Es. 21.1

COSTO TOTALE OFFICINA MECCANICA AUTO

$$C(s) = 2s^2 + 100 \quad [s: \text{auto}]$$

a) $AC = ?$

$$AC = \frac{C(s)}{s} = \frac{2s^2 + 100}{s} = 2s + \frac{100}{s}$$

$$AVC = 2s$$

$$MC = 4s$$

b) $\begin{cases} \text{se } P_s = 20 & s = ? \\ \text{se } P_s = 40 & s = ? \end{cases}$

$\rightarrow$ curva di offerta $MC > AVC$

$$4s > 2s \rightarrow s > 0$$

$$\begin{cases} P_s = 4s \\ \boxed{s = \dfrac{P_s}{4}} \end{cases}$$

$$\begin{cases} \text{se } P_s = 20 \rightarrow s = \dfrac{20}{4} = 5 \\ \text{se } P_s = 40 \rightarrow s = \dfrac{40}{4} = 10 \end{cases}$$

Es. 21.2

$$c(y) = y^3 - 8y^2 + 30y + 5$$

a) $MC(y) = 3y^2 - 16y + 30$

b) $AVC(y) = y^2 - 8y + 30$

d) $\frac{dAVC}{dy} = 2y - 8 \qquad \frac{d^2AVC}{d^2y} = 2 > 0$

$\min_y AVC \rightarrow 2y - 8 = 0$

$$\boxed{y = 4}$$

AVC diminuisce per $y<4$, sale per $y>4$

e) $MC = AVC$ per quale y?

$AVC = y^2 - 8y + 30 = 3y^2 - 16y + 30$

$3y^2 - 16y + 30 - y^2 + 8y - 30 = 0$

$2y^2 - 8y = 0$

$y^2 - 4y = 0$

$y(y-4) = 0 \rightarrow \begin{cases} y = 0 \\ \boxed{y = 4} : \min AVC \end{cases}$

f) impresa cessa di produrre quando
$P < \min AVC$

→ curva di offerta:

