Es. 4.1

$U = x_A \cdot x_B$ $\quad X = (A, B)$

$X = (40, 5)$

x_A	x_B
10	20
5	40
10	20
20	10
40	5

$U(40, 5) = 40 \cdot 5 = 200$

Eq. curva di indifferenza / Funzione di ut

$\to x_A \cdot x_B = 200 \to x_B = \frac{200}{x_A}$

Offerta di scambio $\to +15\ B \ -25\ A$

$\to$ il nuovo paniere è

$(40-25, 5+15) = (15, 20) \to U = ?$

$U(15, 20) = 15 \cdot 20 = 300 > U(40, 5) = 200$

$\to$ accetta lo scambio!

Al massimo si cede nel $x = 30$!

$(40-x)(5+15) = 200$

$(40-x)(20) = 200$

$40 - x = 10$

$\boxed{x = 40 - 10 = 30}$

Es. 4.3

$$U(x_1,x_2) = (x_1+2)(x_2+6)$$

$$X = (\text{BISCOTTI}, \text{LATTE}) = (4,6)$$

a) → inclinazione curva di indifferenza?

$$\boxed{MRS = -\frac{MU_1}{MU_2}}$$

$$MU_1 = \frac{\partial U}{\partial x_1} \qquad MU_2 = \frac{\partial U}{\partial x_2}$$

$\boxed{\text{CI in } X=(4,6)}$

$$U = (4+2)(6+6) = 6 \cdot 12 = 72$$

$$U = x_1 x_2 + 6x_1 + 2x_2 + 12$$

$$\left.\begin{aligned} \frac{\partial U}{\partial x_1} &= x_2 + 6 \\ \frac{\partial U}{\partial x_2} &= x_1 + 2 \end{aligned}\right\} \rightarrow MRS = -\frac{x_2+6}{x_1+2}$$

$$\boxed{MRS = -\frac{6+6}{4+2} = -\frac{12}{6} = -2}$$

Es. 4.3 2

b) $U = 72$

x_1	x_2	x_1+2	x_2+6	$x_1 \cdot x_2$
4	6	6	12	
10	0	12	6	72
7	2	9	8	
2	12	4	18	

equazione CI per $U=72$

$$(x_1+2)(x_2+6) = 72$$

$$x_2+6 = \frac{72}{x_1+2}$$

$$x_2 = \frac{72}{x_1+2} - 6$$

c) si offre lo scambio $+9L$ $-3B$

→ rapporto di scambio: $\boxed{\frac{\Delta x_2}{\Delta x_1} = \frac{\Delta L}{\Delta B} = \frac{9}{-3} = -3}$

È vantaggioso?

→ NUOVO PANIERE? → $X(4-3,\ 6+9) = (1, 15)$

$U(1,15) = (1+2)(15+6) = 3 \cdot 21 = 66 < U(4,6) = 72!$

<u>NO, lo SCAMBIO NON CONVIENE!</u>

Es. 4.3 (3)

$$\left.\begin{array}{l} \Delta L = 3 \\ \Delta B = -1 \end{array}\right\} X = (4-1, 6+3) = (3, 9)$$

$$U(3, 9) = (3+2)(9+6) = 5 \cdot 15 = 75$$

→ SCAMBIO VANTAGGIOSO! $75 > 72$

$$\left.\begin{array}{l} \Delta L = 6 \\ \Delta B = -2 \end{array}\right\} \rightarrow X = (4-2, 6+6) = 2, 12$$

$$U(2, 12) = (2+2)(12+6) = 4 \cdot 18 = 72$$

SCAMBIO INDIFFERENTE!

Es. 4.12

a) $U(x_1, x_2) = x_1^2 + 2x_1x_2 + x_2^2$

$$MRS = -\frac{MU_1}{MU_2} \quad ?$$

$$MU_1 = \frac{\partial U}{\partial x_1} = 2x_1 + 2x_2$$

$$MU_2 = \frac{\partial U}{\partial x_2} = 2x_1 + 2x_2$$

$$\boxed{MRS = -\frac{2(x_1 + x_2)}{2(x_1 + x_2)} = -1}$$

b) $v(x_1, x_2) = x_2 + x_1$

$MRS = ?$

$\bar{v} = x_2 + x_1 \rightarrow x_2 = \bar{v} - x_1$

$MRS = -1$

oppure

$$MRS = -\frac{MU_1}{MU_2} = -\frac{1}{1} = 1$$

c) $U(x_1, x_2) = x_1^2 + 2x_1^2 x_2^2 + x_2^2 = \left(x_1 + x_2\right)^{2\ 2}$

$$U(x_1, x_2) = \left(V(x_1, x_2)\right)^2$$

$\rightarrow$ U è trasformazione monotona di V.

