasing in x_i. With disclosure rule $\phi = D$, reputational effects grow stronger as the type increases, because it is the high types that have a greater chance of winning and therefore to have their type revealed. With disclosure rule $\phi = T$, on the other hand, reputational effects are more uniform across types. So, in the former case the bidding function is "steeper". The converse ranking is obtained, but only for the reputational components of the second-price bidding functions, $\widetilde{\psi}^{\phi}(x)$, when V is strictly decreasing, for exactly the opposite reasons (see part IIIa): with $\phi = D$, high types have a significantly lower reason to underbid than low types.

A relationship between T and E disclosure rules that echoes the one we just described between the D and T disclosure rules also obtains (see parts IIb and IIIb) and again the intuition follows in a similar way: $\phi = E$ auctions provide reputational incentives that are relatively stronger for low types than in the $\phi = T$, so that if V is strictly decreasing (resp. increasing) low types have a relatively higher incentive to underbid (resp. overbid).

A very interesting implication of the above proposition is:

Corollary 1 *Full disclosure is dominated in terms of guaranteeing the allocation of the object*

[19] Actually, for given subsets of these properties, log-concavity of f_V is stronger than what is needed. For example, if f_V is strictly decreasing, then IIa and IIIb obtain while if f_V is strictly increasing, then IIb and IIIa obtain.

to the bidder with the highest user-value/type by non-disclosure of bids and, if f_V is log-concave, by non-disclosure of each and every losing (resp. winning) bid when underbidding (resp. overbidding) takes place.

Finally, we note that a relative ranking in terms of efficiency or monotonicity between P and E, T, D auctions that depends on intuitive properties of our primitives is difficult to obtain.

4.2. Revenue Comparisons

Assuming a PBE exists for all our disclosure rules, we now turn to (expected) revenue comparisons. Denote by $ER(\phi)$ the expected revenue associated with a specific disclosure rule ϕ. The result below allows us to compare revenue properties:

Proposition 5 *Assume the relevant symmetric and monotone PBE exist. Then:*

I $ER(T) = ER(E)$

II $ER(D) - ER(S)$ *is positive (resp. zero, negative) and both* $ER(D)$, $ER(S)$ *are decreasing (resp. constant, increasing) in* N, *if* f_V *is strictly increasing (resp. constant, decreasing)*

III Whenever $V_x > 0$ *for all* $x > \underline{\omega}$ *then*

a.

$$\begin{aligned} ER(T) - ER(D) &> 0 \\ ER(E) - ER(P) &> 0 \\ ER(P) - ER(D) &> 0 \end{aligned}$$

b. $ER(T)$ *(and hence* $ER(E)$*) is strictly increasing in* N .

c. For large enough N, $ER(T) - ER(S) > 0$.

IV Whenever $V_x < 0$ *for all* $x > \underline{\omega}$ *then*

a.

$$\begin{aligned} ER(T) - ER(D) &< 0 \\ ER(E) - ER(P) &< 0 \\ ER(P) - ER(D) &< 0 \end{aligned}$$

b. $ER(T)$ *(and hence* $ER(E)$*) is strictly decreasing in* N*.*

c. For large enough N*,* $ER(T) - ER(S) < 0$*.*

Proof See Appendix.[20]■

Proposition 5 provides a simple and comprehensive set of conditions for analyzing the revenue properties of T, D, E, P and S auctions. In the first instance, point I of the proposition tells us that revenue equivalence applies to T and E auctions. To see how this might come about, note, after some straightforward manipulations, that

$$\psi^E(x_i) = \psi^T(x_i) + T(x_i) - V(x_i) - \frac{G(x_i)}{g(x_i)} V_x(x_i).$$

So, if $V(x)$ is strictly increasing, then for $x_i = \underline{\omega}$ the bidding function for rule $\phi = E$ is above that for the rule $\phi = T$, while for $x_i = \overline{\omega}$ the reverse occurs. This is clearly visible in figure 1 above. The intuition is that with disclosure rule $\phi = E$, bidders with low valuations have a stronger incentive to overbid, in a PBE. The reason is that such bidders are more likely to lose and $\phi = E$ implies disclosure of losing bids. Conversely, high types have a much higher chance of winning in a PBE. Thus, they do not have an incentive to overbid by much, as their high type is likely to not be disclosed. This logic does not apply to auctions with $\phi = T$ because, in this case, all types are revealed in a PBE. Our result, then, shows that the additional incentives to overbid for low types and the more muted incentives to overbid for high types that one finds with disclosure rule E cancel out in expectation. Similarly, if $V(x)$ is strictly decreasing.

Point II focuses on the comparison between the expected revenues of $\phi = D$ and $\phi = S$ auctions. We have that auctions with disclosure rule $\phi = D$ provide a higher expected revenue than auctions with disclosure rule $\phi = S$, whenever high values of the random variable V are more likely, and vice versa. The intuition here is that with the disclosure rule $\phi = S$ reputational

[20] In the appendix, we actually prove the stronger result that

$$\begin{aligned} E_G\left[\psi^T(Y)\,|Y < x_i\right] - E_G\left[\psi^D(Y)\,|Y < x_i\right] &> 0 \\ E_G\left[\psi^E(Y)\,|Y < x_i\right] - E_G\left[\psi^P(Y)\,|Y < x_i\right] &> 0 \end{aligned}$$

for all $x_i > \underline{\omega}$ whenever $V_x > 0$ and vice versa. This clearly implies the expected revenue results described in the proposition. However, the result is in effect stronger when first-price auctions are considered. The reason is that the above implies also a ranking for revenues ex post.

incentives are uniform across types, while disclosure rule $\phi = D$ gives stronger (resp. lower) incentives for overbidding to higher types when $V(x)$ is strictly increasing (resp. decreasing). Thus, it is not surprising that whenever higher values of V are more likely, we expect more revenues from the disclosure rule $\phi = D$. To finish with point II., we have that over/underbidding, due to reputational incentives, under $\phi = S$ increases with the number of bidders if f_V is decreasing. The latter condition guarantees that $T(x) - Z(x)$ is an increasing function, which, in turn, means that the expected reputational value of winning the auction increases in x. Thus, as the number of bidders increase, the incentive to overbid must also increase. Conversely if f_V is decreasing, and analogously for $\phi = D$.

Points IIIa. and IVa. address the comparisons in terms of expected revenues that depend on the monotonicity of V. We begin by discussing the difference in expected revenues between $\phi = T$ and $\phi = D$ auctions on the one hand and $\phi = E$ and $\phi = P$ auctions on the other. Our results tell us that when reputational effects are increasing in type then T or E auctions induce higher expected revenues than D or P auctions, respectively; conversely, if reputational effects are decreasing in type the opposite results hold. There is a symmetry in the relationship between T and D auctions on the one hand and E and P auctions on the other: the difference between T and D auctions is that the latter reveals the winner's bid while the former reveals the winner's bid and the bids of all the lower bidders. Similarly, the difference between E and P auctions is that the latter reveals the highest loser's bid while the former reveals the highest loser's bid and the bids of all the lower bidders. Given this symmetry the simple intuition that comes from our results is that adding lower bidders to the highest disclosed bid increases reputational incentives (whether they are increasing or decreasing in type) because lower types now faces a higher chance of their bids being revealed.

