The Krugman model with product innovation

- Two countries: A is the innovator, B is the follower
- A produces “new” goods
- When a good becomes “old” it is produced and exported by country B
- One factor of production: labour

- Consumers have the following utility function:

$$U = \left[\sum_{i=1}^{n} c(i)^{\theta} \right]^{1/\theta} \quad 0 < \theta < 1$$

- n is the total number of available goods (new + old)

- When Δn "new" goods are produced, then consumers maximize

$$U = \left[\sum_{i=1}^{n+\Delta n} c(i)^{\theta} \right]^{1/\theta}$$

- subject to the income constraint

$$M = p_A c_A + p_B c_B$$

•One unit of labour produces one unit of good (MPL = 1)

•Because of perfect competition

$$p_A = w_A \quad p_B = w_B$$

•A produces "new" goods and B produces "old" goods because

$$\frac{w_A}{w_B} > 1$$

•Total number of goods is $n = n_A + n_B$

•Let c_A and c_B denote consumption in countries A and B

- We obtain demand function maximizing the lagrangian

$$V = \left[c_A^\theta + c_B^\theta \right]^{1/\theta} - \lambda \left(M - p_A c_A - p_B c_B \right)$$

- First order condition (foc) are:

$$\frac{\delta V}{\delta C_A} = \frac{1}{\theta} \left(c_A^\theta + c_B^\theta \right)^{(1-\theta)/\theta} \cdot \theta c_A^{\theta-1} - \lambda p_A = 0$$

$$\frac{\delta V}{\delta C_B} = \frac{1}{\theta} \left(c_A^\theta + c_B^\theta \right)^{(1-\theta)/\theta} \cdot \theta c_B^{\theta-1} - \lambda p_B = 0$$

$$\frac{\delta V}{\delta \lambda} = M - p_A c_A - p_B c_B = 0$$

- From the first two foc we get

$$\frac{c_A^{\theta-1}}{c_B^{\theta-1}} = \frac{p_A}{p_B} \quad \rightarrow \frac{c_A}{c_b} = \left(\frac{p_A}{p_B}\right)^{-1/(1-\theta)} = \left(\frac{w_A}{w_B}\right)^{-1/(1-\theta)}$$

- Demand for labour depends on both goods demand and the number of available goods

$$L_A = n_A c_A \quad L_B = n_B c_B$$

- From labour and goods demand we get the relative demand for labour

$$c_A = \frac{L_A}{n_A} \quad c_B = \frac{L_B}{n_B}$$

$$\frac{L_A}{L_B} = \left(\frac{n_A}{n_B}\right)\left(\frac{w_A}{w_B}\right)^{-1/(1-\theta)}$$

- From relative demand for labour we find relative wage

$$\frac{w_A}{w_B} = \left(\frac{n_A}{n_B}\right)^{1-\theta}\left(\frac{L_A}{L_B}\right)^{\theta-1}$$

- Wage differential between A and B depends upon the ratio between “new” and “old” goods

- Wage differential in increasing in the speed A introduces “new” goods

Dynamic equilibrium of the model

- Innovation and imitation continuolsy occur through time
- The overall stock of "new" and "old" goods depends on innovation and imitation processes
- Because of continuos innovation n grows according to $\dot{n} = vn$
- Imitation process is measured by $\dot{n}_B = tn_A$
- v and t are positive constant $v > o \quad t > o$
- Imitation lag is equal to $\frac{1}{t}$

- Average growth rate of “new” product is

$$\dot{n}_A = \dot{n} - \dot{n}_B = vn - tn_A$$

- We have a two equation dynamic system

$$\dot{n} = vn$$

$$\dot{n}_A = vn - tn_A$$

- The system is “instable” and “explosive”
- The number of “new” products continuously increases because of continuos innovation

- A phase diagram:

