Expectations and speculative bubbles in the monetary model of exchange rate

Monetary model of exchange rate

1) $m_t - p_t = \phi y_t - \eta i_t$

2) $m_t^* - p_t^* = \phi y_t^* - \eta i_t^*$

3) $s_t = p_t - p_t^*$

4) $s_{t+1}^e - s_t = i_t - i_t^*$

5) $p_t = m_t - \phi y_t + \eta i_t$

6) $p_t^* = m_t^* - \phi y_t^* + \eta i_t^*$

Solving the model for the exchange rate S yelds:

$$\boxed{s_t = \left(m_t - m_t^*\right) - \phi\left(y_t - y_t^*\right) + \eta\left(i_t - i_t^*\right)}$$

Solving the model with expectations

$$E_t s_{t+1} - s_t = i_t - i_t^*$$

$$s_t = \left(m_t - m_t^*\right) - \phi\left(y_t - y_t^*\right) + \eta E_t\left(s_{t+1} - s_t\right)$$

$$s_t = \frac{1}{1+\eta}\left[\left(m_t - m_t^*\right) - \phi\left(y_t - y_t^*\right) + \eta E s_{t+1}\right]$$

$$s_{t+1} = \frac{1}{1+\eta}\left[\left(m_{t+1} - m_{t+1}^*\right) - \phi\left(y_{t+1} - y_{t+1}^*\right) + \eta E s_{t+2}\right]$$

$$s_t = \frac{1}{1+\eta}\left[\left(m_t - m_t^*\right) - \phi\left(y_t - y_t^*\right)\right] +$$

$$+ \frac{\eta}{1+\eta}\left[\left(m_{t+1} - m_{t+1}^*\right) - \phi\left(y_{t+1} - y_{t+1}^*\right) + \eta E s_{t+2}\right]$$

Solving the model with expectations

$$s_t = \frac{1}{1+\eta}\left[\left(m_t - m_t^*\right) - \phi\left(y_t - y_t^*\right)\right] +$$

$$+\frac{\eta}{1+\eta}\left[\left(m_{t+1} - m_{t+1}^*\right) - \phi\left(y_{t+1} - y_{t+1}^*\right) + \eta E s_{t+2}\right]$$

$$s_t = \frac{1}{1+\eta}\sum_{i=0}^{n} E_t\left(\frac{\eta}{1+\eta}\right)^i\left[\left(m_{t+1} - m_{t+1}^*\right) - \phi\left(y_{t+1} - y_{t+1}^*\right)\right]$$

$$+\left(\frac{\eta}{1+\eta}\right)^{n+1} E_t s_{t+n+1} \qquad \lim_{n\to\infty}\left(\frac{\eta}{1+\eta}\right)^{n+1} E_t s_{t+n+1} = 0$$

$$s_t = \frac{1}{1+\eta}\sum_{i=0}^{\infty} E_t\left(\frac{\eta}{1+\eta}\right)^i\left[\left(m_{t+1} - m_{t+1}^*\right) - \phi\left(y_{t+1} - y_{t+1}^*\right)\right]$$

Long run neutrality of money

$$s_t = \frac{1}{1+\eta}\sum_{i=0}^{\infty} E_t\left(\frac{\eta}{1+\eta}\right)^i \left[\left(m_{t+1} - m_{t+1}^*\right) - \phi\left(y_{t+1} - y_{t+1}^*\right)\right]$$

Spot exchange rate depends on relative money supply, relative output and their expected future values

Any revision of expectations changes the current spot exchange rate

Money is completely neutral in the long run: a 5% increase in money supply depreciate the exchange rate by 5%

$$\frac{1}{1+\eta}\sum_{i=0}^{\infty}\left(\frac{\eta}{1+\eta}\right)^i = 1$$

$$\sum_{i=0}^{\infty}\left(\frac{\eta}{1+\eta}\right)^i = \frac{1}{1-\dfrac{\eta}{1+\eta}} = 1+\eta$$

Expectations and speculative bubbles

The solution of the monetary model of exchange rate

$$s_t = \frac{1}{1+\eta}\sum_{i=0}^{\infty} E_t\left(\frac{\eta}{1+\eta}\right)^i \left[\left(m_{t+1} - m_{t+1}^*\right) - \phi\left(y_{t+1} - y_{t+1}^*\right)\right]$$

Is obtained from

$$s_t = \frac{1}{1+\eta}\sum_{i=0}^{n} E_t\left(\frac{\eta}{1+\eta}\right)^i \left[\left(m_{t+1} - m_{t+1}^*\right) - \phi\left(y_{t+1} - y_{t+1}^*\right)\right] + \left(\frac{\eta}{1+\eta}\right)^{n+1} E_t s_{t+n+1}$$

imposing the transversality condition $\lim_{n\to\infty}\left(\frac{\eta}{1+\eta}\right)^{n+1} E_t s_{t+n+1} = 0$

that rules out “speculative bubbles in the foreign exchange market

Expectations and speculative bubbles

Suppose the transversality condition does not hold
Then, the exchange rate may be written as the sum of two parts

$$s_t = s_t^f + b$$

The first part is driven by "fundamentals"
The second part is a speculative bubble that follows the stocastic process

$$b_t = \lambda E_t b_{t+1}$$

$$b_{t+1} = \frac{1}{\lambda} b_t + \varepsilon_{t+1} \quad \text{with probability } \rho$$

$$b_{t+1} = \varepsilon_{t+1} \qquad \text{with probability } 1-\rho \qquad \lambda = \frac{\eta}{1+\eta}, \quad \varepsilon \sim (0,\sigma)$$

Expectations and speculative bubbles

ρ is the probability the bubble grows

$1-\rho$ is the probability the bubble burns

The expected future exchange rate is therefore

$$E_t s_{t+1} = \rho s_{t+1}^b + (1-\rho)\bar{s}$$

And since $s_t = \rho s_t + \left(1-\rho\right) s_t$

$$E_t s_{t+1} - s_t = \rho\left(s_{t+1}^b - s_t\right) + (1-\rho)\left(\bar{s} - s_t\right)$$

$\rho\left(s_{t+1}^b - s_t\right)$ is the gain from betting on the bubble

$(1-\rho)\left(\bar{s} - s_t\right)$ Is the capital losse in the case the bubble burns

Expectations and speculative bubbles

Using the uncovered interest parity condition

$$Es_{t+1} - s_t = i - i^*$$

$$E_t s_{t+1} - s_t = \rho\left(s_{t+1}^b - s_t\right) + (1-\rho)\left(\overline{s} - s_t\right)$$

$$i - i^* = \rho\left(s_{t+1}^b - s_t\right) + (1-\rho)\left(\overline{s} - s_t\right)$$

$$\rho\left(s_{t+1}^b - s_t\right) = i - i^* - (1-\rho)\left(\overline{s} - s_t\right)$$

$$s_{t+1}^b - s_t = \frac{1}{\rho}\left(i - i^*\right) + \frac{(1-\rho)}{\rho}(s_t - \overline{s})$$

Expectations and speculative bubbles

$$s_{t+1}^{b} - s_t = \frac{1}{\rho}\left(i - i^*\right) + \frac{(1-\rho)}{\rho}(s_t - \overline{s})$$

The equation says that the expected change in the exchange rate is

Proportional to the interest rate differential

positively depends on the deviation of current exchange rate from its "fundamental" equilibrium value