

# Intra-Industry Trade

- The Krugman model

- n varieties of the same good exist
- labour L is the only factor of production

$$l_i = \alpha + \beta x_i$$

- $l_i$  is the amount of labour used to produce a quantity  $x$  of the good (variety)  $i$ ;  $\alpha$  is fixed cost;  $\beta$  is marginal cost

- The utility function  $U = \sum_{i=1}^n v(c_i)$

represents consumers preferences. All n varieties of the good enter the utility function. (Dixit-Stiglitz approach)

- Full employment is assumed, therefore

$$L = \sum_{i=1}^n l_i = \sum_{i=1}^n [\alpha + \beta x_i]$$

- In equilibrium, demand for each variety is equal to supply

$$x_i = Lc_i$$

- The model must be solved for three variables:
  - output for each good (variety)  $x_i$
  - the ratio  $\frac{p}{w}$ 
    - ( $w$  is determined in a competitive market)
  - the number (variety) of goods  $n$

# Consumers equilibrium

- Consumers maximize the utility function

$$U = \sum_{i=1}^n v(c_i)$$

- subject to the budget constraint

$$w = \sum_{i=1}^n p_i c_i$$

- The Lagrangian to maximize is

$$\ell = \sum_{i=1}^n v(c_i) + \lambda \left[ w - \sum_{i=1}^n p_i c_i \right]$$

- Maximizing the Lagrangian with respect to  $c_i$  we get the first order condition (FOC)

$$\ell = \sum_{i=1}^n v(c_i) + \lambda \left[ w - \sum_{i=1}^n p_i c_i \right]$$

$$\frac{\delta \ell}{\delta c_i} = v'(c_i) - \lambda p_i = 0$$

$$v'(c_i) = \lambda p_i$$

- Combining FOC with  $x_i = Lc_i \rightarrow c_i = \frac{x_i}{L}$

- we obtain the inverse demand function  $p_i = \frac{v'(x_i/L)}{\lambda}$

# Firms

- Firms maximize the profit function:

$$\pi = p_i x_i - w l_i$$

$$p_i = \frac{v'(x_i/L)}{\lambda} \rightarrow \pi = \frac{v'(x_i/L)}{\lambda} x_i - w(\alpha + \beta x_i)$$

- Maximizing with respect to output  $x_i$  :

$$\frac{d\pi}{dx_i} = \frac{v''(x_i/L) \frac{1}{L} x_i + v'(x_i/L)}{\lambda} - w\beta = 0$$