Es. 5.1

$U(x_A, x_B) = x_A \cdot x_B$

$P_A = 1$

$P_B = 2$

$m = 40$

$$\begin{cases} x_A + 2x_B = 40 \\ V.B \rightarrow x_B = 20 - \frac{1}{2}x_A \end{cases}$$

a) $U = 300 = x_A \cdot x_B$

x_A	x_B
10	30
15	20
20	15
30	10

b) Sì

c) No

d) $(20, 10)$

e) $MRS = -\frac{x_B}{x_A}$ $\boxed{-\frac{P_1}{P_2} = -\frac{1}{2}}$ → inclinazione retta di bilancio

f) Condizione di Tangenza

$$\begin{cases} MRS = \frac{P_1}{P_2} \rightarrow \frac{x_B}{x_A} = \frac{1}{2} \\ x_B = \frac{1}{2}A \end{cases}$$

g.) $\begin{cases} \text{punto di incrocio fra } VB \text{ e } x_B = \frac{1}{2}A \\ \frac{1}{2}x_A = 20 - \frac{1}{2}x_A \rightarrow x_A = 20,\ x_B = 10 \end{cases}$

Es. 5.1 2

h) $U(20,10) = 20 \cdot 10 = 200$

Ex. 5.2

$$U(x,y) = (x+2)(y+1)$$

a) CI por $(x,y) = (2,8)$?

$$\begin{cases} U(2,8) = (2+2)(8+1) = 4 \cdot 9 = 36 \\ 36 = (x+2)(y+1) \\ x+2 = \dfrac{36}{y+1} \\ \boxed{x = \dfrac{36}{y+1} - 2} \end{cases} \longrightarrow \boxed{y = \frac{36}{x+2} - 1}$$

b) $U = 36 \longrightarrow$

x	y
0	17
1	~~36~~ 11
2	8
4	5
6	3,5
8	2,6
10	2
16	1
34	0

Es. 5-2 (2)

b) $P_1 = P_2 = 1$, $m = 11$

$$P_1 x + P_2 y = 11 = x + y$$

$$\boxed{Y = \frac{11}{\cancel{}} - x} \leftarrow \text{retta di bilancio}$$

$\boxed{U = 36 \text{ raggiungibile?}} \longrightarrow \boxed{SI}$!

c) MRS = ?

$$MRS = -\frac{MU_x}{MU_y}$$

$MU_x = Y + 1$

$MU_y = x + 2$

$$\begin{cases} U = xy + x + 2y + 2 \\ U_x = y + 1 \\ U_y = x + 2 \end{cases}$$

$$\boxed{MRS = -\frac{Y+1}{x+2}}$$

d) $MRS = \frac{P_1}{P_2} \rightarrow \frac{Y+1}{x+2} = 1$

e) retta di bilancio?

$$\boxed{Y = 11 - x}$$

Es. 5-2 (3)

*) domanda di X e y ?

$$\begin{cases} \dfrac{y+1}{x+2} = 1 \\ y = 11 - x \end{cases}$$

$y + 1 = x + 2$

$11 - x + 1 = x + 2$

$12 - 2 = 2x \longrightarrow 10 = 2x$

$\boxed{x = \frac{10}{2} = 5}$

$\boxed{y = 11 - 5 = 6}$

$$\begin{cases} x = 5 \\ y = 6 \end{cases} \Bigg\| \longleftarrow$$

CI $U(5,6) = 49 = (x+2)(y+1)$

6.4 s = francobolli

τ = plumkake

1

$$U = s + \ln\tau \qquad P_s s + P_\tau \tau = m$$

a) $$\frac{MU_\tau}{MU_s} = \frac{P_\tau}{P_s}$$

$$MU_\tau = \frac{1}{\tau} \qquad MU_s = 1$$

$$\frac{\frac{1}{\tau}}{1} = \frac{P_\tau}{P_s} \rightarrow \boxed{\frac{1}{\tau} = \frac{P_\tau}{P_s}}$$

b) $\tau(P_\tau, P_s, m) = ?$

$$\boxed{\tau = \frac{P_s}{P_\tau}}$$

c) ~~domanda di gusto~~ quantità di m spesa in plumpcake?