$$\begin{cases} P = MC \\ P = 3y^2 - 16y + 30 \end{cases}$$

Per $y = 4 \rightarrow \min AVC$

$$P = 3 \cdot 4^2 - 16 \cdot 4 + 30 = 14$$

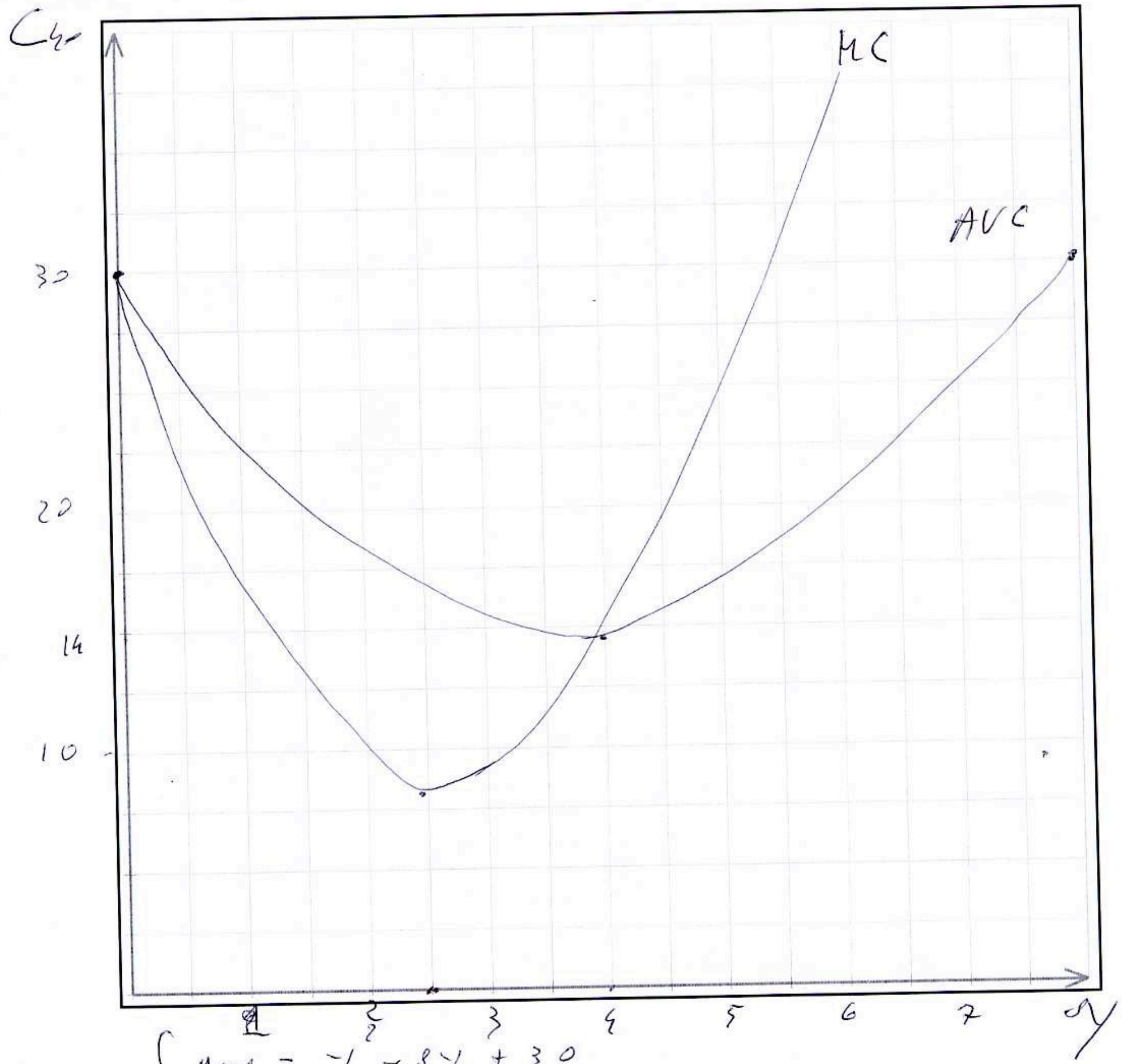
→ Quando $P < 14$ si cessa di produrre

g) Quantità minima di offerta non nulla è
y per cui $MC = AVC$
→ $\boxed{y = 4}$

Se $y = 6$, $P = ?$

$$\boxed{P = 3 \cdot 6^2 - 16 \cdot 6 + 30 = 42}$$

Es. 21.2



$$\begin{cases} AVC = y - 8y + 30 \\ y = 0 \rightarrow AVC = 30 \\ \text{min } AVC : y = 4 \ , \ AVC = 14 \\ y = 8 \quad AVC = 30 \end{cases}$$

$MC = 3y^2 - 16y + 30$

$y = 0 \rightarrow MC = 30$

min MC ~~2~~,6 → $y = 2,6 \quad MC = 8,68$

21,4 $\quad C(y) = y^2 + 10 \quad y > 0 \quad C(0) = 0$

a) $MC = ?$

$$\boxed{MC = 2y}$$

$$AC = \frac{y^2}{y} + \frac{10}{y} = y + \frac{10}{y}$$

b) $y \to MC = AC$?

$$y + \frac{10}{y} = 2y$$

$$10 = y^2$$

$$y = \pm\sqrt{10} \to \boxed{y = \sqrt{10}}$$

$y \to \min AC = ?$

$\to y = \sqrt{10}$ $[MC = AC \to y \to \min AC]$

$$\min_{y} AC = y + \frac{10}{y} = y + 10y^{-1}$$

$$\frac{dAC}{dy} = 1 - \frac{10}{y^2} = 0$$

$$\boxed{\begin{gathered} y^2 = 10 \\ y = \sqrt{10} \end{gathered}}$$

c) P_{min} per $y > 0$ e $y \geq \sqrt{10}$?

$$\begin{cases} P = MC = 2y \\ y = \frac{P}{2} \quad \text{[curva di offerta]} \end{cases}$$

$$\boxed{P = 2\sqrt{10}} \quad e \quad \boxed{y = \frac{2\sqrt{10}}{2} = \sqrt{10}}$$

21.7

$$\begin{cases} f(x_1, x_2) = x_1^{\frac{1}{3}} \, x_2^{\frac{1}{3}} & \begin{cases} x_1 = \text{limon} \\ x_2 = \text{ore di lavoro} \end{cases} \\ c(w_1, w_2, y) = 2 w_1^{\frac{1}{2}} w_2^{\frac{1}{2}} y^{\frac{3}{2}} & \{ y = \text{litri di limonata} \end{cases}$$

(1)

a) Se $w_1 = 1$ e $w_2 = 1$

$MC = ?$

$c(1,1,y) = 2 \cdot 1^{\frac{1}{2}} \cdot 1^{\frac{1}{2}} y^{\frac{3}{2}} = 2y^{\frac{3}{2}}$

$$\boxed{MC = \frac{3}{2} \cdot 2y^{\frac{3}{2}-1} = 3y^{\frac{1}{2}} = 3\sqrt{y}}$$

offerta:

$p = MC$

$p = 3\sqrt{y}$

$p^2 = 9y$ $\qquad$ $\min AC = 0 \to y = 0$

$$\boxed{y = \frac{p^2}{9}} \quad y > 0$$

Se $w_1 = 4$ e $w_2 = 9$

$c = 2 \cdot 4^{\frac{1}{2}} \cdot 9^{\frac{1}{2}} \cdot y^{\frac{3}{2}} = 2\sqrt{4}\sqrt{9}\, y^{\frac{3}{2}} = 2 \cdot 2 \cdot 3 y^{\frac{3}{2}} = 12 y^{\frac{3}{2}}$

$MC = \frac{3}{2} \cdot 12 y^{\frac{1}{2}} = 18\sqrt{y}$

$18\sqrt{y} = p$ $\qquad$ $\boxed{y = \frac{p^2}{18^2}}$

b) $MC = \frac{3}{4} \cdot 4\sqrt{w_1} \cdot \sqrt{w_2}\sqrt{y}$

offerta:

$$3\sqrt{w_1} \cdot \sqrt{w_2}\sqrt{y} = p$$

$$\sqrt{y} = \frac{p}{3\sqrt{w_1}\sqrt{w_2}}$$

$$\boxed{y = \frac{p^2}{9 \cdot w_1 \cdot w_2}}$$

1

21.10 Y = BARILI DI BENZINA

$$C(Y) = \frac{Y^2}{2} + P_0 Y$$

P_0: prezzo di un barile di greggio

a) $MC(P_0) = +Y$

$$MC(Y) = \frac{2}{2}Y - P_0 = Y + P_0$$

b) $\begin{cases} P_0 = 5 & Y \leq 50 \\ P_0 = 15 & Y > 50 \end{cases}$

$\begin{cases} MC(Y \leq 50) = Y + 5 \\ MC(Y > 50) = Y + 15 \end{cases}$

d) $D = P = 30$, $S = ?$

$$AVC = \frac{1}{2}\frac{Y^2}{Y} + \frac{P_0 Y}{Y} = \frac{1}{2}Y + P_0$$

min AVC = $\cancel{0}$, $Y = 0$

$\begin{cases} P = Y + P_0 & \text{[curva di offerta]} \\ P = Y + 5 & [Y \leq 50] \\ P = Y + 15 & [Y > 50] \end{cases}$

Per $y = 50$

$$p = 50 + 5 = 55$$

per cui se $D = p = 30$ si ottiene

$$\begin{cases} 30 = y + 5 \\ \boxed{y = 25} \end{cases}$$

e) Se $P_0 = 15$ $y > 0$ si ottiene sempre

$p = y + 15$ e quindi per $p = 30$

$$\begin{cases} 30 = y + 15 \\ \boxed{y = 15} \end{cases}$$

f) Se per ogni barile pagato 15 se ne può acquistare 1 a 5, 2 barili costano 20 e 1 barile 10

$$p = y + 10$$

$\boxed{y = p - 10}$ → nuova curva di offerta

Se $p = 30$ → $30 = y + 10$ $\boxed{y = 20}$

22.0

(1)

a) $S_1(P) = P$, $S_2(P) = 2P$, $S_3(P) = 3P$

→ Tutte e 3 le industrie offrono per $P > 0$ perciò

$$\begin{cases} S = S_1(P) + S_2(P) + S_3(P) = P + 2P + 3P = 6P \\ S = \sum_{i=1}^{3} S_i(P) = 6P \end{cases}$$

b) $S_1(P) = 2P$, $S_2(P) = P - 1$

$S_1(P) > 0$ se $P > 0$

$S_2(P) > 0$ se $P - 1 > 0 \rightarrow P > 1$

$$\begin{cases} S = S_1(P) = 2P & \text{per } P \leq 1 \\ S = S_1(P) + S_2(P) = 2P + P - 1 = 3P - 1 & \text{per } P > 1 \end{cases}$$

c) 200 imprese $S_1(P) = 2P - 8$ $[n_1 = 200]$

100 imprese $S_2(P) = P - 3$ $[n_2 = 100]$

$$\begin{cases} S_1(P) > 0 \rightarrow 2P - 8 > 0 \rightarrow P > \frac{8}{2} = 4 \\ S_2(P) > 0 \rightarrow P - 3 > 0 \rightarrow P > 3 \end{cases}$$

→

(2)

