The Krugman model with product innovation

- Two countries: A is the innovator, B is the follower
- A produces “new” goods
- When a good becomes “old” country B produces and exports it
- One factor of production: labour

- Consumers have the following utility function:

$$U = \left[\sum_{i=1}^{n} c(i)^{\theta} \right]^{1/\theta} \quad 0 < \theta < 1$$

- n is the total number of available goods (new + old)

- When Δn "new" goods are produced, then consumers maximize

$$U = \left[\sum_{i=1}^{n+\Delta n} c(i)^{\theta} \right]^{1/\theta}$$

- subject to the income constraint

$$M = p_A c_A + p_B c_B$$

- One unit of labour produces one unit of good (MPL = 1)

- Because of perfect competition

$$p_A = w_A \quad p_B = w_B$$

- A produces "new" goods and B produces "old" goods because

$$\frac{w_A}{w_B} > 1$$

- Total number of goods is $n = n_A + n_B$

- Let c_A and c_B denote consumption in countries A and B

•We obtain the relative demand function maximizing the lagrangian $\dfrac{c_A}{c_b} = \left(\dfrac{p_A}{p_B}\right)^{-1/(1-\theta)}$

$$V = \left[c_A^{\theta} + c_B^{\theta}\right]^{1/\theta} - \lambda\left(M - p_A c_A - p_B c_B\right)$$

•First order conditions (f.o.c.) are:

$$\frac{\delta V}{\delta C_A} = \frac{1}{\theta}\left(c_A^{\theta} + c_B^{\theta}\right)^{(1-\theta)/\theta} \cdot \theta c_A^{\theta-1} - \lambda p_A = 0$$

$$\frac{\delta V}{\delta C_B} = \frac{1}{\theta}\left(c_A^{\theta} + c_B^{\theta}\right)^{(1-\theta)/\theta} \cdot \theta c_B^{\theta-1} - \lambda p_B = 0$$

$$\frac{\delta V}{\delta \lambda} = M - p_A c_A - p_B c_B = 0$$

- From the first two f.o.c. we get

$$\frac{c_A^{\theta-1}}{c_B^{\theta-1}} = \frac{p_A}{p_B} \quad \rightarrow \frac{c_A}{c_b} = \left(\frac{p_A}{p_B}\right)^{-1/(1-\theta)} = \left(\frac{w_A}{w_B}\right)^{-1/(1-\theta)}$$

- Demand for labour depends on both goods demand and the number of available goods

$$L_A = n_A c_A \quad L_B = n_B c_B$$

- From labour and goods demand we get the relative demand for labour

$$c_A = \frac{L_A}{n_A} \quad c_B = \frac{L_B}{n_B}$$

$$\frac{L_A}{L_B} = \left(\frac{n_A}{n_B}\right)\left(\frac{w_A}{w_B}\right)^{-1/(1-\theta)}$$

•From relative demand for labour we finally find the relative wage

$$\frac{w_A}{w_B} = \left(\frac{n_A}{n_B}\right)^{1-\theta} \left(\frac{L_A}{L_B}\right)^{\theta-1}$$

•Wage differential between A and B depends upon the ratio between “new” and “old” goods

•Wage differential is increasing in the speed A introduces “new” goods

Dynamic equilibrium of the model

- Innovation and imitation continuously occur through time
- The overall stock of “new” and “old” goods depends on innovation and imitation processes
- Because of continuos innovation, n grows according to $\dot{n} = vn$
- Imitation process is measured by $\dot{n}_B = tn_A$
- v and t are positive constant $v > o \quad t > o$
- Imitation lag is equal to $\dfrac{1}{t}$