$\hookrightarrow P_\tau \tau$ $\quad$ da $\frac{1}{\tau} = \frac{P_\tau}{P_s} \rightarrow \boxed{P_\tau \cdot \tau = P_s}$

d) $P_\tau \cdot \tau = m - P_s s$

$$s = \frac{m}{P_s} - \frac{P_\tau}{P_s}\tau = \frac{m}{P_s} - \frac{P_\tau}{P_s} \cdot \frac{P_s}{P_\tau} = \boxed{\frac{m}{P_s} - 1}$$

$$P_\tau \cdot \tau = m - P_s\left(\frac{m}{P_s} - 1\right) = \cancel{m} - \cancel{m} + P_s = P_s$$

e) Se $m < P_s$ $s < 0$

→ quindi lomorda di $s = 0$

$$\boxed{t = \frac{m}{P_t}}$$ (ottimo di frontiera)

2

f) Se $\Delta m = 1$ $\Delta s > 0$?

$\boxed{Sì}$, perchè $s = \frac{m}{P_s} - 1 \rightarrow \frac{ds}{dm} = \frac{1}{P_s} > 0$

g) curve di Engel per $P_t = 2$, $P_s = 1$

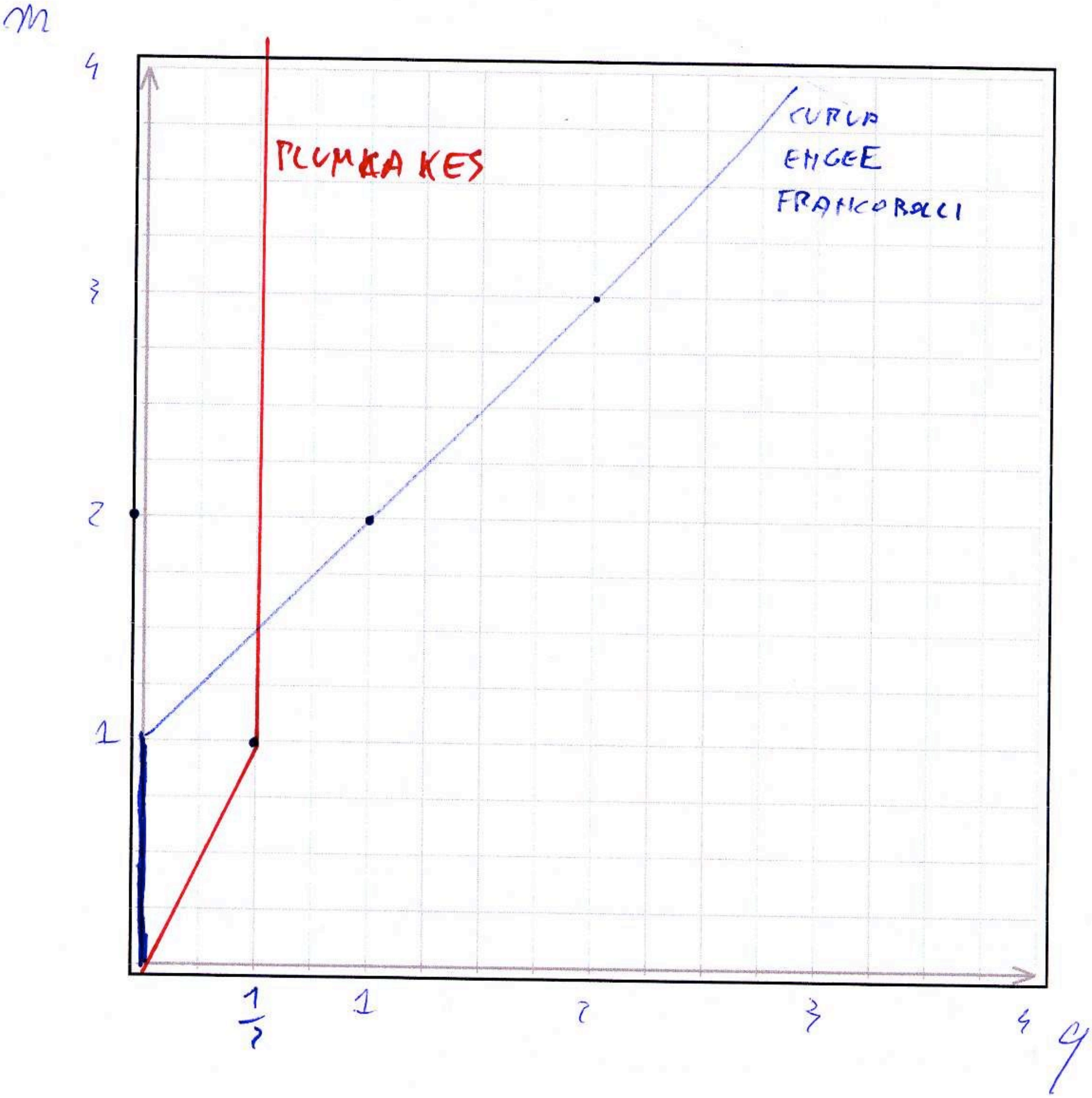
m	s
	$\frac{m}{P_s} - 1$
0	-1 [0]
1	$1 - 1 = 0$
2	$2 - 1 = 1$
3	$3 - 1 = 2$