With regards to the comparison between $\phi = P$ and $\phi = D$ we wish to emphasize that this is particularly important because it sheds some light on the relationship between what one should expect from first- versus second-price sealed-bid auctions where only the price and the identities of the two highest bidders are revealed. Interestingly, the results suggest that reputational incentives are stronger in P than in D auctions. This follows from the fact that low types have a higher chance of being the highest loser than the winner while the difference between the probability of being the winner or the highest losers is not so significant for high types. In other words, although both disclosure rules disclose only one bid, reputational effects are stronger when bidders feel they have a higher chance of being that one bidder whose bid is disclosed.

Finally, points IIIc. and IVc. tell us that if $V(x)$ is strictly increasing, then reputational effects with the disclosure rule $\phi = T$ (and hence $\phi = E$) will eventually dominate revenue-wise those with disclosure rule $\phi = S$, and vice versa. Intuitively, when the number of bidders is high, there are much stronger reputational incentives for losers in T auctions. The reason is that their bids will be revealed in T auctions regardless of the number of bidders, while in an S auction only the fact that they lost will be revealed. The latter implies that the reputational returns upon losing are decreasing with the number of bidders in S auctions. Conversely if $V(x)$ is strictly decreasing. Similar intuition explains also the results in points IIIb. and IVb., which deal with changes in expected revenues as a function of the number of bidders for disclosure rules $\phi = T, E$.

A very interesting implication of the above proposition, given the much celebrated revenue equivalence of first- and second-price sealed-bid auctions when reputational incentives are absent is that

Corollary 2 *Consider a first-price and a second-price sealed-bid auction where only the price, the corresponding bidder and winner are disclosed. Whenever $V_x > 0$ for all $x > \underline{\omega}$ the second-price auction generates more expected revenues than the first-price auction, and vice versa.*

Proof Directly from (a) the fact that the second-price auction is de facto a P auction and the first-price auction is de facto a D auction, (b) the revenue equivalence of FP-D and SP-D, and (c) the comparison of P and D auctions in parts IIIa and IVa of the above proposition.■

Further points deserve to be made that compare our results in proposition 4 with those above:

- The result in proposition 5 IIIa. and IVa. above provides a striking contrast with that in proposition 4 because it may reverse the implicit ranking we had in that proposition: if $V_x > 0$, we are more likely to have a PBE where the disclosure rule is $\phi = D$ than where the disclosure rule is $\phi = T$, but, conditional on existence of PBE in both cases, expected revenues will be higher with $\phi = T$. Thus, if overbidding takes place, then a government might choose a different disclosure rule from $\phi = D, T$ than a profit-maximizing seller.

- It is interesting that the linkage principle - obtained for single-object auctions by Milgrom and Weber [18] - has been broadly interpreted as implying that more public information raises prices and revenue. Several of our results above, however, could be interpreted as a failure of such interpretation of the linkage principle in environments where reputational effects are in place, rather than interdependent, correlated values.[21] This is, in particular, emphasized by:

Corollary 3 *Suppose that underbidding takes place. Full disclosure is dominated in terms of expected revenues by non-disclosure of bids if the number of bidders is sufficiently high, and by the disclosure rules $\phi = D, P$.*

Proof Directly from the relevant comparisons in part IV of the above proposition.■

Corollary 1 and 3, in turn, imply the following interesting result in terms of optimality of full-disclosure:

Corollary 4 *Suppose that underbidding takes place. Full disclosure is dominated by another disclosure rule both in terms of expected revenues and guaranteeing that the object is sold to the bidder with the highest type/user-value.*

4.3. Winner's only reputational concerns

To develop further understanding of the above results, we now provide an analysis of the case where it is only winners who have reputational concerns. Analysis is much simpler now because disclosure rules only matter to the extent that they provide different information about the winner's type. Indeed, since

$$v^{\phi l}(y, z_i) \equiv 0$$

[21] For a similar argument in multi-unit sequential auction with unit-demands and interdependent types/signals see [16].

one can easily see that theorems 1 and 2 hold to this setting as well, after using the appropriate effective valuations. In particular, now we have

$$\begin{aligned}\psi^T(x_i) &= \psi^D(x_i) = x_i + V(x_i) + \frac{G(x_i)}{g(x_i)}V_x(x_i),\\ \psi^E(x_i) &= \psi^P(x_i) = x_i + T(x_i)\\ \psi^S(x_i) &= x_i + E_G[T(Y)]\end{aligned}$$

Not surprisingly, we have that $\psi^T(x_i) = \psi^D(x_i)$ since they both reveal the winner's type. Similarly, $\psi^E(x_i) = \psi^P(x_i)$ since they both reveal the highest loser's type. Given the strategic equivalence between T and D on the one hand and E and P on the other, we now focus on disclosure rules T, E and S. Figure 2 below shows the relevant reputational effects with the same parametrization as in Figure 1:

[INSERT FIGURE 2 HERE]

With regards to existence of PBE, our first result is that:

Proposition 6 *If V is strictly increasing then bidding functions for disclosure rules E and S are strictly increasing as is the first-price bidding function for disclosure rule T.*[22]

Proof. For E and S the result follows immediately from the fact that $\psi^E(x_i)$ and $\psi^S(x_i)$ are strictly increasing. For T we just need to observe that

$$E\left[\psi^T(Y)\,|Y < x\right] = \frac{1}{G(x)}\int_{\underline{\omega}}^{x} ydG(y) + V(x_i)$$

which is clearly strictly increasing. ■

In other words, when only winners have reputational concerns and these reputational concerns reward high types, using a first-price auction guarantees existence of PBE regardless of disclosure rule. This is a very different result from the corresponding case where losers also have reputational concerns; in that case efficiency was much more difficult to achieve.[23] Intuitively

[22]Since the SP bidding function being strictly increasing implies that the FP is also strictly increasing (but the reverse does not necessarily hold) then FP bidding functions for E and S are also strictly increasing.

[23]If V is decreasing then, of course, monotonicity is not guaranteed here either. Simple inspection of the bidding functions for T and E provides precise conditions.

this is because when only winners have reputational concerns, low types are relatively unconcerned with reputational effects and do not overbid much, while when they do have very strong reputational concerns they will have very strong incentives to overbid that bidding functions might actually be decreasing initially.

Our results might change significantly with regard to revenue ranking too. In particular, we have:

Proposition 7 *In a PBE,*

1. $ER(T) = ER(E)$
2. *If* V *is strictly increasing then* $ER(S) > ER(T)$ *while if* V *is strictly decreasing then* $ER(S) < ER(T)$.

Proof. See appendix. ■

The first part of this result is not particularly striking in that compared with the other case where losers have reputational concerns, T and E provide the same information for losers. In other words, once you know losers' types, knowing the winner's type is irrelevant for expected revenue, whether the losers have reputational concerns or not. The second result is much more interesting because in combination with the first, provides a clear prediction: if reputational concerns are such that being perceived of a high type is good, then revealing the winning bid or the highest loser's type will reduce expected revenues. The intuition is simple. Since losing brings no reputational returns, low types have relatively lower reputational incentives in a T auction, compared to S auctions. This follows from the fact that winning with a low type is not particularly good news in T auctions, while in a S auction, where only the fact that one won the auction is reported, is better news. Conversely, if reputational concerns are decreasing in type, revealing information such information is preferable to revealing just the identity of the winner.