- Simplifying notation, the FOC becomes

$$\frac{v''c_i + v'}{\lambda} = w\beta$$

- or, after some algebraic manipulation, recalling that

$$p_i = \frac{v'(x_i/L)}{\lambda} = \frac{v'}{\lambda}$$

$$\frac{v'}{\lambda} \left( \frac{v''c_i}{v'} + 1 \right) = w\beta$$

$$p_i \left( \frac{v''c_i}{v'} + 1 \right) = w\beta$$

- Defining  $e = -\frac{v'}{v''c_i}$

- then,  $p_i \left( \frac{v''c_i}{v'} + 1 \right) = w\beta$  may be written as

$$p \left( 1 - \frac{1}{e} \right) = w\beta \quad [\text{marginal cost}]$$

$$\frac{p}{w} = \frac{e}{e-1} \beta$$

- $e$  is the income elasticity of demand

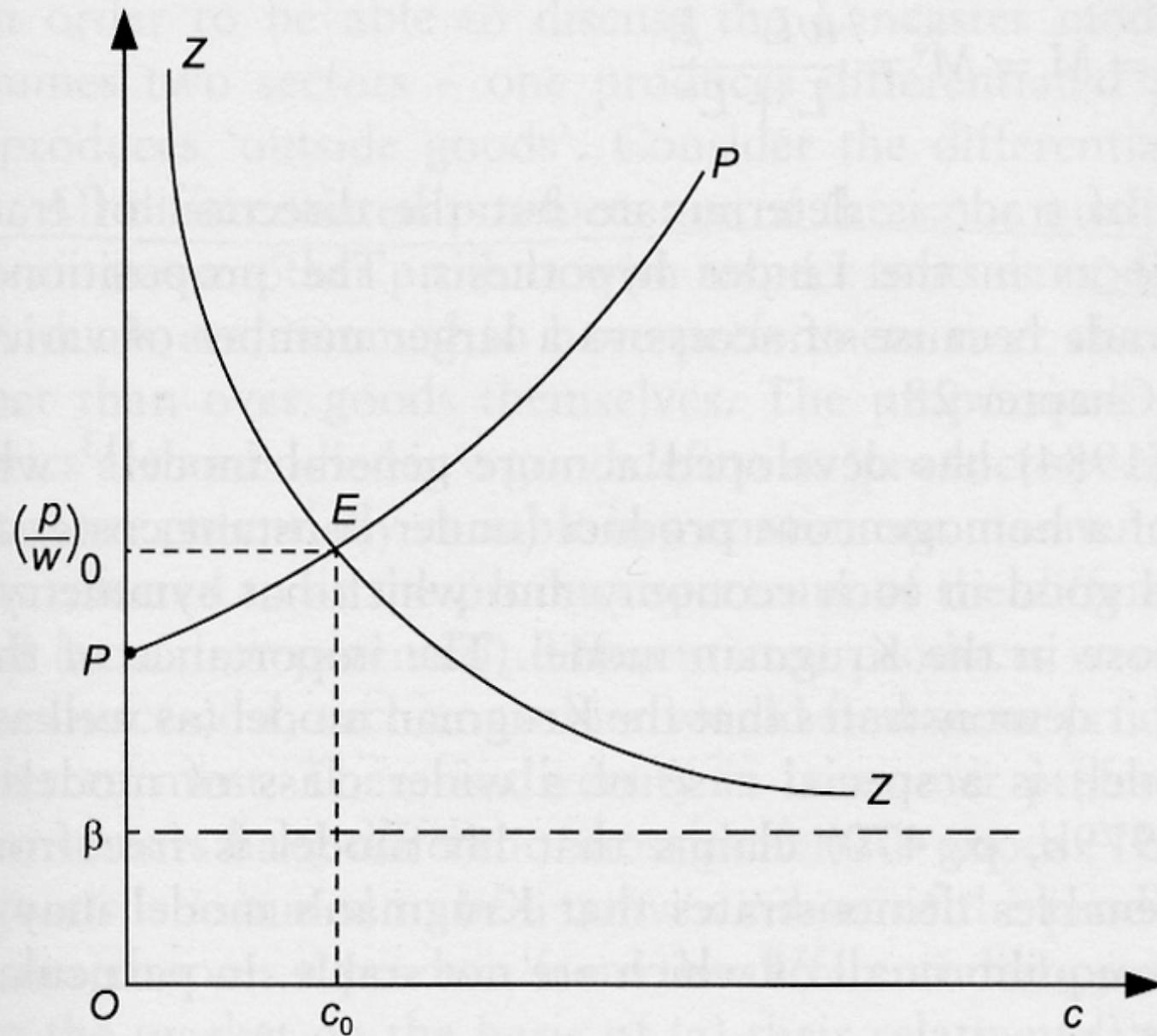
- when  $\pi = 0$ , market is in equilibrium, otherwise
- if  $\pi > 0$  new firms enter the market
- if  $\pi < 0$  firms exit the market
- the above equilibrium condition implies that

$$px - w(\alpha + \beta x) = 0$$

- or 
$$\frac{p}{w} = \beta + \frac{\alpha}{x} = \beta + \frac{\alpha}{Lc}$$

# Autarkic equilibrium

- PP line  $\frac{p}{w} = \frac{e}{e-1} \beta$
- ZZ line  $\frac{p}{w} = \beta + \frac{\alpha}{x} = \beta + \frac{\alpha}{Lc}$
- Intersection of the two lines determines equilibrium consumption  $c$  and as a consequence the number  $n$  of goods (variety) that are produced



$$n = \frac{L}{l_i}$$

$$l_i = \alpha + \beta x_i$$

$$n = \frac{L}{\alpha + \beta x}$$

$$x_i = Lc_i$$

$$n = \frac{L}{\alpha + \beta Lc}$$

# International equilibrium

- There are no transportation costs
- Symmetry among countries implies that wages and prices are equal at the international level
- International trade increases firms dimension
- The variety of goods available for consumption increases
- Direction of trade flows is not determined
- Trade volumes are determined

# International equilibrium

- With international trade, utility function is

$$U = \sum_{i=1}^n v(c_i) + \sum_{i=n+1}^{n+n^*} v(c_i)$$

- The number of goods produced in the two countries is proportional to labour endowment

$$n = \frac{L}{\alpha + \beta x} \quad n^* = \frac{L^*}{\alpha + \beta x}$$

# International equilibrium

- The share of goods produced in the two countries in the world economy are

$$\frac{n^*}{n + n^*} = \frac{L^*}{L + L^*}; \quad \frac{n}{n + n^*} = \frac{L}{L + L^*}$$

- We have balanced trade when value of import of a country is equal to value of export by the other country ( $wL$  is income of home country)

$$\frac{wL \cdot L^*}{L + L^*} = M = M^* = \frac{wL^* \cdot L}{L + L^*}$$

- We may determine trade volume but not trade direction.