→ c)

$$\begin{cases} S_1 = n_2 \cdot S_2(P) = 100(P-3) \left[4 \geq P > 3\right] \\ S_2 = n_1 S_1(P) + n_2 \cdot S_2(P) = 200(2P-8) + 100(P-3) \\ \left[P > 4\right] \end{cases}$$

$$\begin{cases} S_1 = 100P - 300 \left[3 < P \leq 4\right] \\ S_2 = 400P - 1600 + 100P - 300 = 500P - 1900 \quad \left[P > 4\right] \end{cases}$$

/ $\underline{22.4}$ $\begin{cases} c(y) = y^2 + 1 \\ y > 0 \end{cases}$ - costo identico per n (grandi) di imprese

$\begin{cases} c(0) = 0 \end{cases}$

$D(p) = 52 - p$ [domanda di mercato]

a) S per singola impresa? S dell'industria?

$MC = 2y$

$\begin{cases} p = 2y \\ y = \frac{1}{2} p \end{cases}$ $\boxed{S = n \cdot \frac{1}{2} p}$

b) $\begin{cases} p_{min} \text{ per } y > 0 \text{ ?} \\ \text{se } y_{min} = 1 \text{ [produce quantità intere]} \\ 1 = \frac{1}{2} p \rightarrow \boxed{p = 2} \end{cases}$

$\boxed{22.4}$ 2

c) n in equilibrio?

Se n cresce P diminuisce

$$P = \frac{2S}{n}$$

ma $P_{min} = 2$ per cui con n molto grande la curva di offerta dell'industria diventa orizzontale

$$\boxed{P = 2}$$

- in equilibrio $D = S$

$$\frac{n}{2} P = 52 - P$$

e per $P = 2$

$$n \cdot \frac{2}{2} = 52 - 2$$

$$\boxed{n = 50}$$

d) P e Y in equilibrio?

$n = 50$

$P = 5$

$$\begin{cases} \frac{50}{2} P = 52 - P \\ 25P + P = 52 \\ 26P = 52 \\ \boxed{P^* = \frac{52}{26} = 2} \end{cases}$$

$S = 25 \cdot 2 = 50 \rightarrow \boxed{Y^* = 50}$

→ output di ogni impresa è $\frac{S}{n} = \frac{50}{50} = 1 = y^*$

e) $\boxed{S^* = 50}$

f) Se $D = 52{,}5 - P$, essendo $P^* = 2$

$n = 52{,}5 - 2 = 50{,}5$

$\begin{cases} \text{ma non può entrare una "mezza" impresa per cui} \\ \text{restano sempre } n = 50 \text{ imprese!} \end{cases}$

→

→ f) Se ci fossero 51 imprese, il prezzo di equilibrio sarebbe

$$S = D$$

$$51 \cdot \frac{1}{2} P = 52,5 - P$$

$$25,5 P + P = 52,5$$

$$26,5 P = 52,5$$

$$\boxed{P = \frac{52,5}{26,5} = 1,98 < 2 !}$$

→ le imprese avrebbero perdite!

g) nuovo P^* e Y^* ? $\pi = ?$

→ poiché $n = 50$, $S = D$ implica

$$25 P = 52,5 - P$$

$$26 P = 52,5$$

$$\boxed{P^* = \frac{52,5}{26} = 2,01}$$

$$\boxed{Y^* = \frac{1}{2} \cdot 2,01 = 1,01}$$

$$S = 50 \cdot 1,01 = 50,5$$

$$\boxed{\pi = 2,01 \cdot 1,01 - 1,01^2 - 1 = 0,01}$$

→

22.4 (5)

h) Se $D = 53 - P \quad n = ?$

Se ora ci fosse un'impresa in più per cui

$n = 51$

$D = S \rightarrow \frac{51}{2} P = 53 - P \rightarrow P = \frac{53}{26{,}5} = 2$!

$\boxed{\text{in equilibrio } n = 51!}$ $\boxed{P^* = 2}$

i) $y = \frac{1}{2} P = \frac{1}{2} \cdot 2 = 1$

$\boxed{\Pi = 2 \cdot 1 - 1^2 - 1 = 0}$

/

$\boxed{22.8}$ $S_i(P) = \frac{P}{2}$

$n = 1, 2, 3, 4$

$$\left\{\begin{array}{l} S_1(P) = \frac{P}{2} \\ S_2 = S_1 + S_2 = 2 \cdot \frac{P}{2} = P \\ S_3 = \frac{3}{2} P \\ S_4 = \frac{4}{2} P \end{array}\right. \quad \left| \begin{array}{l} P = 2y \\ P = y \\ P = \frac{2}{3} y \\ P = \frac{1}{2} y \end{array}\right\}$$

a) Se $\pi < 0$ per $P < 3$ e $D(P) = 3,5$, P^*, S^*?