5. Conclusions

This paper is a step towards the study of auction theory when bidders have reputational concerns. We study a set up with private independent types/signals and our analysis shows how, rather surprisingly, given the previous literature, many of the general results follow in a similar fashion to the basic results of the familiar independent values set up, with the important qualification that here effective valuations, not just types/user-values need to be used. We show how disclosure rules and not price mechanisms are really relevant in this context and discuss the relative implications of using different disclosure rules for guaranteeing existence of symmetric and monotone equilibria and for maximizing the seller's expected revenue.

The analysis we have presented here is just a first step towards understanding the effect of external incentives on bidding behavior. Research in progress is concerned with extending the results presented here. First of all, external incentives distort bidding behavior in a world where the revenue equivalence theorem would otherwise apply. One would want to investigate how these incentives would affect revenue ranking results in the case of affiliated signals and/or multi-object auctions and/or bidders who are asymmetrically informed initially. Also, one could ask how external incentives and disclosure rules affect the decision to participate in the auction. Finally, it would be interesting to investigate the case where bidders are agents for principals (say managers bidding on behalf of shareholders) and whether a) explicit incentives can help to reduce the extent of the problem and b) credible communication between bidders and shareholders is possible.

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6. Proofs

Effective Valuation for P auctions We have:

$$\begin{aligned} v^{Pw}(y,z_i) &= T(y) \\ v^{Pl}(y,z_i) &= \int_{\underline{\omega}}^{z_i} V(z_i)dL(y_2|y) + \int_{z_i}^{\overline{\omega}} Z(y_2)\,dL(y_2|y) \end{aligned}$$

where

$$l(y_2|y) = \begin{cases} \frac{(N-2)F(y_2)^{N-3}f(y_2)}{F(y)^{N-2}} & if \quad y_2 \leq y \\ 0 & if \quad y_2 > y \end{cases}$$

$$L(y_2|y) = \begin{cases} \frac{F(y_2)^{N-2}}{F(y)^{N-2}} & if \quad y_2 \leq y \\ 1 & if \quad y_2 > y \end{cases}.$$

Given the above

$$\begin{aligned} v_z^{Pw}(y,z_i) &= 0 \\ v_z^{Pl}(y,z_i) &= L(z_i|y)\,V_x(z_i) + V(z_i)\,l(z_i|y) - Z(z_i)\,l(z_i|y) \end{aligned}$$

so that

$$\begin{aligned} \Psi^P(x_i,x_i) &\equiv \psi^P(x_i) = x_i + T(x_i) - V(x_i)L(x_i|x_i) - \int_{x_i}^{\overline{\omega}} Z(y_2)dL(y_2|x_i) \\ &+ \frac{V_x(x_i)}{g(x_i)}\int_{x_i}^{\overline{\omega}} L(x_i|t)\,dG(t) + \frac{[V(x_i)-Z(x_i)]}{g(x_i)}\int_{x_i}^{\overline{\omega}} l(x_i|t)\,dG(t) \end{aligned}$$

But

$$L(x_i|x_i) = 1$$
$$\text{and} \int_{x_i}^{\overline{\omega}} Z(y_2)dL(y_2|x_i) = 0.$$

Also

$$\begin{aligned} \int_{x_i}^{\overline{\omega}} L(x_i|t)\,dG(t) &= \int_{x_i}^{\overline{\omega}} \frac{F(x_i)^{N-2}}{F(t)^{N-2}} dF(t)^{N-1} \\ &= (N-1)\,F(x_i)^{N-2}\int_{x_i}^{\overline{\omega}} f(t)\,dt \\ &= (N-1)\,F(x_i)^{N-2}(1-F(x_i)) \\ &= \frac{(1-F(x_i))\,g(x_i)}{f(x_i)} \end{aligned}$$

and

$$
\begin{aligned}
\int_{x_i}^{\varpi} l\left(x_i|t\right) dG\left(t\right) &= \int_{x_i}^{\varpi} \frac{(N-2)F\left(x_i\right)^{N-3} f(x_i)}{F\left(t\right)^{N-2}} dF\left(t\right)^{N-1} \\
&= \left(N-1\right)\left(N-2\right) F\left(x_i\right)^{N-3} f(x_i) \int_{x_i}^{\varpi} f\left(t\right) dt \\
&= \left(N-2\right) \frac{g(x_i)}{F(x_i)} \left(1-F\left(x_i\right)\right).
\end{aligned}
$$

Using these results we have

$$
\psi^P\left(x_i\right) = x_i + T(x_i) - V(x_i) + \frac{\left[V_x\left(x_i\right) + \left(N-2\right) Z_x\left(x_i\right)\right]}{f\left(x_i\right)} \left(1-F\left(x_i\right)\right)
$$

where we have used

$$
Z_x\left(x_i\right) = \frac{f\left(x_i\right)}{F\left(x_i\right)} \left[V\left(x_i\right) - Z\left(x_i\right)\right]
$$

from Lemma in the main text.■

Proof of Proposition 4:

I. This is trivial.

IIa and IIb

Recall that

$$
\begin{aligned}
\psi^T\left(x\right) &= x + \frac{V_x\left(x\right)}{g\left(x\right)} \\
\psi^D\left(x\right) &= x + V\left(x\right) - Z\left(x\right) + G(x)\frac{V_x\left(x\right)}{g\left(x\right)} \\
\psi^E\left(x\right) &= x + T\left(x\right) - V\left(x\right) + \left(1-G(x)\right) \frac{V_x\left(x\right)}{g\left(x\right)}
\end{aligned}
$$

Thus,

$$
\begin{aligned}
\psi_x^D\left(x\right) &= 1 - G(x) + V_x\left(x\right) - Z_x\left(x\right) + g(x)\widetilde{\psi}^T\left(x\right) + G(x)\psi_x^T\left(x\right) \\
\psi_x^E\left(x\right) &= G(x) + T_x\left(x\right) - V_x\left(x\right) - g(x)\widetilde{\psi}^T\left(x\right) + \left(1-G(x)\right)\psi_x^T\left(x\right)
\end{aligned}
$$

Clearly, then, if $V_x > 0$ (and hence $\widetilde{\psi}^T\left(x\right) > 0$) and $V_x\left(x\right) > Z_x\left(x\right)$, then $\psi_x^T\left(x\right) > 0$ implies that $\psi_x^D\left(x\right) > 0$. Similarly, if $V_x < 0$ (and hence $\widetilde{\psi}^T\left(x\right) < 0$) and $V_x\left(x\right) < T_x\left(x\right)$, then $\psi_x^T\left(x\right) > 0$ implies that $\psi_x^E\left(x\right) > 0$.