la curva di Engel per t è verticale per $t = \frac{1}{2}$ quando $m > 1$

	m	t
	0	$\frac{m}{2} = 0$
$m < 1$	1	$\frac{1}{2} = \frac{1}{2}$
$m > 1$	2	$\frac{P_s}{P_t} = \frac{1}{2}$
	3	.
	4	.

Parameters:	Plotting Area	Axes Division	Printing Format
x-Axis:	[0.0 | 16.0]	1E = automatic	1E = 1 cm
y-Axis:	[0.0 | 16.0]	1E = automatic	1E = 1 cm

Functions:



$\boxed{6.1}$

$U(x_A, x_B) = x_A x_B$

$p_A x_A + p_B x_B = m$

$$MRS = ? \quad -\frac{MU_1}{MU_2} = -\frac{x_B}{x_A}$$

inclinazione vincolo bilancio $\rightarrow -\frac{p_A}{p_B}$

$$x_B = \frac{m}{p_B} - \frac{p_A}{p_B} x_A$$

$$\boxed{\begin{gathered} U = x^c \cdot x^d \\ \begin{cases} x_A = \frac{c}{c+d} \frac{m}{p_A} \\ x_B = \frac{d}{c+d} \frac{m}{p_B} \end{cases} \end{gathered}}$$

$$\begin{cases} x_A(p_A, p_B, m) \; ? \\ MRS = \frac{p_A}{p_B} \\ \frac{x_B}{x_A} = \frac{p_A}{p_B} \rightarrow x_A = \frac{p_B}{p_A} x_B \\ x_B(p_A, p_B, m) \; ? \rightarrow x_B = \frac{p_A}{p_B} x_A \end{cases}$$

$$x_A = \frac{p_B}{p_A} \cdot \frac{1}{\cancel{p_B}} (m - p_A x_A) = \frac{1}{p_A} \cdot m (1 - p_A x_A)$$

$$x_A = \frac{m}{p_A} - x_A$$

$$2x_A = \frac{m}{p_A} \qquad \boxed{x_A = \frac{1}{2} \cdot \frac{m}{p_A}} \quad \boxed{x_B = \frac{1}{2} \frac{m}{p_B}}$$

6.2 $U(x_1, x_2) = x_1^2 x_2^3$

$$p_1 x_1 + p_2 x_2 = m \rightarrow x_2 = \frac{m}{p_2} - \frac{p_1}{p_2} x_1$$

a) Inclinazione della CI in (x_1, x_2)?

