The assets approach to exchange rate determination

Flexible price models and exchange rate bubbles

General features

- Exchange rate is the relative price of two national monies
  - Expectations affects exchange rates
  - International financial market disequilibria generate exchange rates movements
- Two main strands of exchange rate models:
  - Monetary models with fixed or variable prices
  - Portfolio models

Monetary models

- Exchange rate is the price that" clears" home and foreign monetary markets
- Domestic and foreign assets (bonds, equities etc.) are perfect substitutes in agents' portfolio
- Uncovered interest parity (UIP) is always satisfied: $s_{t+1}^{e} - s_{t} = i_{t} - i_{t}^{*}$

Portfolio models

- Assets are not perfect substitutes in agents' portfolio
- Uncovered interest parity (UIP) is not satisfied
- Exchange rates are determined by both money and assets stocks

A "base" model

$$m_t - p_t = \phi y_t - \eta i_t$$

$$y_t^d = \mu + \delta q_t - \sigma r_t + \gamma g_t + \tau y_t^*$$

$$r_t = i_t - \left(p_{t+1}^e - p_t \right)$$

$$q_t = s_t - p_t + p_t^*$$

$$s_{t+1}^e - s_t = i_t - i_t^*$$

Exogenous variables: g_t, m_t, y^*, p^*, i^*

Expected variables: s_{t+1}^e, p_{t+1}^e

Six endogenous variables but only 5 independent equations: y, r, q, i, p, s^e

A “base” model

- The “base” model may be “closed” in three different ways:
  1. Short run: fixed prices
  2. Short run: “sticky prices”
  3. Long run: flexible prices

Case 1 implies $p_t = c$

Case 2 implies $p_{t+1} - p_t = \psi\left(y_t^d - \bar{y}\right)$

Case 3 implies $y = \hat{y}$

Monetary models with flexible prices

- Model assumptions:
  - Prices in goods market are perfectly flexible
  - Purchasing power parity is continuosly valid
- The model:

$$m_t - p_t = \phi y_t - \eta i_t$$

$$m_t^* - p_t^* = \phi y_t^* - \eta i_t^*$$

$$s_t = p_t - p_t^*$$

$$s_{t+1}^e - s_t = i_t - i_t^*$$

Monetary models with flexible prices

- From equations 1 and 2

$$m_t - p_t = \phi y_t - \eta i_t \qquad m_t^* - p_t^* = \phi y_t^* - \eta i_t^*$$

We get

$$p_t = m_t - \phi y_t + \eta i_t \qquad p_t^* = m_t^* - \phi y_t^* + \eta i_t^*$$

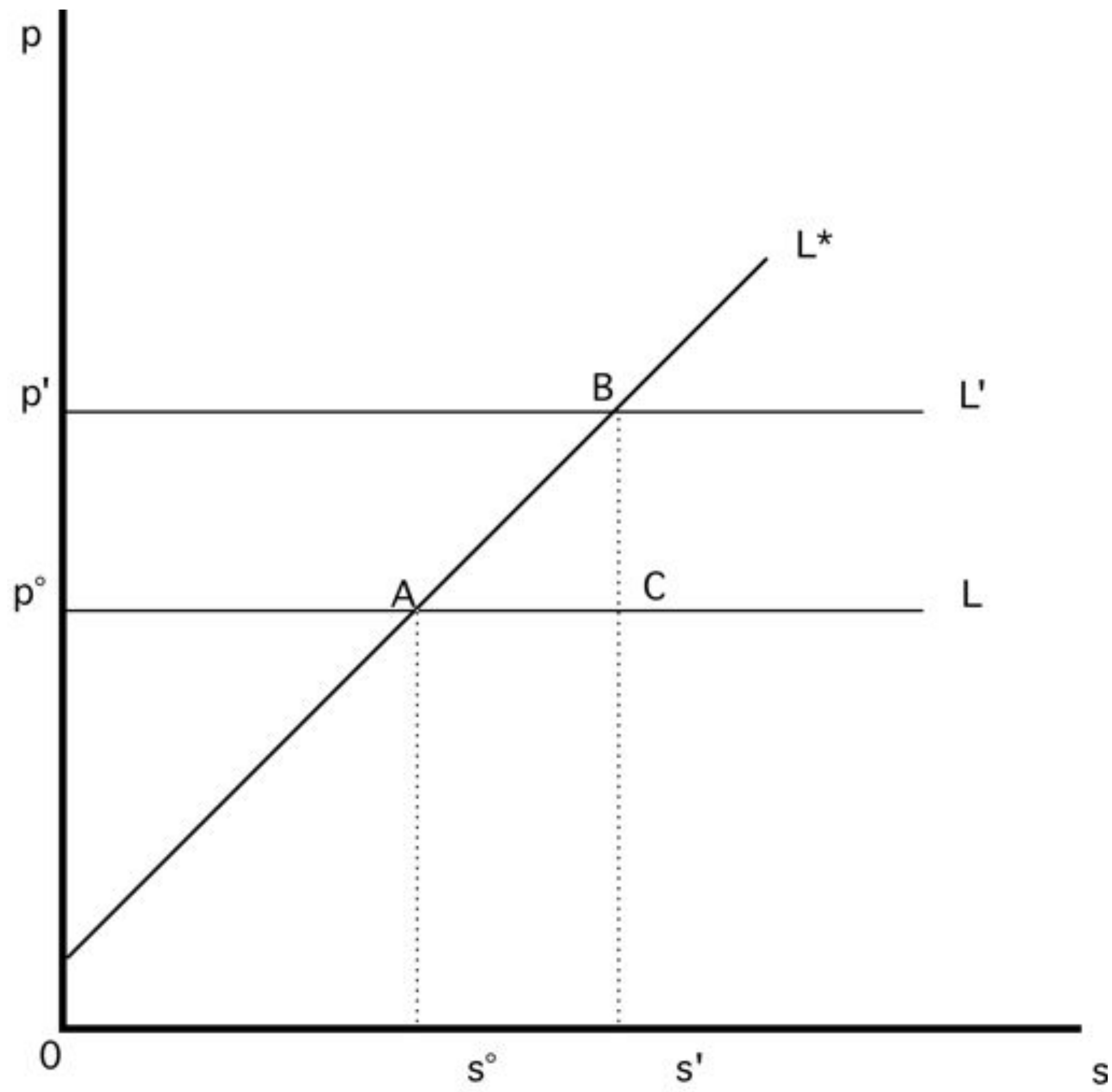
combining with PPP equation $s_t = p_t - p_t^*$