To complete the proof define first

$$\begin{aligned} Z_V(v) &= E_{F_V}[V|V \leq v] \\ T_V(v) &= E_{F_V}[V|V \geq v], \end{aligned}$$

and note (a) if $V_x > 0$, then $V_x(x) - Z(x)$ has the sign of $\frac{d}{dv}(v - Z_V(v))$, while $V_x(x) - T_x(x)$ has the sign of $\frac{d}{dv}(v - T_V(v))$, and (b) if $V_x < 0$, then $V_x(x) - Z(x)$ has the sign of $\frac{d}{dv}(T_V(v) - v)$, while $V_x(x) - T_x(x)$ has the sign of $\frac{d}{dv}(Z_V(v) - v)$. Finally, note that (a) $\frac{d}{dv}(v - Z_V(v))$ be positive is equivalent to the condition that $\int_{V(\underline{\omega})}^{v} F_V(s)\,ds$ is log-concave (Bagnoli and Bergstrom [2], Lemma 1), (b) $\frac{d}{dv}(v - T_V(v))$ be positive is equivalent to the condition that $\int_{v}^{V(\overline{\omega})}(1 - F_V(s))\,ds$ is log-concave (Bagnoli and Bergstrom [2], Lemma 2), and (c) $\int_{V(\underline{\omega})}^{v} F_V(s)\,ds$ and $\int_{v}^{V(\overline{\omega})}(1 - F_V(s))\,ds$ are log-concave whenever f_V is log-concave (Bagnoli and Bergstrom [2], Theorems 1 and 3).

IIIa and IIIb

We have

$$\begin{aligned} \widetilde{\psi}_x^D(x) &= V_x(x) - Z_x(x) + g(x)\widetilde{\psi}^T(x) + G(x)\widetilde{\psi}_x^T(x) \\ &\Leftrightarrow \widetilde{\psi}_x^T(x) = \frac{1}{G(x)}\left[\widetilde{\psi}_x^D(x) - V_x(x) + Z_x(x) - g(x)\widetilde{\psi}^T(x)\right] \\ \widetilde{\psi}_x^E(x) &= T_x(x) - V_x(x) - g(x)\widetilde{\psi}^T(x) + (1 - G(x))\,\widetilde{\psi}_x^T(x) \\ &\Leftrightarrow \widetilde{\psi}_x^T(x) = \frac{1}{1 - G(x)}\left[\widetilde{\psi}_x^E(x) - T_x(x) + V_x(x) + g(x)\widetilde{\psi}^T(x)\right] \end{aligned}$$

The proof is complete after recalling from above the relationships between the signs of V_x, $V_x - Z_x$ and $T_x - V_x$ when f_V is log-concave.■

Proof of Proposition 5: Given our bidding functions are separable between a non-reputational component and a reputational component, and given that the former is the same across disclosure rules, we can restrict attention to the reputational component of expected revenue for each disclosure rule. This is defined as $\widetilde{ER}(\phi)$ and for T auctions we have

$$\widetilde{ER}(T) = N\int_{\underline{\omega}}^{\overline{\omega}}\int_{\underline{\omega}}^{x_i}\frac{V_x(y)}{g(y)}dG(y)\,dF(x_i),$$

for D auctions

$$\widetilde{ER}(D) = N\int_{\underline{\omega}}^{\overline{\omega}}\int_{\underline{\omega}}^{x_i}\left[V(y) - Z(y) + \frac{G(y)}{g(y)}V_x(y)\right]dG(y)\,dF(x_i),$$

for E auctions

$$\widetilde{ER}(E) = N \int_{\underline{\omega}}^{\overline{\omega}} \int_{\underline{\omega}}^{x_i} \left[T(y) - V(y) + \frac{1 - G(y)}{g(y)} V_x(y) \right] dG(y)\, dF(x_i),$$

for P auctions

$$\widetilde{ER}(P) = N \int_{\underline{\omega}}^{\overline{\omega}} \int_{\underline{\omega}}^{x_i} \left[T(y) - V(y) + \frac{1 - F(y)}{f(y)} \left(V_x(y) + (N-2) Z_x(y) \right) \right] dG(y)\, dF(x_i),$$

and, finally, for S auctions

$$\begin{aligned}
\widetilde{ER}(S) &= N \int_{\underline{\omega}}^{\overline{\omega}} \int_{\underline{\omega}}^{x_i} \int_{\underline{\omega}}^{\overline{\omega}} (T(y) - Z(y))\, dG(y)\, dG(y)\, dF(x_i) \\
&= \left(\int_{\underline{\omega}}^{\overline{\omega}} (T(y) - Z(y))\, dG(y) \right) \int_{\underline{\omega}}^{\overline{\omega}} N G(x_i)\, dF(x_i) \\
&= \int_{\underline{\omega}}^{\overline{\omega}} (T(y) - Z(y))\, dG(y) \int_{\underline{\omega}}^{\overline{\omega}} dF^N(x_i) \\
&= \int_{\underline{\omega}}^{\overline{\omega}} (T(y) - Z(y))\, dG(y).
\end{aligned}$$

I. We have that

$$\widetilde{ER}(T) - \widetilde{ER}(E) = N \int_{\underline{\omega}}^{\overline{\omega}} \int_{\underline{\omega}}^{x_i} \left[V(y) - T(y) + \frac{G(y)}{g(y)} V_x(y) \right] dG(y)\, dF(x_i)$$

The proof proceeds in two steps. We first show that the difference is independent of N and then show revenue equivalence for $N = 2$. We begin by noting that

$$\begin{aligned}
& \int_{\underline{\omega}}^{\overline{\omega}} \int_{\underline{\omega}}^{x_i} \frac{G(y)}{g(y)} V_x(y)\, dG(y)\, dF(x_i) \\
=\ & \int_{\underline{\omega}}^{\overline{\omega}} \int_{\underline{\omega}}^{x_i} \int_{\underline{\omega}}^{y} \frac{1}{g(y)} V_x(y)\, dG(s)\, dG(y)\, dF(x_i) \\
=\ & \int_{\underline{\omega}}^{\overline{\omega}} \int_{\underline{\omega}}^{x_i} \int_{s}^{x_i} V_x(y)\, dy dG(s)\, dF(x_i) \\
=\ & \int_{\underline{\omega}}^{\overline{\omega}} \int_{\underline{\omega}}^{x_i} [V(x_i) - V(s)]\, dG(s)\, dF(x_i) \\
=\ & \int_{\underline{\omega}}^{\overline{\omega}} \int_{\underline{\omega}}^{x_i} [V(x_i) - V(y)]\, dG(y)\, dF(x_i).
\end{aligned}$$

So,

$$\begin{aligned}
\widetilde{ER}(T) - \widetilde{ER}(E) &= N \int_{\underline{\omega}}^{\overline{\omega}} \int_{\underline{\omega}}^{x_i} [V(x_i) - T(y)]\, dG(y)\, dF(x_i) \\
&= N \int_{\underline{\omega}}^{\overline{\omega}} \int_{\underline{\omega}}^{x_i} V(x_i)\, dG(y)\, dF(x_i) - N \int_{\underline{\omega}}^{\overline{\omega}} \int_{y}^{\overline{\omega}} T(y)\, dF(x_i)\, dG(y) \\
&= N \int_{\underline{\omega}}^{\overline{\omega}} V(x_i)\, G(x_i)\, dF(x_i) - N \int_{\underline{\omega}}^{\overline{\omega}} T(x_i)\, (1 - F(x_i))\, dG(x_i).
\end{aligned}$$