$$\dot{n} = vn$$

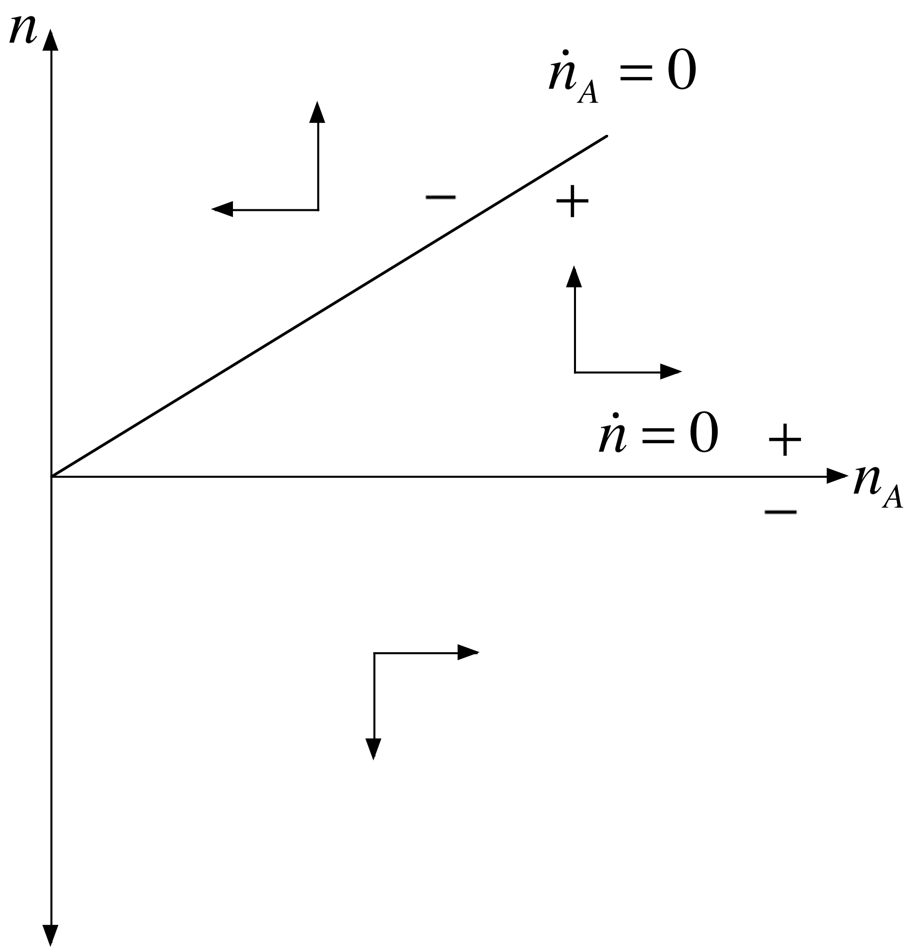
$$\frac{\delta \dot{n}}{\delta n} = v > 0$$

$$\dot{n} = 0 \rightarrow vn = 0$$

$$\dot{n}_A = vn - tn_A$$

$$\frac{\delta \dot{n}_A}{\delta n_A} = \frac{t}{v} > 0$$

$$\dot{n}_A = 0 \rightarrow n = \frac{t}{v} n_A$$



- n increases without bounds but the ratio between "new" and "old" goods is stationary
- $\sigma = {}^{n_A}\!/_{n}$ is the share of "new" goods on the total. By time differentiation we get:

$$\ln \sigma = \ln n_A - \ln n \qquad d \ln \sigma = \frac{1}{n_A} dn_A - \frac{1}{n} dn$$

$$\frac{d \ln \sigma}{dt} = \frac{1}{n_A} \frac{dn_A}{dt} - \frac{1}{n} \frac{dn}{dt} \qquad \frac{1}{\sigma} \frac{d\sigma}{dt} = \frac{1}{n_A} \dot{n}_A - \frac{1}{n} \dot{n}$$

$$\dot{\sigma} = \frac{\sigma}{n_A} \dot{n}_A - \frac{\sigma}{n} \dot{n}$$

$$\dot{\sigma} = \frac{\dot{n}_A}{n} - \sigma \frac{\dot{n}}{n}$$

- Since $\dot{n} = vn \quad \dot{n}_A = vn - tn_A$ from

$$\dot{\sigma} = \frac{\dot{n}_A}{n} - \sigma \frac{\dot{n}}{n}$$

- we obtain $\dot{\sigma} = v - (t + v)\sigma$

- therefore, steady state $\dot{\sigma} = 0$ implies

$$\sigma = \frac{v}{(t + v)}$$

- recalling that $n = n_A + n_B$
- we finally get $$\frac{n_A}{n_B} = \frac{\sigma}{(1-\sigma)} = \frac{v}{t}$$

- in steady state the ratio between “new” and “old” goods is an increasing function of innovation rate and imitation lag.

- Because relative wage depends on the “new” and “old” goods ratio,“it is an increasing function of innovation rate and imitation lag too.

- International trade pattern does not change and A keep exporting “new” product.