$$s_t = \left(m_t - m_t^*\right) - \phi\left(y_t - y_t^*\right) + \eta\left(i_t - i_t^*\right)$$

Monetary models with flexible prices

$$s_t = \left(m_t - m_t^*\right) - \phi\left(y_t - y_t^*\right) + \eta\left(i_t - i_t^*\right)$$

- According to the above equation
- An increase of domestic money supply depreciates exchange rate
- An increase in domestic GDP appreciates exchange rate
- An increase in domestic interest rate depreciates exchange rate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Expectations in the monetary model of exchange rate determination

- Substituting uncovered interest rates parity condition $E_t s_{t+1} - s_t = i_t - i_t^*$ for $i_t - i_t^*$ in the exchange rate equation

$$s_t = \left(m_t - m_t^*\right) - \phi\left(y_t - y_t^*\right) + \eta\left(i_t - i_t^*\right)$$

We get

$$s_t = \left(m_t - m_t^*\right) - \phi\left(y_t - y_t^*\right) + \eta E_t\left(s_{t+1} - s_t\right)$$

$$s_t = \frac{1}{1+\eta}\left[\left(m_t - m_t^*\right) - \phi\left(y_t - y_t^*\right) + \eta E s_{t+1}\right]$$

Expectations in the monetary model of exchange rate determination

$$s_t = \frac{1}{1+\eta}\left[\left(m_t - m_t^*\right) - \phi\left(y_t - y_t^*\right) + \eta E s_{t+1}\right]$$

- Current exchange rate depends on expected future exchange rate.
- What is the value of future expected exchange rate?
- We may find it using an iterative procedure

Expectations in the monetary model of exchange rate determination

- Move current exchange rate equation forward one period
- Solve it for next period exchange rate
- Next period exchange rate solution gives us the expected exchange rate
- Use that value in current exchange rate equation

Expectations in the monetary model of exchange rate determination

$$s_t = \frac{1}{1+\eta}\left[\left(m_t - m_t^*\right) - \phi\left(y_t - y_t^*\right) + \eta E s_{t+1}\right]$$

$$s_{t+1} = \frac{1}{1+\eta}\left[\left(m_{t+1} - m_{t+1}^*\right) - \phi\left(y_{t+1} - y_{t+1}^*\right) + \eta E s_{t+2}\right]$$

$$s_t = \frac{1}{1+\eta}\left[\left(m_t - m_t^*\right) - \phi\left(y_t - y_t^*\right)\right] +$$

$$+\frac{\eta}{1+\eta}\left[\left(m_{t+1} - m_{t+1}^*\right) - \phi\left(y_{t+1} - y_{t+1}^*\right) + \eta E s_{t+2}\right]$$