Using the definition of $G = F^{N-1}$ we have directly that

$$N \int_{\underline{\omega}}^{\overline{\omega}} \{V(x_i) F(x_i) - (N-1) T(x_i)(1 - F(x_i))\} F^{N-2}(x_i) \, dF(x_i).$$

Therefore, the sign of $\widetilde{ER}(T) - \widetilde{ER}(E)$ is the sign of

$$\Theta(N) \equiv \int_{\underline{\omega}}^{\overline{\omega}} \{V(x_i) F(x_i) - (N-1) T(x_i)(1 - F(x_i))\} F^{N-2}(x_i) \, dF(x_i).$$

The proof of 1. follows from the fact that $\Theta(2) = 0$ and $\Theta(N+1) = \Theta(N)$ for any $N > 1$. We show these next. First, note that

$$\begin{aligned}
\Theta(N+1) - \Theta(N+1) = & - \int_{\underline{\omega}}^{\overline{\omega}} V(x_i)(1 - F(x_i)) F^{N-1}(x_i) \, dF(x_i) + \int_{\underline{\omega}}^{\overline{\omega}} T(x_i)(1 - F(x_i)) dF^{N-1}(x_i) \\
& - \int_{\underline{\omega}}^{\overline{\omega}} T(x_i)(1 - F(x_i)) dF^{N}(x_i) \\
= & - \int_{\underline{\omega}}^{\overline{\omega}} V(x_i)(1 - F(x_i)) F^{N-1}(x_i) \, dF(x_i) \\
& + \int_{\underline{\omega}}^{\overline{\omega}} \left(F^{N}(x_i) - F^{N-1}(x_i)\right) \left[T_x(x_i)(1 - F(x_i)) - f(x_i) T(x_i)\right] dx_i,
\end{aligned}$$

where the last equality follows from integration by parts. Given that

$$T_x(x) = \frac{f(x)}{1 - F(x)} \left(T(x) - V(x)\right),$$

(recall the Lemma) we have that

$$\begin{aligned}
& \Theta(N+1) - \Theta(N) \\
= & - \int_{\underline{\omega}}^{\overline{\omega}} V(x_i)(1 - F(x_i)) F^{N-1}(x_i) \, dF(x_i) - \int_{\underline{\omega}}^{\overline{\omega}} \left(F^{N}(x_i) - F^{N-1}(x_i)\right) V(x_i) \, dF(x_i) \\
= & - \int_{\underline{\omega}}^{\overline{\omega}} V(x_i) \left[F^{N}(x_i) - F^{N-1}(x_i) + (1 - F(x_i)) F^{N-1}(x_i)\right] dF(x_i) = 0.
\end{aligned}$$

We now consider the case of two bidders. In that case, we have that

$$\begin{aligned}
\Theta(2) = & \int_{\underline{\omega}}^{\overline{\omega}} \{V(x_i) F(x_i) - T(x_i)(1 - F(x_i))\} \, dF(x_i) \\
= & \int_{\underline{\omega}}^{\overline{\omega}} V(x_i) F(x_i) \, dF(x_i) + \int_{\underline{\omega}}^{\overline{\omega}} F(x_i) \, d[T(x_i)(1 - F(x_i))]\} \\
= & \int_{\underline{\omega}}^{\overline{\omega}} F(x_i) \left[V(x_i) f(x_i) + (1 - F(x_i)) T_x(x_i) - f(x_i) T(x_i)\right] dx_i = 0.
\end{aligned}$$

as desired, where the second equality follows from integration by parts and the last equality follows from the Lemma.

II. We will need first to prove the following: if $V(\bullet)$ is strictly increasing or decreasing, then $T(x) - Z(x)$ has the opposite monotonicity of f_V. To prove this, note first from Jewitt [10] that $E_F[X|X \geq x] - E_F[X|X < x]$ has the opposite monotonicity of f_X. Note now that if $V(\bullet)$ is strictly increasing, with $v \equiv V(x)$, then

$$\begin{aligned}
& T(x) - Z(x) \\
= \;& E_F[V(X)|X \geq x] - E_F[V(X)|X \leq x] \\
= \;& E_F[V(X)|V(X) \geq v] - E_F[V(X)|V(X) \leq v] \\
= \;& E_{F_V}[V|V \geq v] - E_{F_V}[V|V \leq v] \\
= \;& T_V(v) - Z_V(v)
\end{aligned}$$

Thus, $T(x) - Z(x)$ has the opposite monotonicity of f_V. Conversely, if $V(\bullet)$ is strictly decreasing then

$$T(x) - Z(x) = Z_V(v) - T_V(v)$$

but then

$$\frac{d[T(x) - Z(x)]}{dx} = \frac{d(Z_V(v) - T_V(v))}{dv}\frac{dv}{dx}$$

Since $\frac{dv}{dx} < 0$ by assumption, we have then that the monotonicity of $T(x) - Z(x)$ has the same sign as the monotonicity of $T_V(v) - Z_V(v)$ and thus the opposite monotonicity of f_V.

Next, we compare expected revenues for disclosure rules $\phi = D$ versus $\phi = S$. We know from 1. that $\widetilde{ER}(E) = \widetilde{ER}(T)$. This means that

$$\begin{aligned}
\widetilde{ER}(D) &= \widetilde{ER}(D) + \widetilde{ER}(E) - \widetilde{ER}(T) \\
&= N\int_{\underline{\omega}}^{\overline{\omega}}\int_{\underline{\omega}}^{x_i}[T(y) - Z(y)]\,dG(y)\,dF(x_i) \\
&= \int_{\underline{\omega}}^{\overline{\omega}}[T(y) - Z(y)]\,N(1 - F(y))\,dG(y) \\
&= \int_{\underline{\omega}}^{\overline{\omega}}[T(y) - Z(y)]\,dF_2^{(N)}(y),
\end{aligned}$$

while, recall that

$$\widetilde{ER}(S) = \int_{\underline{\omega}}^{\overline{\omega}}[T(y) - Z(y)]\,dG(y).$$

So, we have that

$$\begin{aligned}\widetilde{ER}(D)-\widetilde{ER}(S) &= \int_{\underline{\omega}}^{\overline{\omega}}[T(y)-Z(y)]\left(f_2^{(N)}(y)-f_1^{(N-1)}(y)\right)dy \\ &= \int_{\underline{\omega}}^{\overline{\omega}}\left(F_1^{(N-1)}(y)-F_2^{(N)}(y)\right)(T_x(y)-Z_x(y))\,dy.\end{aligned}$$

But

$$\begin{aligned} & F_1^{(N-1)}(y)-F_2^{(N)}(y) \\ = {} & F^{N-1}(y)-NF^{N-1}(y)+(N-1)F^N(y) \\ = {} & (N-1)\left(F^N(y)-F^{N-1}(y)\right)<0 \text{ a.e.}\end{aligned}$$

So, we have, after recalling our result above on the properties of $T_x(y)-Z_x(y)$ that

$$\widetilde{ER}(D)-\widetilde{ER}(S)\begin{array}{lll} >0 & if & f_V \text{ increasing} \\ =0 & if & f_V \text{ uniform} \\ <0 & if & f_V \text{ decreasing}\end{array}.$$

