Intra-Industry Trade

- The Krugman model

- n varieties of the same good exist
- labour L is the only factor of production

$$l_i = \alpha + \beta x_i$$

- l_i is the amount of labour used to produce a quantity x of the good (variety) i; α is fixed cost; β is marginal cost
- The utility function $U = \sum_{i=1}^{n} v(c_i)$ represents consumers preferences. All n varieties of the good enter the utility function. (Dixit-Stiglitz approach)

- Full employment is assumed, therefore

$$L = \sum_{i=1}^{n} l_i = \sum_{i=1}^{n} \left[\alpha + \beta x_i \right]$$

- In equilibrium, demand for each variety is equal to supply

$$x_i = L c_i$$

- The model must be solved for three variables:
  - output for each good (variety) x_i
  - the ratio $\dfrac{p}{w}$
    - (w is determined in a competitive market)
  - the number (variety) of goods n

Consumers equilibrium

- Consumers maximize the utility function

$$U = \sum_{i=1}^{n} v(c_i)$$

- subject to the budget constraint

$$w = \sum_{i=1}^{n} p_i c_i$$

- The Lagrangian to maximize is

$$\ell = \sum_{i=1}^{n} v(c_i) + \lambda \left[w - \sum_{i=1}^{n} p_i c_i \right]$$

- Maximizing the Lagrangian with respect to c_i we get the first order condition (FOC)

$$\ell = \sum_{i=1}^{n} v(c_i) + \lambda\left[w - \sum_{i=1}^{n} p_i c_i\right]$$

$$\frac{\delta \ell}{\delta c_i} = v'(c_i) - \lambda p_i = 0$$

$$v'(c_i) = \lambda p_i$$

- Combining FOC with $x_i = Lc_i \rightarrow c_i = \dfrac{x_i}{L}$

- we obtain the inverse demand function $p_i = \dfrac{v'\left(x_i/L\right)}{\lambda}$

Firms

- Firms maximize the profit function:

$$\pi = p_i x_i - w l_i$$

$$p_i = \frac{v'\left(x_i/L\right)}{\lambda} \rightarrow \qquad \pi = \frac{v'\left(x_i/L\right)}{\lambda} x_i - w\left(\alpha + \beta x_i\right)$$

- Maximizing with respect to output x_i :

$$\frac{d\pi}{dx_i} = \frac{v''\left(x_i/L\right)\frac{1}{L}x_i + v'\left(x_i/L\right)}{\lambda} - w\beta = 0$$

- Simplifying notation, the FOC becomes

$$\frac{v''c_i + v'}{\lambda} = w\beta$$

- or, after some algebraic manipulation, recalling that

$$p_i = \frac{v'\left(x_i/L\right)}{\lambda}$$

$$\frac{v'}{\lambda}\left(\frac{v''c_i}{v'} + 1\right) = w\beta$$

$$p_i\left(\frac{v''c_i}{v'} + 1\right) = w\beta$$

- Defining $e = -\dfrac{v'}{v'' c_i}$

- then, $p_i\left(\dfrac{v'' c_i}{v'} + 1\right) = w\beta$ may be written as

$$p\left(1 - \frac{1}{e}\right) = w\beta \quad [\text{marginal cost}]$$

$$\frac{p}{w} = \frac{e}{e-1}\beta$$

- e is the income elasticity of demand

- when $\pi = 0$, market is in equilibrium, otherwise
- if $\pi > 0$ new firms enter the market
- if $\pi < 0$ firms exit the market
- the above equilibrium condition implies that