Expectations in the monetary model of exchange rate determination

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$$+\frac{\eta}{1+\eta}\left[\left(m_{t+1} - m_{t+1}^*\right) - \phi\left(y_{t+1} - y_{t+1}^*\right) + \eta E s_{t+2}\right]$$

$$s_t = \frac{1}{1+\eta}\sum_{i=0}^{n} E_t\left(\frac{\eta}{1+\eta}\right)^i \left[\left(m_{t+1} - m_{t+1}^*\right) - \phi\left(y_{t+1} - y_{t+1}^*\right)\right]$$

$$+\left(\frac{\eta}{1+\eta}\right)^{n+1} E_t s_{t+n+1}$$

Expectations in the monetary model of exchange rate determination

$$s_t = \frac{1}{1+\eta}\sum_{i=0}^{n} E_t\left(\frac{\eta}{1+\eta}\right)^i \left[\left(m_{t+1} - m_{t+1}^*\right) - \phi\left(y_{t+1} - y_{t+1}^*\right)\right]$$

$$+\left(\frac{\eta}{1+\eta}\right)^{n+1} E_t s_{t+n+1}$$

- Finally, assuming infinite horizon and using the transversality condition

$$\lim_{n\to\infty}\left(\frac{\eta}{1+\eta}\right)^{n+1} E_t s_{t+n+1} = 0$$

$$s_t = \frac{1}{1+\eta}\sum_{i=0}^{\infty} E_t\left(\frac{\eta}{1+\eta}\right)^i \left[\left(m_{t+1} - m_{t+1}^*\right) - \phi\left(y_{t+1} - y_{t+1}^*\right)\right]$$

•Current exchange rate depends on current output and money supply

•But also on their future expected values!

$$s_t = \frac{1}{1+\eta}\sum_{i=0}^{\infty} E_t\left(\frac{\eta}{1+\eta}\right)^i \left[\left(m_{t+1} - m_{t+1}^*\right) - \phi\left(y_{t+1} - y_{t+1}^*\right)\right]$$

- Implications:
  - Changes in expected values result in exchange rates movements
  - Money is *neutral* in the long run because

$$\sum_{i=0}^{\infty}\left(\frac{\eta}{1+\eta}\right)^i = \frac{1}{1-\frac{\eta}{1+\eta}} = 1+\eta$$

so that $$\frac{1}{1+\eta}\sum_{i=0}^{\infty}\left(\frac{\eta}{1+\eta}\right)^i = 1$$

Exchange rates bubbles

- A speculative bubbles occurs when exchange rate continuosly ”*diverges*” from “*fundamentals*” originating an “*explosive*” dynamics
- In such cases, the transversality condition does not hold, e.g.:

$$\lim_{n\to\infty}\left(\frac{\eta}{1+\eta}\right)^{n+1} E_t s_{t+n+1} \neq 0$$

A model of exchange rate bubbles

- Exchange rate may be written as the sum of two parts:
  - One depending on *"fundamentals"*
  - One depending on *speculative* behaviour (bubble)

$$s_t = s_t^f + b \qquad\qquad b_t = \lambda E_t b_{t+1}$$

We assume that the speculative part follows a stochastic process

A model of exchange rate bubbles

$$b_t = \lambda E_t b_{t+1}$$

$$b_{t+1} = \frac{1}{\lambda} b_t + \varepsilon_{t+1} \quad \text{with probability } \rho \text{ that } s^b_{t+1} > s_t$$

$$b_{t+1} = \varepsilon_{t+1} \quad \text{with probability } 1-\rho \text{ that } s_{t+1} = \overline{s}$$

$$\lambda = \frac{\eta}{1+\eta}, \quad \varepsilon \sim (0,\sigma)$$

- Expected exchange rate therefore is

$$E_t s_{t+1} = \rho s^b_{t+1} + (1-\rho)\overline{s}$$

•And expected exchange rate depreciation is

$$E_t s_{t+1} - s_t = \rho\left(s^b_{t+1} - s_t\right) + (1-\rho)\left(\overline{s} - s_t\right)$$

A model of exchange rate bubbles

$$E_t s_{t+1} - s_t = \rho\left(s_{t+1}^b - s_t\right) + (1-\rho)\left(\overline{s} - s_t\right)$$

- The first element on the right hand side of the equation is the *capital gain from betting on the bubble*
- The second element is the loss in case the bubble *"burns"*

A model of exchange rate bubbles

$$E_t s_{t+1} - s_t = \rho\left(s_{t+1}^b - s_t\right) + (1-\rho)\left(\overline{s} - s_t\right)$$

- From the UIP condition we may write the above equation as

$$i - i^* = \rho\left(s_{t+1}^b - s_t\right) + (1-\rho)\left(\overline{s} - s_t\right)$$

$$\rho\left(s_{t+1}^b - s_t\right) = i - i^* - (1-\rho)\left(\overline{s} - s_t\right)$$

$$s_{t+1}^b - s_t = \frac{1}{\rho}\left(i - i^*\right) + \frac{(1-\rho)}{\rho}(s_t - \overline{s})$$

- The bubble growth depends on interest rates differential and the degree of under-valuation
- Speculative bubbles partially explains deviations of exchange rates from "*fundamentals*"