Next, we show how these expected revenues change with the number of bidders. We show this for $\phi = S$ and the proof for $\phi = D$ is entirely analogous. With some abuse of notation, let us introduce the dependence of the expected revenues on the number of bidders. Then, for N bidders, we have

$$\begin{aligned}\widetilde{ER}(S,N) &= \int_{\underline{\omega}}^{\overline{\omega}}[T(y)-Z(y)]\,dF_1^{(N-1)}(y) \\ &= [T(\overline{\omega})-Z(\overline{\omega})]-\int_{\underline{\omega}}^{\overline{\omega}}[T_x(y)-Z_x(y)]\,F_1^{(N-1)}(y)\,dy.\end{aligned}$$

So, we have

$$\begin{aligned} & \widetilde{ER}(S,N+1)-\widetilde{ER}(S,N) \\ = {} & \int_{\underline{\omega}}^{\overline{\omega}}[T_x(y)-Z_x(y)]\,F^{N-1}(y)\,[1-F(y)]\,dy.\end{aligned}$$

Thus, the sign of $\widetilde{ER}(S,N+1)-\widetilde{ER}(S,N)$ is dictated by the sign of $T_x(y)-Z_x(y)$. Clearly, then,

$$\widetilde{ER}(S,N+1)-\widetilde{ER}(S,N)\begin{array}{lll} >0 & if & f_V \text{ decreasing} \\ =0 & if & f_V \text{ uniform} \\ <0 & if & f_V \text{ increasing.}\end{array}$$

IIIa and IVa.

We provide the proof by comparing $E_G\left[\psi^{\phi}(Y)|Y<x_i\right]$ across for the relevant rules for T vs D and for E vs. P. This establishes that for FP auctions with V strictly increasing, T and E provide higher revenues than D and P respectively. For the comparison between P and D, on the other hand, our result only applies to expected revenue.

We begin with the comparison between T and D.

$$\begin{aligned} & E_G\left[\psi^{T}(Y)|Y<x_i\right]-E_G\left[\psi^{D}(Y)|Y<x_i\right] \\ = & \frac{1}{G(x_i)}\int_{\underline{\omega}}^{x_i}\left[Z(y)-V(y)+\frac{1-G(y)}{g(y)}V_x(y)\right]dG(y). \end{aligned}$$

But

$$\begin{aligned} & \int_{\underline{\omega}}^{x_i}\frac{1-G(y)}{g(y)}V_x(y)\,dG(y) \\ = & \int_{\underline{\omega}}^{x_i}\int_{y}^{\overline{\omega}}\frac{1}{g(y)}V_x(y)\,dG(s)\,dG(y) \\ = & \int_{\underline{\omega}}^{x_i}\int_{\underline{\omega}}^{s}V_x(y)\,dydG(s)+\int_{x_i}^{\overline{\omega}}\int_{\underline{\omega}}^{x_i}V_x(y)\,dydG(s) \\ = & \int_{\underline{\omega}}^{x_i}\left[V(s)-V(\underline{\omega})\right]dG(s)+\int_{x_i}^{\overline{\omega}}\left[V(x_i)-V(\underline{\omega})\right]dG(s) \\ = & \int_{\underline{\omega}}^{x_i}V(y)\,dG(y)+(1-G(x_i))\,V(x_i)-V(\underline{\omega}), \end{aligned}$$

and so we have

$$\begin{aligned} & \int_{\underline{\omega}}^{x_i}\left[Z(y)-V(y)+\frac{1-G(y)}{g(y)}V_x(y)\right]dG(y) \\ = & \int_{\underline{\omega}}^{x_i}Z(y)\,dG(y)+\int_{x_i}^{\overline{\omega}}V(x_i)\,dG(y)-V(\underline{\omega}). \end{aligned}$$

Clearly, if $V(\bullet)$ is strictly increasing then $V(x_i)>V(\underline{\omega})$ and $Z(y)>V(\underline{\omega})$ for any $x_i,y>\underline{\omega}$, while if $V(\bullet)$ is strictly decreasing then $V(\underline{\omega})>V(x_i)$ and $V(\underline{\omega})>Z(y)$ for any $x_i,y>\underline{\omega}$, and the result follows directly.

Now consider the comparison between E and P.

$$\begin{aligned} & E_G\left[\psi^{E}(Y)|Y<x_i\right]-E_G\left[\psi^{P}(Y)|Y<x_i\right] \\ = & \frac{1}{G(x_i)}\int_{\underline{\omega}}^{x_i}\left[\frac{1-G(y)}{g(y)}V_x(y)-\frac{1-F(y)}{f(y)}\left[V_x(y)+(N-2)Z_x(y)\right]\right]dG(y). \end{aligned}$$

Clearly, this is zero for $N=2$. For $N\geq 3$, we already know that

$$\int_{\underline{\omega}}^{x_i}\frac{1-G(y)}{g(y)}V_x(y)\,dG(y)=\int_{\underline{\omega}}^{x_i}V(y)\,dG(y)+(1-G(x_i))\,V(x_i)-V(\underline{\omega})$$

Now,

$$
\begin{aligned}
& \int_{\underline{\omega}}^{x_i} \frac{1-F(y)}{f(y)} V_x(y)\, dG(y) \\
= \;& \int_{\underline{\omega}}^{x_i} \int_{y}^{\overline{\omega}} \frac{1}{f(y)} V_x(y)\, dF(s)\, dG(y) \\
= \;& (N-1) \left[\int_{\underline{\omega}}^{x_i} \int_{\underline{\omega}}^{s} V_x(y)\, F(y)^{N-2}\, dy dF(s) + \int_{x_i}^{\overline{\omega}} \int_{\underline{\omega}}^{x_i} V_x(y)\, F(y)^{N-2}\, dy dF(s) \right] \\
= \;& (N-1) \left[\begin{array}{c} \int_{\underline{\omega}}^{x_i} \left[F(s)^{N-2} V(s) - \int_{\underline{\omega}}^{s} V(y)\, dF(y)^{N-2} \right] dF(s) \\ + \int_{x_i}^{\overline{\omega}} \left[F(x_i)^{N-2} V(x_i) - \int_{\underline{\omega}}^{x_i} V(y)\, dF(y)^{N-2} \right] dF(s) \end{array} \right] \\
= \;& \int_{\underline{\omega}}^{x_i} V(y)\, dG(y) + (N-1)(1-F(x_i)) \left[F(x_i)^{N-2} V(x_i) - \int_{\underline{\omega}}^{x_i} V(y)\, dF(y)^{N-2} \right] \\
& -(N-1) \int_{\underline{\omega}}^{x_i} \int_{\underline{\omega}}^{s} V(y)\, dF(y)^{N-2}\, dF(s) \\
= \;& \int_{\underline{\omega}}^{x_i} V(y)\, dG(y) + (N-1)(1-F(x_i)) \left[F(x_i)^{N-2} V(x_i) - \int_{\underline{\omega}}^{x_i} V(y)\, dF(y)^{N-2} \right] \\
& -(N-1) \int_{\underline{\omega}}^{x_i} V(y)\, (F(x_i) - F(y))\, dF(y)^{N-2} \\
= \;& \int_{\underline{\omega}}^{x_i} V(y)\, dG(y) + (N-1) \left[(1-F(x_i))\, F(x_i)^{N-2} V(x_i) - \int_{\underline{\omega}}^{x_i} V(y)\, (1-F(y))\, dF(y)^{N-2} \right]
\end{aligned}
$$