$\left\{\right.$ Se $n = 4$, $S = \frac{4}{2} P$, $D = S \to 3,5 = \frac{4}{2} P \to P = \frac{7}{4} < 3!$
$\to$ ci sarebbero perdite, un'impresa esce

$\left\{\right.$ Se $n = 3$ $S = \frac{3}{2} P$, $D = S \to 3,5 = \frac{3}{2} P \to P = 2,3 < 3!$
$\to$ ci sarebbero perdite

Se $n = 2$, $S = P$ $\to D = S \to \boxed{3,5 = P^* > 3}$ ok!

$\to$ RESTANO 2 IMPRESE e $\boxed{S^* = 3,5}$

In generale $S = n \cdot \frac{P}{2} = 3,5 \to P = \frac{2 \cdot 3,5}{n} > 3$

$7 > 3 \cdot n \to 2,3 > n \to n = 2!$

$\longrightarrow$

$\boxed{22{,}8}$ 2

→ b) Se $D(P) = 8 - P$, P^*, S^*? $n = ?$

$$\frac{n}{2}P = 8 - P$$

$$\frac{n}{2}P + P = 8$$

$$P\left(\frac{n}{2}+1\right) = 8$$

$$P = \frac{2 \cdot 8}{n+2} > 3$$

$$16 > 3(n+2)$$

$$16 > 3n + 6$$

$$10 > 3n$$

$$3{,}3 > n \rightarrow \boxed{n = 3}!$$

per $n = 3$ $D = 5$ $\frac{3}{2}P = 8 - P$

$$\frac{3}{2}P + P = 8$$

$$P\left(\frac{3}{2}+1\right) = 8$$

$$P \cdot \frac{5}{2} = 8$$

$$\boxed{P^* = \frac{16}{5} = 3{,}2}$$

$$\boxed{S^* = 3{,}2 \cdot \frac{3}{2} = 4{,}8}$$

23.4) $\begin{cases} C(y) = y^2 \\ P(y) = 12 - y \end{cases}$

a) $y^* = ?$

$$\begin{cases} MC = 2y \\ MR = 12 - 2y \end{cases} \rightarrow \quad 2y = 12 - 2y$$

$$4y = 12 \rightarrow \boxed{y^* = \frac{12}{4} = 3}$$

b) $\tau = 2$ per unità prodotta

$$C(y, \tau) = y^2 + 2 \cdot y$$

$$MC = 2y + 2$$

$$MC = MR \rightarrow 2y + 2 = 12 - 2y$$

$$4y = 10$$

$$\boxed{y^* = \frac{10}{4}}$$

c) Se $\tau = 10\%$ sui profitti

$$\begin{cases} \max_{y} (1 - 0{,}1)(R - C) \\ \rightarrow \text{COME IN (a) non cambia } y^* ! \end{cases}$$

28.3

$$U_I = B_I + 2\sqrt{P_I}$$

$$U_M = B_M + 4\sqrt{P_M}$$

$\omega_I = (8, 12)$

$\omega_M = (8, 4)$

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a) MRS_I , MRS_M ?

$$MRS = -\frac{MU_B}{MU_M}$$

1) I : $\begin{cases} MU_B = 1 \quad , \quad MU_M = \frac{1}{2} \cdot 2 \, P_I^{-\frac{1}{2}} = \frac{1}{P_I} \\ MRS_I = \frac{1}{\frac{1}{P_I}} = -P_I \end{cases}$

2) M : $\begin{cases} MU_B = 1 \quad , \quad MU_M = \frac{1}{2} \, 4 \, P_M^{-\frac{1}{2}} = \frac{2}{P_M} \\ MRS_M = \frac{1}{\frac{2}{P_M}} = -\frac{P_M}{2} \end{cases}$

$\longrightarrow$

$\boxed{28.3}$ 2

b) $MRS_I = MRS_M$

$$\boxed{P_I = \frac{P_M}{2}}$$

c) $\frac{P_I}{P_M} = \frac{1}{2}$

d) $P_I + P_M = 12 + 4 = 16$

e) $P_I = ?$ $P_M = ?$

$$\begin{cases} P_I = \frac{P_M}{2} \\ P_I + P_M = 16 \end{cases} \rightarrow \frac{1}{2} P_M + P_M = 16$$

$$P_M + 2P_M = 32$$

$$3P_M = 32$$

$$\boxed{P_M = \frac{32}{3}}$$

$$\boxed{P_I = \frac{32}{6}}$$