$$px - w(\alpha + \beta x) = 0$$

- or

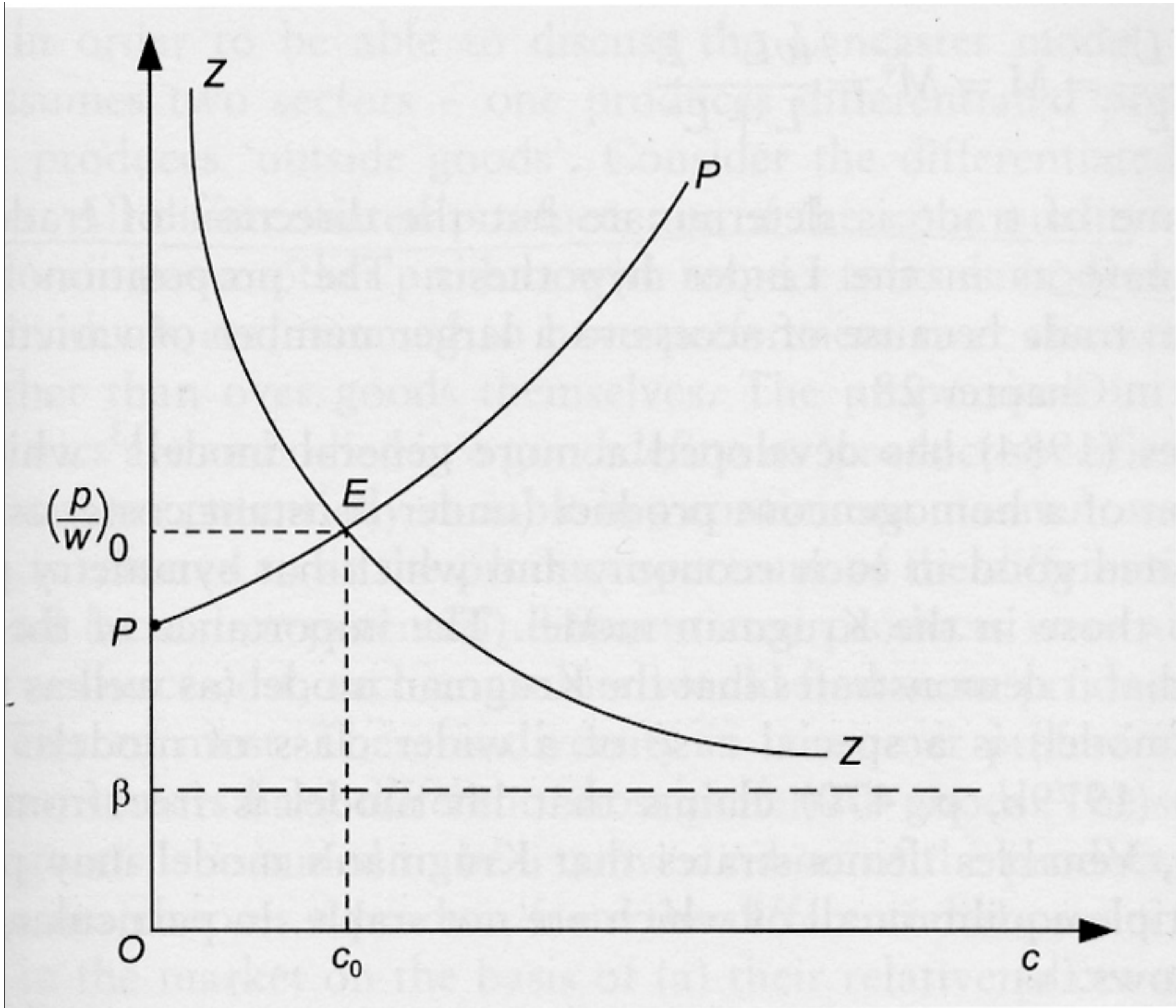
$$\frac{p}{w} = \beta + \frac{\alpha}{x} = \beta + \frac{\alpha}{Lc}$$

Autarkic equilibrium

- PP line $$\frac{p}{w} = \frac{e}{e-1}\beta$$

- ZZ line $$\frac{p}{w} = \beta + \frac{\alpha}{x} = \beta + \frac{\alpha}{Lc}$$

- Intersection of the two lines determines equilibrium consumption c and as a consequence the number n of goods (variety) that are produces



$$n = \frac{L}{l_i}$$

$$l_i = \alpha + \beta x_i$$

$$n = \frac{L}{\alpha + \beta x}$$

$$x_i = L c_i$$

$$n = \frac{L}{\alpha + \beta L c}$$

International equilibrium

- There are no transportation costs
- Symmetry among countries implies that wages and prices are equal at the international level
- International trade increases firms dimension
- The variety of goods available for consumption increases
- Direction of trade flows in not determined
- Trade volumes are determined

International equilibrium

- With international trade, utility function is

$$U = \sum_{i=1}^{n} v(c_i) + \sum_{i=n+1}^{n+n^*} v(c_i)$$

- The number of goods produced in the two countries is proportional to labour endowment

$$n = \frac{L}{\alpha + \beta x} \qquad n^* = \frac{L^*}{\alpha + \beta x}$$

International equilibrium

- The share of goods produced in the two countries in the world economy are

$$\frac{n^*}{n+n^*}=\frac{L^*}{L+L^*};\frac{n}{n+n^*}=\frac{L}{L+L^*}$$

- We have balanced trade when value of import of a country is equal to value of export by the other country (wL is income of home country)

$$\frac{wL\cdot L^*}{L+L^*}=M=M*=\frac{wL^*\cdot L}{L+L^*}$$

- We may determine trade volume but not trade direction.