Finally, from the Lemma,

$$
\begin{aligned}
& \int_{\underline{\omega}}^{x_i} (N-2)\, Z_x(y)\, \frac{1-F(y)}{f(y)} dG(y) \\
= \;& \int_{\underline{\omega}}^{x_i} (N-2)\, [V(y) - Z(y)]\, \frac{1-F(y)}{F(y)} dG(y) \\
= \;& (N-1) \int_{\underline{\omega}}^{x_i} [V(y) - Z(y)]\, (1-F(y))\, dF(y)^{N-2}
\end{aligned}
$$

which gives us

$$
\begin{aligned}
& \int_{\underline{\omega}}^{x_i} \left[\frac{1-G(y)}{g(y)} V_x(y) - \frac{1-F(y)}{f(y)} \left[V_x(y) + (N-2) Z_x(y) \right] \right] dG(y) \\
&= (1-G(x_i))\, V(x_i) - (N-1) \left((1-F(x_i))\, V(x_i)\, F(x_i)^{N-2} - \int_{\underline{\omega}}^{x_i} Z(y)\, (1-F(y))\, dF(y)^{N-2} \right) - V(\underline{\omega}) \\
&= V(x_i) \left(1 - F(x)^{N-1} \right) - V(\underline{\omega}) - V(x_i) \left(F_2^{(N-1)}(x_i) - F(x)^{N-1} \right) + \int_{\underline{\omega}}^{x_i} Z(y)\, dF_2^{(N-1)}(y) \\
&= V(x_i) \left(1 - F_2^{(N-1)}(x_i) \right) + \int_{\underline{\omega}}^{x_i} Z(y)\, dF_2^{(N-1)}(y) - V(\underline{\omega}) \\
&= \int_{\underline{\omega}}^{x_i} Z(y)\, dF_2^{(N-1)}(y) + \int_{x_i}^{\overline{\omega}} V(x_i)\, dF_2^{(N-1)}(y) - V(\underline{\omega})
\end{aligned}
$$

The above is positive for $V_x > 0$ since then $V(x), Z(x) > V(\underline{\omega})$ for any $x > \underline{\omega}$. If $V_x < 0$ the above is now negative since $V(x), Z(x) < V(\underline{\omega})$ for any $x > \underline{\omega}$.

Finally, we focus on the comparison between P and D. We know that P and E are equivalent for $N = 2$ and so the result holds for that case, from I, IIIa and IVa. Now we assume $N \geq 3$. Recall that

$$\widetilde{ER}(D) = N \int_{\underline{\omega}}^{\overline{\omega}} \int_{\underline{\omega}}^{x_i} [T(y) - Z(y)]\, dG(y)\, dF(x_i),$$

from II above.

Now,

$$\widetilde{ER}(P) - \widetilde{ER}(D) = N \int_{\underline{\omega}}^{\overline{\omega}} \int_{\underline{\omega}}^{x_i} \left[Z(y) - V(y) + \frac{1 - F(y)}{f(y)} \left(V_x(y) + (N-2) Z_x(y) \right) \right] dG(y)\, dF(x_i)$$

Using that

$$\int_{\underline{\omega}}^{x_i} \frac{1 - F(y)}{f(y)} V_x(y)\, dG(y) =$$
$$\int_{\underline{\omega}}^{x_i} V(y)\, dG(y) + (N-1) \left[(1 - F(x_i)) F(x_i)^{N-2} V(x_i) - \int_{\underline{\omega}}^{x_i} V(y) (1 - F(y))\, dF(y)^{N-2} \right],$$

we can manipulate the term

$$N \int_{\underline{\omega}}^{\overline{\omega}} \int_{\underline{\omega}}^{x_i} \left[Z(y) - V(y) + \frac{1 - F(y)}{f(y)} V_x(y) \right] dG(y)\, dF(x_i)$$

into

$$N(N-1) \{ \int_{\underline{\omega}}^{\overline{\omega}} (1 - F(x_i)) V(x_i) F(x_i)^{N-2} dF(x_i)$$
$$+ \int_{\underline{\omega}}^{\overline{\omega}} \int_{\underline{\omega}}^{x_i} \left[Z(y) F^{N-2}(y) - (N-2) V(y) (1 - F(y)) F(y)^{N-3} \right] dF(y)\, dF(x_i) \}$$
$$= \int_{\underline{\omega}}^{\overline{\omega}} V(x_i)\, dF_2^{(N)}(x_i)$$
$$+ N(N-1) \int_{\underline{\omega}}^{\overline{\omega}} (1 - F(y)) \left[Z(y) F^{N-2}(y) - (N-2) V(y) (1 - F(y)) F(y)^{N-3} \right] dF(y)$$
$$= \int_{\underline{\omega}}^{\overline{\omega}} [V(s) + Z(s)]\, dF_2^{(N)}(s) - 2 \int_{\underline{\omega}}^{\overline{\omega}} V(s)\, dF_3^{(N)}(s),$$

where the last equality follows from noting that

$$\begin{aligned} f_3^{(N)}(x) &= \frac{N!}{(N-3)!2!} F(x)^{N-3} (1 - F(x))^2 f(x) \\ &= \frac{N(N-1)(N-2)}{2} F(x)^{N-3} (1 - F(x))^2 f(x) \end{aligned}$$

Now, we focus on the remaining component of expected revenue:

$$N\int_{\underline{\omega}}^{\overline{\omega}}\int_{\underline{\omega}}^{x_i}\left[\frac{1-F(y)}{f(y)}(N-2)Z_x(y)\right]dG(y)\,dF(x_i) =$$

$$N\int_{\underline{\omega}}^{\overline{\omega}}\left[\frac{(1-F(y))^2}{f(y)}(N-2)Z_x(y)\right]dG(y) =$$

$$= \int_{\underline{\omega}}^{\overline{\omega}} N(N-1)(N-2)F(s)^{N-3}(1-F(s))^2 f(s)[V(s)-Z(s)]\,ds$$

$$= 2\int_{\underline{\omega}}^{\overline{\omega}}[V(s)-Z(s)]\,dF_3^{(N)}(s).$$

Thus,

$$\widetilde{ER}(P)-\widetilde{ER}(D)$$

$$= \int_{\underline{\omega}}^{\overline{\omega}}[V(s)+Z(s)]\,dF_2^{(N)}(s) - 2\int_{\underline{\omega}}^{\overline{\omega}} Z(s)\,dF_3^{(N)}(s)$$

$$> 2\int_{\underline{\omega}}^{\overline{\omega}} Z(s)\,dF_2^{(N)}(s) - 2\int_{\underline{\omega}}^{\overline{\omega}} Z(s)\,dF_3^{(N)}(s)$$

$$= 2\int_{\underline{\omega}}^{\overline{\omega}}\left(F_3^{(N)}(s)-F_2^{(N)}(s)\right)Z_x(s)\,ds > 0$$

with this inequality being true if $V_x > 0$ and hence (from the Lemma) $Z_x > 0$. The last equality follows from integration by parts. The argument is symmetric if $V_x < 0$ ■

IIIb. and IVb. We have that

$$\widetilde{ER}(T) = N\int_{\underline{\omega}}^{\overline{\omega}}[V(x_i)-V(\underline{\omega})]\,dF(x_i).$$

This is clearly positive and unboundedly increasing in N, if $V(\bullet)$ is strictly increasing. Conversely, if $V(\bullet)$ is strictly decreasing.