$$MRS = -\frac{MU_1}{MU_2}$$

$$\ln U = 2 \ln x_1 + 3 \ln x_2$$

$$MU_1 = 2 \frac{1}{x_1}, \quad MU_2 = 3 \frac{1}{x_2}$$

$$\boxed{MRS = -\frac{\frac{2}{x_1}}{\frac{3}{x_2}} = -\frac{2}{3} \frac{x_2}{x_1}}$$

b) $\frac{p_1}{p_2} \cdot \frac{x_1}{x_2} = ?$

$$\frac{p_1}{p_2} = |MRS| = \frac{2}{3} \frac{x_2}{x_1}$$

$$\boxed{\frac{p_1}{p_2} \cdot \frac{x_1}{x_2} = \frac{2}{3}}$$

c) Quale frazione di reddito spende su x_1?

$$\boxed{\frac{c}{c+d} = \frac{2}{5}}$$

$$x_1(p_1, p_2, m) = ?$$

$$\frac{p_1}{\cancel{p_2}} = \frac{2}{3}\frac{(m - p_1 x_1)}{x_1}\frac{1}{\cancel{p_2}}$$

$$\frac{3}{2} p_1 x_1 = m - p_1 x_1$$

$$p_1 x_1 \left(\frac{3}{2} + 1\right) = m$$

$$\frac{5}{2} x_1 = \frac{m}{p_1}$$

$$\boxed{x_1 = \frac{2}{5}\frac{m}{p_1}}$$

d) Se $U(x_1, x_2) = c\, x_1^a x_2^b$ -> quota spesa in x_1?

$$\boxed{\frac{a}{a+b}}$$

-> indipendente da c che produce una trasformazione lineare che non modifica le preferenze!

6.3 $U = 4\sqrt{x_1} + x_2$ $\begin{cases} x_1 = \text{noci} \\ x_2 = \text{lacch} \end{cases}$ 1

a) $x_1(P_1, P_2, m) = ?$

$$MRS(x_1, x_2) = -\frac{MU_1}{MU_2} = -\frac{2}{\sqrt{x_1}}$$

$$\rightarrow U = 4x_1^{\frac{1}{2}} + x_2$$

$$U'_{x_1} = \frac{1}{2} \cdot 4x_1^{\frac{1}{2}-1} = 2x_1^{-\frac{1}{2}} = 2\frac{1}{\sqrt{x_1}}$$

$$\frac{P_1}{P_2} = \frac{2}{\sqrt{x_1}}$$

$$\sqrt{x_1} = 2\frac{P_2}{P_1}$$

$$\boxed{x_1 = 4\left(\frac{P_2}{P_1}\right)^2}$$

b) $x_2(P_1, P_2, m) = ?$

$$P_1 x_1 + P_2 x_2 = m$$

$$\boxed{x_2 = \frac{m}{P_2} - \frac{\cancel{P_1}}{\cancel{P_2}} \cdot \frac{4P_2^{\cancel{2}}}{P_1^{\cancel{2}}} = \frac{m}{P_2} - 4\frac{P_2}{P_1}}$$

---

c) Se $P_1 = 1$, $P_2 = 2$, $m = 9$ $x_1 = 4\left(\frac{2}{1}\right)^2 = 16$

SOLUZIONE DI FRONTIERA! $\rightarrow x_2 = 0$! $x_2 = \frac{9}{2} - 4 \cdot 2 = \frac{9}{2} - 8 = \frac{9-16}{2} = -7 \rightarrow$

x_2 e $x_1 > 0$ se $x_2 > 0$

$$x_2 = \frac{m}{P_2} - 4\frac{P_2}{P_1} > 0$$

se $P_1 = 1$, $P_2 = 2$

$$\frac{m}{2} - 4 \cdot 2 > 0$$

$$\frac{m}{2} > 8$$

$$\boxed{m > 16}$$

6.6 $U = \cancel{\ldots} \min\left(T, \frac{1}{2}L\right)$ $\begin{cases} \text{THE} \\ \text{LIMONE} \end{cases}$

$$\begin{cases} P_L = 0{,}75 \\ P_T = 1 \\ m = 20 \end{cases} \qquad L = \frac{20}{0{,}75} - \frac{1}{0{,}75} T = 26{,}67 - 1{,}33\, T$$