- Average growth rate of “new” product is

$$\dot{n}_A = \dot{n} - \dot{n}_B = vn - tn_A$$

- We have a dynamic system with two equations

$$\dot{n} = vn$$

$$\dot{n}_A = vn - tn_A$$

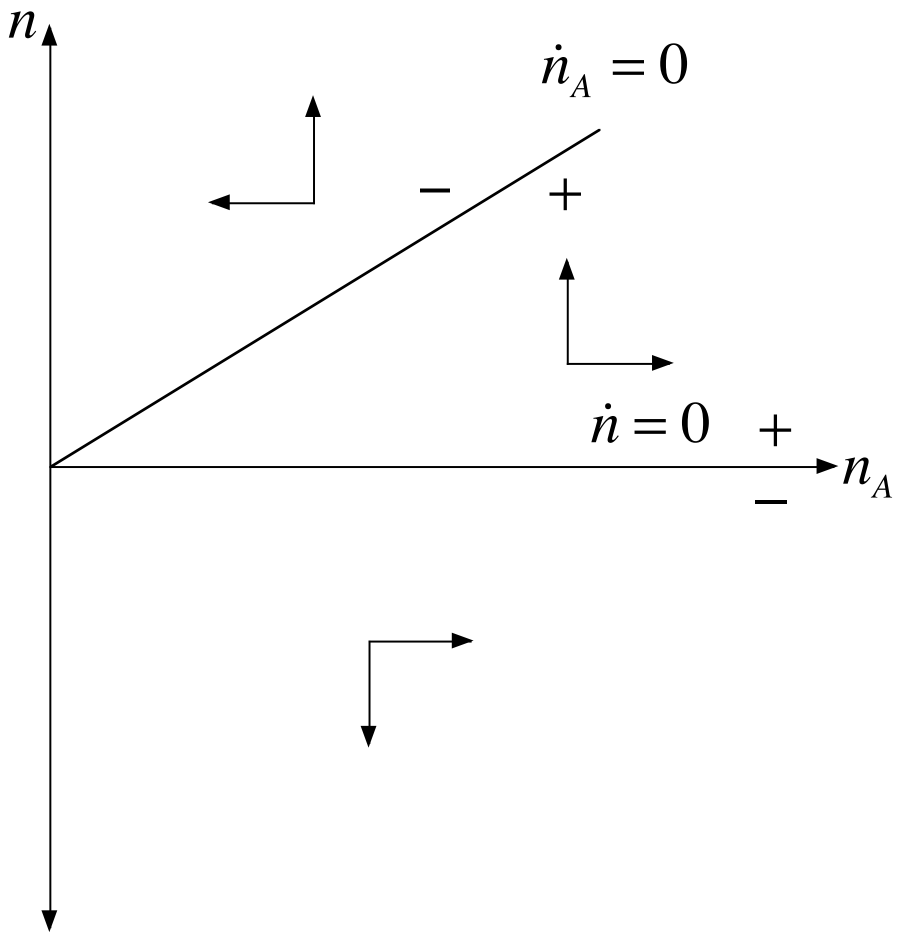
- The system is “instable” and “explosive”
- The number of “new” products continuously increases because of continuos innovation

- A phase diagram:

$$\dot{n} = vn$$

$$\dot{n} = 0 \rightarrow vn = 0 \rightarrow n = 0$$

$$\frac{\delta \dot{n}}{\delta n} = v > 0$$



$$\dot{n}_A = vn - tn_A$$

$$\dot{n}_A = 0 \rightarrow n = \frac{t}{v} n_A$$

$$\frac{\delta \dot{n}_A}{\delta n_A} = \frac{t}{v} > 0$$

- n increases without bounds but the ratio between "new" and "old" goods is stationary $\frac{n_A}{n_B} = \frac{v}{t}$
- $\sigma = {}^{n_A}\!/_{n}$ is the share of "new" goods on the total. Using logarithms, by time differentiation we get:

$$\ln \sigma = \ln n_A - \ln n \qquad d \ln \sigma = \frac{1}{n_A} dn_A - \frac{1}{n} dn$$

$$\frac{d \ln \sigma}{dt} = \frac{1}{n_A}\frac{dn_A}{dt} - \frac{1}{n}\frac{dn}{dt} \qquad \frac{1}{\sigma}\frac{d\sigma}{dt} = \frac{1}{n_A}\dot{n}_A - \frac{1}{n}\dot{n}$$

Multiplying both sides by σ we get $\dot{\sigma} = \frac{\sigma}{n_A}\dot{n}_A - \frac{\sigma}{n}\dot{n}$

$$\dot{\sigma} = \frac{n_A}{n}\frac{\dot{n}_A}{n_A} - \sigma\frac{\dot{n}}{n} \qquad \dot{\sigma} = \frac{\dot{n}_A}{n} - \sigma\frac{\dot{n}}{n}$$

•Since $\dot{n} = vn$ and $\dot{n}_A = vn - tn_A$, from

$$\dot{\sigma} = \frac{\dot{n}_A}{n} - \sigma\frac{\dot{n}}{n} \quad \rightarrow \quad \dot{\sigma} = \frac{vn - tn_A}{n} - \sigma\frac{vn}{n}$$

$$\dot{\sigma} = v - \frac{tn_A}{n} - \sigma v \quad \rightarrow \quad \dot{\sigma} = v - \sigma t - \sigma v$$

•we obtain $\dot{\sigma} = v - (t + v)\sigma$

•therefore, steady state condition $\dot{\sigma} = 0$ implies

$$\sigma = \frac{v}{(t + v)}$$

- recalling that $n = n_A + n_B \qquad \sigma = \dfrac{n_A}{n}$

$$\frac{n_A}{n_B} = \frac{n - n_B}{n_B} \qquad\qquad \frac{n_A}{n_B} = \frac{n_A + n_B - n_B}{n_B} = \frac{n_A}{n - n_A}$$

$$\frac{n_A}{n_B} = \frac{\frac{n_A}{n}}{1 - \frac{n_A}{n}} = \frac{\sigma}{1-\sigma} \qquad \frac{n_A}{n_B} = \frac{\sigma}{1-\sigma} = \frac{\frac{v}{t+v}}{1 - \frac{v}{t+v}} = \frac{\frac{v}{t+v}}{\frac{t+v-v}{t+v}}$$

- we finally get $\dfrac{n_A}{n_B} = \dfrac{\sigma}{(1-\sigma)} = \dfrac{v}{t}$

$$\frac{n_A}{n_B} = \frac{\sigma}{(1-\sigma)} = \frac{v}{t}$$

• in steady state the ratio between "new" and "old" goods is an increasing function of innovation rate and imitation lag.

• Because relative wage depends on the "new" and "old" goods ratio,"it is an increasing function of innovation rate and imitation lag too.

• International trade pattern does not change and A keep exporting "new" product.