IIIc. and IVc. We know that

$$\widetilde{ER}(S) = \int_{\underline{\omega}}^{\overline{\omega}}[T(y)-Z(y)]\,dF^{N-1}(y).$$

If $V(\bullet)$ is strictly increasing, then $T(y) > Z(y)$ almost everywhere, and hence $\widetilde{ER}(S)$ is bounded from above by $\max_y\{T(y)-Z(y)\}$. IIIc. follows after recalling from the proof of II above that here $\widetilde{ER}(T)$ is unboundedly increasing in N. Conversely, if $V(\bullet)$ is strictly decreasing.■

Proof of Proposition 7

1. Follows using the same exact method as in I in Proposition 5.

2. We know from 1. that $\widetilde{ER}(T) = \widetilde{ER}(E)$, but

$$\begin{aligned}
\widetilde{ER}(E) - \widetilde{ER}(S) &= N \int_{\underline{\omega}}^{\overline{\omega}} \int_{\underline{\omega}}^{x_i} T(y)\, dG(y)\, dF(x_i) - \int_{\underline{\omega}}^{\overline{\omega}} T(s)\, dG(s) \\
&= N \int_{\underline{\omega}}^{\overline{\omega}} \int_{y}^{\overline{\omega}} T(y)\, dF(x_i)\, dG(y) - \int_{\underline{\omega}}^{\overline{\omega}} T(s)\, dG(s) \\
&= \int_{\underline{\omega}}^{\overline{\omega}} T(s)\, N(1 - F(s))\, dG(s) - \int_{\underline{\omega}}^{\overline{\omega}} T(s)\, dG(s) \\
&= \int_{\underline{\omega}}^{\overline{\omega}} T(s)\, dF_2^{(N)}(s) - \int_{\underline{\omega}}^{\overline{\omega}} T(s)\, dF_1^{(N-1)}(s)
\end{aligned}$$

Thus,

$$\widetilde{ER}(E) - \widetilde{ER}(S) = \int_{\underline{\omega}}^{\overline{\omega}} T(s) \left(f_2^{(N)}(s) - f_1^{(N-1)}(s) \right) ds$$
$$= \int_{\underline{\omega}}^{\overline{\omega}} \left(F_1^{(N-1)}(s) - F_2^{(N)}(s) \right) T_x(s)\, ds$$

but

$$\begin{aligned}
& F_1^{(N-1)}(s) - F_2^{(N)}(s) \\
= \;& F^{N-1}(s) - NF^{N-1}(s) + (N-1)F^{N}(s) \\
= \;& (N-1)\left(F^{N}(s) - F^{N-1}(s) \right) < 0
\end{aligned}$$

which means that whenever $V_x > 0$, and hence $T_x > 0$, then the difference is negative, and vice-versa.■

7. Figures

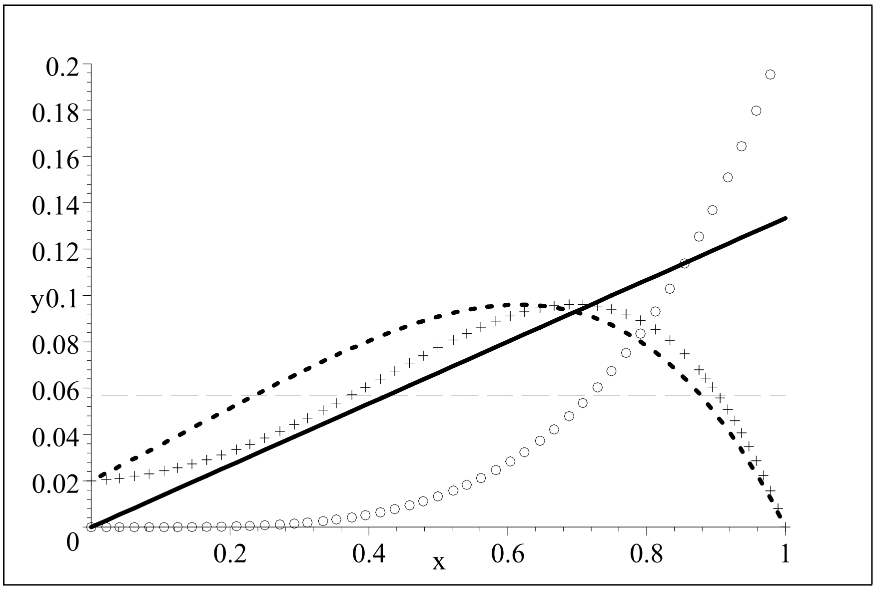

Figure 1. $\widetilde{\psi}^{\phi}(x)$ for our parametrization. The solid line is the case $\phi = T$, the dashed line is $\phi = S$, the circles represent $\phi = D$, the dots $\phi = E$ and the crosses $\phi = P$.

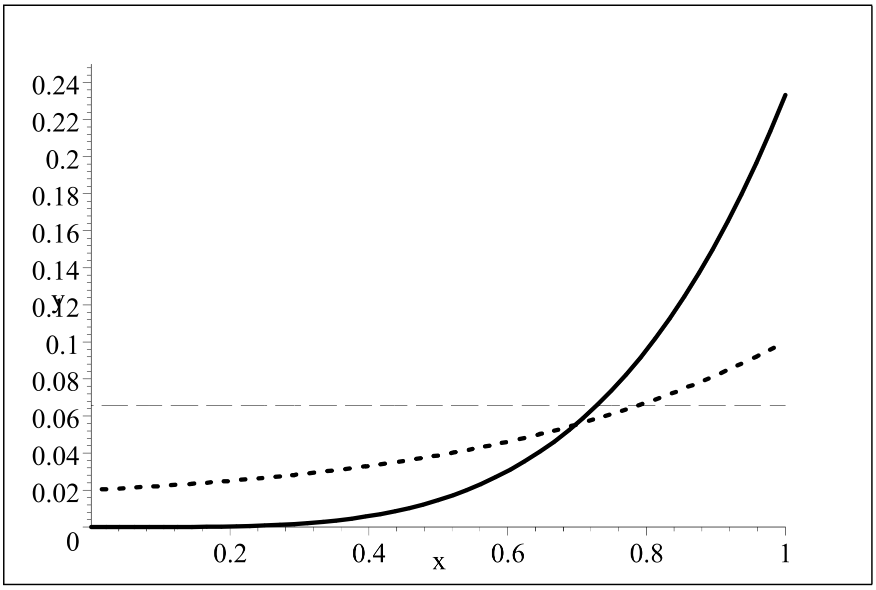

Figure 2. $\widetilde{\psi}^{\phi}(x)$ for our parametrization when only winners have reputational concerns The solid line is the case $\phi = T$, the dashed line is $\phi = S$ and the dotted line represents $\phi = E$.