a) $\left.\begin{matrix} T = ? \\ L = ? \end{matrix}\right\}$ dal grafico risulta $\begin{matrix} T = 8 \\ L = 16 \end{matrix}$

b) Funzione di domanda di $L(P_T, P_L, m) = ?$

$$\begin{cases} L = \dfrac{m}{P_L} - \dfrac{P_T}{P_L} T \\ L = 2T \rightarrow T = \dfrac{1}{2} L \end{cases}$$

$$L = \frac{m}{P_L} - \frac{P_T}{P_L} \cdot \frac{1}{2} L$$

$$L\left(1 + \frac{P_T}{P_L} \cdot \frac{1}{2}\right) = \frac{m}{P_L}$$

$$L\left(\frac{2P_L + P_T}{2\cancel{P_L}}\right) = \frac{m}{\cancel{P_L}} \rightarrow L = \frac{2m}{2P_L + P_T} = \boxed{\frac{m}{P_L + \frac{1}{2} P_T}}$$

$$L = \frac{m}{\rho_L} - \frac{\rho_T}{\rho_L} \cdot \frac{1}{2} L$$

500-

$$L + \frac{\rho_T}{\rho_L} \cdot \frac{1}{2} L = \frac{m}{\rho_L}$$

$$L \left(1 + \frac{\rho_T}{2 \rho_L} \right) = \frac{m}{\rho_L}$$

$$L \; \frac{2 \rho_L + \rho_T}{2 \cancel{\rho_L}} = \frac{m}{\cancel{\rho_L}}$$

$$\boxed{L = \frac{2\, m}{\rho_T + 2 \rho_L}} = \frac{m}{\rho_L + \frac{1}{2} \rho_T}$$

$$\begin{cases} b = \text{campanule} \\ c = \text{convolvoli} \\ 500 \; mt^2 \text{ di giardino} \\ 1 \text{ campanula} = 1 \; m^2 \\ 1 \text{ convolvolo} = 4 \; m^2 \end{cases}$$

(6.7)

$$U = b + 100c - c^2$$

VINCOLO DI BILANCIO $\rightarrow$ $1 \cdot b + 4 \cdot c = 500$

a) $b = ?$ $c = ?$ $b = 500 - 4c$

$$MRS = -\frac{MU_c}{MU_b}$$

$$MU_b = 1 \quad , \quad MU_c = 100 - 2c$$

$$MRS = -\frac{100 - 2c}{1} = -\frac{4}{1}$$

$$100 - 2c = 4$$

$$-2c = 4 - 100$$

$$2c = 96$$

$$\boxed{c = \frac{96}{2} = 43}$$

$$\boxed{b = 500 - 43 \cdot 4 = 328}$$

(6.7) 2

b) Se $\Delta M^2 = 100$ $b = ?$ $c = ?$

v. B $\rightarrow$ $b = 600 - 4c$

$\rightarrow$ non cambia la quantità di consumo

$c = 43$

$b = 600 - 4 \cdot 43 = 428$

$$\begin{cases} \Delta b = 100 \\ \Delta c = 0 \end{cases}$$

(6.10) $\boxed{\begin{array}{ll} a = \text{BIRRA} & m = 100 \\ c = \text{SALSICCE} & P_a = 1 \end{array}}$

domanda di salsicce:

$$q_c = m - 30P_c + 20P_a$$

a) SOSTITUTO perché $\frac{\Delta q}{\Delta P_a} = 20 > 0$

b) $q_c = 100 - 30P_c + 20 = \overset{120}{\cancel{80}} - 30P_c$

c) <u>DOMANDA INVERSA</u>:

$30P_c = 1\cancel{8}0 - q_c$

$$\boxed{P_c = 4 - \frac{1}{30} q_c}$$

$$\boxed{\text{Se } q_c = 30 \rightarrow P_c = 4 - \frac{1}{30} \cdot 30 = 4 - 1 = 3}$$

d) Se $P_a = 2{,}50$

$q_c = 100 - 30P_c + 20 \cdot 2{,}50 = 150 - 30P_c$

$$\boxed{P_c = 5 - \frac{1}{30} q_c}$$